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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

Nguyen Van Hieu

SOME EXPERIMENTAL CONSEQUENCES OF THE ANALYTICITY OF THE FORMFACTOR

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Нгуен Ван Хьеу E2-3729

Некоторые экспериментальные следствия аналитичности формфактора

На основе аналитических свойств формфактора изучается связь между его поведениями в физических областях каналов рассеяния и аннигиляции.

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Some Experimental Consequences of the Analyticity of the Formfactor

On the basis of the analyticty of the formfactor we study the connection between its behaviour in the physical regions of the scattering and annihilation channels. We establish also some lower bound for the formfactor in the annihilation channel.

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Nguyen Van Hieu

SOME EXPERIMENTAL CONSEQUENCES OF THE ANALYTICITY OF THE FORMFACTOR

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Consider a certain formfactor $F(t)$ and suppose that it is an analytic function in the complex t plane with a cut from $t=4m_\pi^2$ to ∞. In the local field theory $F(t)$ can increase only more slowly than any linear exponential of $\sqrt{t}$:

$$|F(t)| \leq \exp\left(\varepsilon\sqrt{|t|}\right) \quad t \to \infty \tag{1}$$

for any $\varepsilon > 0$ /1,2/. In a series of papers /3-8/ it was shown that from the analytic properties of the formfactor we can get many experimental consequences. In this work we study some other consequences.

1. We note firstly that at $t \to +\infty$ (in the physical region of the annihilation channel) $F(t)$ cannot decrease faster than exp. $\left[-\text{const.}\sqrt{t}\right]$, namely there exists such a sequence $t_n \to +\infty$ that on which the following inequality holds:

$$|F(t_n)| \geqslant \text{const} \exp\left[-a\sqrt{t_n}\right], \; t_n \to +\infty, \; a>0. \tag{2}$$

To prove this statement we put $t = z^2$ and then apply the following theorem to the function $f(z) \equiv F(t)$ analytic in the upper z halfplane.

<u>Theorem</u>

Let function $f(z)$ be analytic in the upper halfplane $\mathrm{Im}\, z > 0$ and be bounded in any finite point of the real axis. If $f(z)$ increases not faster than some linear exponential in the upper halfplane

$$|f(z)| \leq \text{const. exp. } \left[l \, |z| \right], \quad l > 0, \; z \to \infty, \; \mathrm{Im}z > 0,$$

and decreases exponentially on the real axis

$$|f(z)| \leq \text{const. exp.} \left[-c \, |z| \right], \quad c > 0, \quad z \to \pm \infty,$$

then $f(z) \equiv 0$.

A similar theorem for the functions analytic also on the real axis was proved in the Titchmarsh's book (see ref./9/, theorem 5.8). The theorem formulated here can be proved analogously if instead of the maximum principle (ref./9/, theorem 5.1) we use the generalized maximum principle (ref./10/,chap.VI, §5).

Inequality (2) can be obtained also for the formfactor $F(t)$ analytic only outside some finite region of the cut plane. For this purpose we apply an appropriate conformal mapping and use the above mentioned theorem.

2. If we suppose further that $F(t)$ at $t \to -\infty$ and $|F(t)|$ at $t \to +\infty$ do not oscillate, but have some regular behavior (that can be checked experimentally), then we can get stronger results. Applying the Phragmen-Lindelöf in the general formulation given, e.g., in refs./5,11/, we can prove that

if

$$F(t) \to \frac{a}{|t|^n}, \qquad t \to -\infty$$

then

$$|F(t)| \geq \frac{|a|}{t^n}, \qquad t \to +\infty;$$

if

$$F(t) \to a \cdot \exp.\left[-b|t|^\alpha\right], \quad b>0, \ 0<\alpha\leq\frac{1}{2}, \quad t \to -\infty.$$

then

$$|F(t)| \geq a \cdot \exp.\left[-b \sin\pi\alpha \cdot t^\alpha\right], \quad t \to +\infty$$

In particular, if the interaction is minimal in the sense of Martin[3] (see also Wu and Yang[12]), i.e.

$$F(t) \to a \ \exp.\left[-b\sqrt{|t|}\right], \quad b>0, \quad t \to -\infty,$$

then

$$|F(t)| \geq |a|, \qquad t \to +\infty.$$

In the case of oscillating $|F(t)|$ at $t \to +\infty$ there exists such a sequence of the points $t_n \to \infty$ that on this sequence one of the above inequalities holds when the corresponding condition concerning $F(t)$ at $t \to -\infty$ is satisfied.

3. Suppose further that $F(t)$ is bounded on the cut by some constant

$$\left| F(t) \right| \leqslant M, \qquad t \geqslant 4m_\pi^2 \quad . \tag{3}$$

We prove now that for the values of $F(t)$ in the region $t < 0$ there exists some lower bound. For this purpose we realize firstly the change of variables $w = \left[t/4m_\pi^2 + \alpha \right]^{1/2}$, where α is some positive constant which can be chosen to be rather big, and put $F(t) = g(w)$. The t cut plane is transformed into the upper w halfplane. Since $g(w)$ is real in the interval $-\sqrt{1+\alpha} < w < \sqrt{1+\alpha}$ then due to the Riemann-Schwartz symmetry principle it can be analytically continued into the lower w halfplane. Thus $g(w)$ is an analytic function in the w plane with two cuts $(-\infty, -\sqrt{1+\alpha})$ and $(\sqrt{1+\alpha}, \infty)$.

By mean of the conformal mapping

$$\xi = \frac{\sqrt{1+\alpha}}{w} \left[\sqrt{1+\alpha} - \sqrt{1+\alpha - w^2} \right]$$

we transform the w plane with two cuts into the circle C with the radius $\sqrt{1+\alpha}$ and the center at $\xi = 0$. The point $w = \sqrt{\alpha}$ is transformed to the point $\xi = a$,

$$a = \frac{\sqrt{1+\alpha}}{\sqrt{\alpha}} \left(\sqrt{1+\alpha} - 1 \right), \tag{4}$$

and the points $w = \pm \sqrt{\alpha - \gamma}$, where $\gamma < \alpha$ is some fixed positive number, are transformed to the points $\xi = \pm b$

$$b = \frac{\sqrt{1+\alpha}}{\sqrt{\alpha-\gamma}} \left(\sqrt{1+\alpha} - \sqrt{1+\gamma} \right). \qquad (5)$$

The circle C contains completly the ellipse E with the foci at $\xi = \pm b$ and the major semi-axis $\sqrt{1+\alpha}$. By mean of the conformal mapping

$$\eta = \frac{1}{b} \left[\xi + \sqrt{\xi^2 - b^2} \right]$$

we transform this ellipse into the ring with the center at $\eta = 0$, the internal radius 1 and the external radius R,

$$R = \frac{1}{b} \left[\sqrt{1+\alpha} - \sqrt{1+\alpha - b^2} \right], \qquad (6)$$

following Cerulus and Martin/13/. The point $\xi = a$ (i.e. $w = \sqrt{\alpha}$ or $t = 0$) is transformed to the point $\eta = r$,

$$r = \frac{1}{b} \left[a + \sqrt{a^2 - b^2} \right]. \qquad (7)$$

We put $h(\eta) \equiv g(w) \equiv F(t)$. According to our assumption (see formula (3))

$$\max_{|\eta| = R} |h(\eta)| \leq M,$$

and by definition $h(r) = F(0) = 1$. From the Hadamard's three-circle theorem (see ref./9/, theorem 5.3) it follows that

$$\max_{|\eta| = R} |h(\eta)| \equiv \max_{-\alpha \leq t/4m_\pi^2 \leq -\gamma} |F(t)| \geq \left(\frac{1}{M} \right)^{\frac{\ln r/\ln R}{1 - \ln r/\ln R}}.$$

Using the expressions (4)-(7) we get at the limit $\alpha \to \infty$

$$\max_{t \leq -4 m_\pi^2 \gamma} |F(t)| \geq \left(\frac{1}{M}\right)^{\phi(\gamma)}, \tag{8}$$

where

$$\phi(\gamma) = \frac{\left[1 - (1+\gamma)^{-1/2}\right]^{1/2}}{1 - \left[(1 - (1+\gamma)^{-1/2}\right]^{1/2}} \tag{9}$$

If F(t) decreases monotonely with increasing $/t/$ in the region $t < 0$ then we have

$$|F(t)| \leq \left(\frac{1}{M}\right)^{\phi(|t|/4m^2)}, \quad t < 0. \tag{10}$$

From this inequality it follows that the formfactor can decrease by e times in the interval $(-t_e, 0)$ only if t_e satisfies the condition

$$t_e \geq \frac{1}{(1+\ln M)^2 - 1} \tag{11}$$

For the charged pions the relations (9), (10) and (11) concern only the measurable quantities and therefore they would be checked experimentally.

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