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Nguyen Van Hieu

A LOWER BOUND OF THE ELASTIC SCATTERING AMPLITUDE AT FIXED MOMENTUM TRANSFER

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Оценка снизу для амплитуды упругого рассеяния при фиксированной передаче импульса

На основе аналитических свойств амплитуды рассеяния, получена нижняя граница убывания сечений в физической области $t < 0$.

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A Lower Bound of the Elastic Scattering Amplitude at Fixed Momentum Transfer

A lower bound of the cross section decrease is obtained in the physical region $t < 0$ on the basis of the analytic properties of the scattering amplitude.

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Nguyen Van Hieu

A LOWER BOUND OF THE ELASTIC SCATTERING AMPLITUDE AT FIXED MOMENTUM TRANSFER

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In this work we establish some restriction on the decrease of the elastic scattering amplitude $F(s,t)$ at $s \to \infty$ for the fixed $t < 0$, where s is the squared total energy in c.m.s., $t = -2k^2(1-\cos\theta)$, k is the value of the threedimensional momentum and θ is the scattering angle in this system of reference. For a series of processes the amplitude $F(s,t)$ is an analytic function of t in the ellipse E_t (called the Martin ellipse) with the foci at $t = 0$ and $t = -4k^2$ and major semiaxis $a = 2k^2 + \gamma$, $\gamma > 0$, as this was shown by Martin[1] and Sommer [2]. We denote the imaginary part of $F(s,t)$ by $A(s,t)$. From the results of Jin and Martin[3] it follows that

$$\max_{t \in E_t} |A(s,t)| \leq \text{const}\, s^{1+\varepsilon}, \quad s \to \infty \tag{1}$$

for some positive $\varepsilon < 1$. Instead of $A(s,t)$ it is convenient to consider the function

$$f(s,t) = \frac{A(s,t)}{A(s,0)}, \tag{2}$$

which has the same analytic properties in t as $A(s,t)$ has. We suppose that the total cross section has the behavior const.s^{ρ} at $s \to \infty$. Then

$$A(s,0) \sim \text{const.}\, s^{1+\rho}, \quad s \to \infty,$$

and we have

$$\max_{t \in E_t} |f(s,t)| \leq \text{const}\, s^{\varepsilon - \rho}, \quad s \to \infty. \tag{3}$$

By the change of the variables $w = t + 2k^2$ we transform E_t into the ellipse E_w with the foci at $w = \pm c$, $c = 2k^2$, and with the same major semiaxis. The minor semiaxis is $b = \sqrt{a^2 - c^2}$. We introduce now an arbitrary positive number $\alpha < c$ and consider the ellipse E'_w with the foci at $w = \pm c'$, $c' = c - \alpha$, and with the minor semiaxis b. This ellipse, of course, is contained inside E_w. The major semiaxis a' of E'_w is determined by equation

$$a'^2 = a^2 + c'^2 - c^2 .$$

We shall choose c' (i.e. α) in such a manner that the points $w = \pm c$ (i.e. $t = 0$ and $t = -4k^2$) are contained inside E'_w . Then the physical region is contained completely inside E'_w. This condition is fulfilled if $a' > c'$, i.e. if the following inequality holds

$$(2k^2+\gamma)^2 + (2k^2-\alpha) - (2k^2)^2 > (2k^2)^2 .$$

For large s from this inequality we get

$$\alpha < \gamma . \tag{4}$$

By means of the conformal mapping

$$\xi = \frac{w + \sqrt{w^2 - c'^2}}{c'}$$

we transform the interval $[-c', c']$ in the w plane into the unite circumference in the ξ plane, following Cerulus and Martin/4/. In this conformal mapping the ellipse E'_w is transformed into the ring with the internal radius 1 and the external radius R,

$$R = \frac{a' + \sqrt{a'^2 - c'^2}}{c'} . \tag{5}$$

The point $w = c$ (i.e. $t = 0$) is transformed to the point $\xi = \tau$,

$$\tau = \frac{c + \sqrt{c^2 - c'^2}}{c'} . \tag{6}$$

Denote by m the maximum of $|f(s,t)|$ in the interval $-c' \le w \le c'$ i.e. in the interval $-4k^2+\alpha \le t \le -\alpha$, and by M that of $|f(s,t)|$ on the boundary of E'_t . According to the Hadamard's three circle theorem (see ref./5/, theorem 5.32) we get

$$\ln|f(s,0)| \le \left(1 - \frac{\ln\tau}{\ln R}\right)\ln m + \frac{\ln\tau}{\ln R}\ln M .$$

Putting into (5) and (6) the values of a' , c , c' we get at the limit $s \to \infty$

$$\frac{\ln\tau}{\ln R} \approx \sqrt{\alpha/\gamma} .$$

On the other hand $f(s,0) = 1$. Therefore we have

$$m \ge \left(\frac{1}{M}\right)^{\frac{\sqrt{\alpha/\gamma}}{1-\sqrt{\alpha/\gamma}}} .$$

For πN-scattering we have $\gamma = 4m_\pi^2$. Using the condition (3) we get now

$$\max_{-4k^2+\alpha \le t \le -\alpha} |f(s,t)| \ge \text{const}\; s^{-(\varepsilon-\rho)\phi(\alpha)} , \tag{7}$$

where

$$\phi(\alpha) = \frac{\sqrt{\alpha/4m_\pi^2}}{1 - \sqrt{\alpha/4m_\pi^2}} . \tag{8}$$

If we assume that $f(s,t)$ is analytic and uniformly polynomially bounded in the whole cut plane (the Mandelstam representation) then we have

$$\max_{-4k^2+\alpha \le t \le -\alpha} |f(s,t)| \ge \text{const}\; s^{-n\psi(\alpha)} , \tag{9}$$

where

$$\psi(\alpha) = \frac{\left[1 - \left(1 + \frac{\alpha}{4m_\pi^2}\right)^{-1/2}\right]^{1/2}}{1 - \left[1 - \left(1 + \frac{\alpha}{4m_\pi^2}\right)^{-1/2}\right]^{1/2}} , \tag{10}$$

and n is such a constant that

$$|f(s,t)| \le \text{const}\; s^n , \quad s \to \infty$$

for any t.

If the amplitude has the Regge behavior then the inequalities (7), (9) are the lower bounds of the corresponding Regge trajectories.

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