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1. Introduction

In paper /1/ Korenman and Eramzhian considered the muon capture process taking into account the hyperfine (hf) structure of mesonic atom levels. The influence of this effect is important x). Recently Bukhvostov and Popov obtained in this case a general expression for the angular correlations among pseudovector of muon polarization and the directions of emission of the neutrino and gamma ray quantum for partial transitions of any order of forbiddenness. They considered in detail the allowed transitions only /3/. In this paper we shall extend the results of refs. /3/ and /4/ by means of a detailed consideration of these angular correlations to the cases both of unique allowed and unique forbidden processes. It is known that in this case the influence of the nuclear structure is rather small. Finally, we applied our formulas to some interesting reactions (e.g. the muon capture by ^{15}N).

x) We notice that the muon capture from definite hf states was experimentally measured /2/.

2. General Analysis

The general form of the angular correlation in the process of muon capture from K- orbit of mesonic atom with the following spin sequence

$$j_0 \xrightarrow{\mu^-} j_1 \xrightarrow{\gamma} j_2$$

for the N-th forbidden transitions in the case of nonzero spin of captured nuclei looks as follows (for allowed transitions N=0)

$$W^N = \sum_{S=0}^{2N+3} a_S^N P_S(\vec{k}\cdot\vec{q}) + \sum_{S=1}^{2N+3} b_S^N \vec{\sigma}\cdot\vec{q}\, P_S'(\vec{k}\cdot\vec{q}) + \qquad (1)$$

$$+ \sum_{S=1}^{2N+4} c_S^N \vec{\sigma}\cdot\vec{k}\, P_S'(\vec{k}\cdot\vec{q}) + \sum_{S=1}^{2N+2} d_S^N [\vec{\sigma}\times\vec{q}]\cdot\vec{k}\, P_S'(\vec{k}\cdot\vec{q}),$$

where a_S^N, b_S^N, c_S^N and d_S^N are the correlation constants depending on the weak interaction form factors, the structure of nuclei and kinematic effects. We normed W^N so that $a_0^N = 1$. We denote by $\vec{\sigma}$, $\vec{q}$ and $\vec{k}$ the following unit vectors: the polarization pseudo-vector of a muon on the K-orbit of the mesonic atom, the vectors in the direction of the neutrino and gamma quantum momenta respectively. The quantities $P_\ell(\vec{k}\cdot\vec{q})$ are the Legendre polynomials, and

$$P_\ell'(x) = \frac{d}{dx} P_\ell(x). \qquad ^{x)}$$

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x) The expansion (1) is convenient because it is directly connected with the expansion of the function W^N in the Legendre hyperpolynomials (see Ref. /4b/, Appendix).

The total rate of the N-th forbidden unique muon capture x) is proportional to the quantity

$$\Lambda^N = I \mid M_I - P_I \mid^2 + \frac{2 j_0 + 1}{j_0 + 1} p \left(\mid M_I \mid^2 + 2I \operatorname{Re} M_I P_I^* \right)$$

for $j_1 = j_0 + N + 1$, and (2)

$$\Lambda^N = I \mid M_I - P_I \mid^2 + \frac{2 j_0 + 1}{j_0} (1-p) \left(\mid M_I \mid^2 + 2I \operatorname{Re} M_I P_I^* \right)$$

for $j_1 = j_0 - N - 1$,

where M_I and P_I are the combinations of weak formfactors and nuclear matrix elements (M.E.) of the Gamov-Teller type only and are defined by formula (10) in Ref. /4b/xx).
The approximate formulas for M_I and P_I are given below (9). The quantity I takes on the following values

$$| j_0 - j_1 | \leq I \leq | j_0 + j_1 | .$$

For the unique transitions we can restrict ourselves only to one value of I and in all our formulas we have

$$I = N + 1 = | j_1 - j_0 | .$$

We denote by p the probability of occupation of the hf level of a mesonic atom with total angular momentum $F_+ = j_0 + \frac{1}{2}$. Then the probability of occupation of the second level $F_- = j_0 - \frac{1}{2}$ is 1-p. When $j_0 = 0$, then $p \equiv 1$. Some of special interesting correlation constants will be considered in next sections.

x) For the process of unique muon capture we have by definition $| j_0 - j_1 | = N+1$ and the parity change is $\Delta \pi = (-1)^N$.

xx) They are the same as the formulas (25-26) in paper /4a/.

3. Capture of Unpolarized Muons

In Ref. /4/ an expression for the gamma-neutrino correlation in the case of statistical population of the hf levels of the K shell of a meso-atom is obtained. Considering the muon capture with any population of hf levels we have

$$W^N = 1 + \sum_{S=1}^{2N+3} a_S^N P_S (\vec{k}\cdot\vec{q}) . \qquad (3)$$

The quantities a_S^N take on the following form in dependence on the spin sequence and the parity of number S. If we average over the polarization of gamma quanta, then S is even only.

i. $j_0 = j_1 - N - 1$, S is an even number

$$a_S^N \Lambda^N = (-1)^I \frac{2I+1}{2I} \frac{\sqrt{2j_1+1}}{j_0+1} C^{S0}_{1010} B_{S\eta} W(j_0 j_1 IS1j_1).$$

$$\{ [|M_I|^2 (2I^2 (j_0+1) + 2I(2j_0+1) - S(S+1)(2j_0+1)) + \qquad (4)$$

$$+ 2I^2 (|P_I|^2 (j_0+1) + 2j_0 \mathrm{Re}\, P_I M_I^*)] p +$$

$$+ 2(j_0+1) I^2 |M_I - P_I|^2 (1-p) \} .$$

ii. $j_0 = j_1 - N - 1$, S is an odd number

$$a_S^N \Lambda^N = (-1)^N \frac{2I+1}{2I} \frac{\sqrt{2j_1+1}}{j_0+1} \left[\frac{(2j_1+S+1)S(S+1)}{(2j_1-S+1)(2I+S+1)(2I+S+2)} \right]^{1/2}$$

$$C^{S0}_{I+1\,0\,1\,0} \quad B_{S\eta} W(j_0 j_1 I\, S-1\, I\, j_1)$$

$$\{ [|M_I|^2 (2I^2(3j_0+2) + 2(3I+1)(2j_0+1) - S(S+1)(2j_0+1)) - \qquad (5)$$

$$- 2I^2 j_0 (|P_I|^2 - 2\,\mathrm{Re}\, M_I P_I^*)] p \; +$$

$$+ \; 2I^2 (j_0+1) | M_I - P_I |^2 (1-p) \} \; .$$

In order to obtain a_N^S for $j_0 = j_1 + N + 1$ we should make the following replacement

$$p \longrightarrow 1 - p$$

and

$$j_0 \longrightarrow - j_0 - 1$$

everywhere for the exception of the Racah functions which do not change.

The quantities $B_{S\eta} \equiv B_{S\eta}(j_1 j_2 L)$ depend on the multipolarity 2^L on the character of the gamma transition and on the parameter of the circular polarization $\eta = \pm 1$ (for right- and left- polarized radiation respectively) of the gamma quantum. For pure electromagnetic transitions of multipolarity 2^L we have

$$B_{S\eta} = [(2S+1)(2L+1)(2j_1+1)]^{1/2} \; C^{L\eta}_{L\eta S0} \; W(j_2 L j_1 S j_1 L) . \qquad (6)$$

It is connected with the $\Lambda^{II}_{s,\eta}$ coefficients defined by the formula (3) in Ref. /4b/, where the generalization on the mixed electromagnetic transitions is given.

$$\Lambda^{II}_{s,\eta} = (2j_1 + 1)^{1/2} \, W(j_0 j_1 1 s 1 j_1) \, B_{s\eta} \quad . \tag{7}$$

When $p = \frac{j_0 + 1}{2j_0 + 1}$ (i.e. when we have the statistical population of the hf levels of a mesonic atom F_+ and F_-) the correlation coefficients are the same as the coefficients a^N_ℓ given in paper /4/.

The allowed transitions are considered in detail by Bukhvostov and Popov /3/.

If we know the correlation coefficients (4) and (5) we can easily find the circular polarization of gamma quanta P^N_γ

$$P^N_\gamma = \frac{\sum^{N+1}_{l=0} a^N_{2l+1} \, P_{2l+1}(\vec{k}\cdot\vec{q})}{\sum^{N+1}_{l=0} a^N_{2l} \, P_{2l}(\vec{k}\cdot\vec{q})} \quad . \tag{8}$$

Below we illustrate our formulas as applied to some reactions assuming pure electromagnetic transitions.

On fig.1 we show the correlation coefficient a^1_4 for muon capture by ^{15}N (unique first-forbidden transition).

$$^{15}N\left(\frac{1}{2}-\right) \xrightarrow{\mu^-} {}^{15}C^*\left(\frac{5}{2}+\right) \xrightarrow{\gamma} {}^{15}C\left(\frac{1}{2}+\right).$$

In this case we have mainly the capture from F_+ state (curve a) because the nuclei ^{15}N has the negative magnetic moment. We compare it with the same coefficient for statistical population of hf levels (curve b). When the capture goes from the F_- state then we get the constant value $a^1_4 = -\frac{4}{7}$.

On all the figures we neglect the influence of the small additional M.E. /4,5/, i.e. we put x)

x) The notation is taken from Ref. /4/.

$$M_I \Rightarrow M \equiv C_A - \frac{q}{2M} C_V (1 + \mu_p - \mu_n)$$

$$P_I \Rightarrow P \equiv C_A + \frac{q}{2M} (C_A - C_P) \qquad (9)$$

and

$$C_A = -1.25\, C_V$$

$$\mu_p - \mu_n = 3.7$$

$$q = 0.1\, M \quad .$$

4. Capture of Polarized Muons

The account of the muon polarization leads to the appearance of new important angular correlations. The polarization of muons in various hf states of mesonic atom structure F_+ and F_- is characterized by the quantities λ_+ and λ_-, respectively. They are defined by the mean value of the muon spin in these states

$$\langle \vec{s}_+ \rangle = \frac{1}{2} \lambda_+ \vec{\sigma}$$

$$\langle \vec{s}_- \rangle = \frac{1}{2} \lambda_- \vec{\sigma} \quad .$$

These quantities are small always because of the strong depolarization of muons in mesonic atom caused by the spin-orbital and hf interactions /6,7/.

Therefore the experimental investigation of angular correlations including the pseudovector of polarization of muons becomes difficult. The very interesting angular correlation is the correlation between the direction of neutrino momentum and the polarization pseudovector of a muon when the gamma quantum distribution is not measured.

$$\frac{1}{4\pi}\int W^N d\Omega_{\vec{k}} = 1 + a^N \vec{\sigma}\cdot\vec{q} .$$

From formula (1) we obtain

$$a^N = \Sigma_{l=1}^{N+2} (b^N_{2l-1} + \frac{1}{3} c^N_{2l}) .$$

When $j_0 = j_1 - I$ we have

$$a^N \Lambda^N = 3 [(2I + 3)(2 j_0 + 3)(j_0 + 1)]^{-1}$$

$$\{ | M_I |^2 [2I^2 (2 j_0 + 1) + I (6 j_0 + 15 j_0 + 5) + 3 (2 j_0^2 + 4 j_0 + 1)] -$$

$$- I | P_I |^2 [2I (2 j_0 + 1) + (2 j_0 + 3)(j_0 + 1)] + \tag{10}$$

$$+ 2 I j_0 (2 j_0 - 1) \, Re \, M_I P_I^* \} p \lambda_+ -$$

$$- \frac{3I}{2I + 3} | P_I - M_I |^2 (1 - p) \lambda_- .$$

For $j_0 = j_1 + I$ we should make in formula (10) the following substitutions

$$p \longrightarrow 1 - p$$

$$j_0 \longrightarrow - j_0 - 1$$

$$\lambda_+ \longleftrightarrow \lambda_- .$$

The same should be made for the coefficient β^N for the correlation $\vec{\sigma} \cdot \vec{k}$ considered below (11).

For the processes

$$^{6}Li(1+) \xrightarrow{\mu^-} {}^{6}He(0+)$$

or

$$^{14}N(1+) \xrightarrow{\mu^-} {}^{14}C(0+)$$

the coefficients a^0 for the muon capture from F_- or F_+ states are constant, namely

$$a^0 = 3\lambda_- \quad \text{for } F_- \text{ state}$$

$$a^0 = -0.6\lambda_+ \quad \text{for } F_+ \text{-state}$$

Because the nuclei ^6Li and ^{14}N have the positive magnetic moments the statistical population of the hf states will be violated in favour of the F_- -state.

We compare on fig.2 the coefficient a^0 for F_- state and the same coefficient for statistical population of hf levels. We used the following theoretical estimates of the quantities λ_- and λ_+ for $j_0 = 1$, , taken from paper /6/

$$\lambda_+ = 0.108$$

$$\lambda_- = 0.01$$

Another simple correlation refers to the case when we do not take into consideration the angular distribution of recoil nuclei. Namely the angular correlation has now the form

$$1 + \beta^N \vec{\sigma} \cdot \vec{q} \,.$$

The formula (1) gives us

$$\beta^N = \Sigma_{i=1}^{N+1} \left(c^N_{2i-1} + \frac{1}{3} b^N_{2i} \right) + c^N_{2N+3}$$

when $j_0 = j_1 - I$ we can easily obtain

$$\beta^N \Lambda^N = \left[\frac{3(j_0 + I + 1)}{j_0 + I} \right]^{1/2} \left[(2j_0 + 3)(j_0 + 1) \right]^{-1} B_{1\eta}$$

$$\{ [I j_0 (2 j_0 + 3)(| M_I |^2 + | P_I |^2) + | (2 j_0 + 1) M_I |^2 + \tag{11}$$

$$+ 2I(2j_0^2 + j_0 + 1) \operatorname{Re} M_I P_I^*] p \lambda_+ -$$

$$- (2j_0 + 3)(j_0 + 1) | P_I - M_I |^2 (1 - p) \lambda_- \} .$$

The quantities $B_{1\eta}$ are defined by formula (7) and (8). This correlation for allowed transitions was considered firstly in Ref. /7/.

For the above proposed reaction of muon capture by ^{15}N the quantity λ_+ is estimated to be /8/

$$\lambda_+ = 0.074 .$$

Of course, λ_- is identically equal to zero. The coefficient β^1 for this reaction is shown on fig.3.

We give also the formula for the angular correlations which are non-invariant under time reversal

$$d_S^N \Lambda^N = (-1)^N 3 (2j_0 + 1)(2j_1 + 1)^{1/2} (2I + 1) B_{S\eta}$$

$$W(j_0 I j_1 S j_1 I) \times \tag{12}$$

$$\times \left(\left[\frac{(2I-S)(S+2I+1)}{S(S+1)} \right]^{1/2} C^{SO}_{I0I-10} + C^{SO}_{I0I0} \right)$$

$$\left(\frac{p\lambda_+}{(j_0+1)(2j_0+3)} \delta_{j_1, j_0+1} - \frac{(1-p)\lambda_-}{j_0(2j_0-1)} \delta_{j_1, j_0+1} \right) \quad (12)$$

$$\mathrm{Im}\ M_I P_I^* ,$$

The case $j_0 = 0$ was considered in Ref. /9/.

5. Some Remarks

We would like to notice that for the unique muon capture from the hf level, namely, from the F_- state for $j_o \xrightarrow{\mu^-} j_o + I$ or from the F_+ state for $j_o \xrightarrow{\mu^-} j_o - I$, all the angular correlation coefficients are constants (not depending both on any weak interaction parameters, i.e. on M_I and P_I, and on any M.E.) because the total angular momentum of a neutrino is only $N + \frac{3}{2}$.

This follows from the conservation of the total angular momentum of mesonic atom in muon capture and from the fact that the other terms belonging to the higher forbidden transitions are neglected. Therefore any interference effects do not appear.

This is the reason for that the noninvariant correlations under time reversal disappear because they are connected only with the interference of terms with different I (in the case of nonunique process) or different total angular momenta of a neutrino.

We would like to stress that in these cases the neglected of higher order forbiddennesses is not always justified. Namely, when $C_P = 0$

then the contribution of the higher forbiddenness is of the same order of nagnitude as the main term.

But for large C_P ($|C_P| \gtrsim 10$) this neglect is valid.

We can see also that the correlation coefficients for the unique muon capture from the remaining hf levels have sometimes a strong dependence on $\frac{C_P}{C_A}$ (see fig.1). It is characteristic that the extremal value of all gamma neutrino correlation coefficients (a_s^N) is now shifted in the direction of large $\frac{C_P}{C_A}$ (see e.g. also fig.1) in comparison to the statistical population where always the maximal point is $P_I = 0$ i.e.

$$\frac{C_P}{C_A} \approx 2 \frac{M}{q} + 1 \approx 21 .$$

The experimental study of the angular correlations can give us an additional information about the muon capture constants and the nuclear structure.

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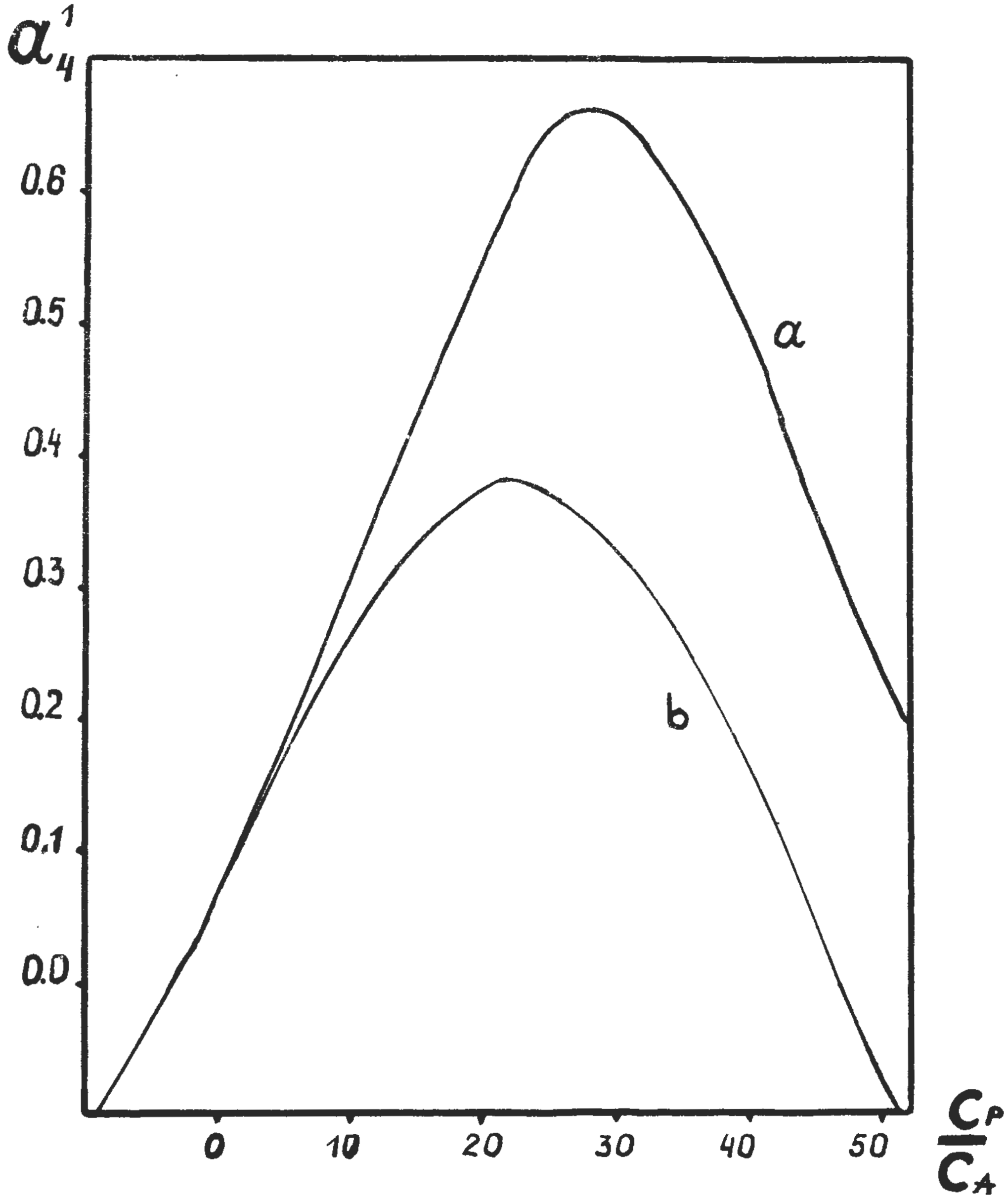

Fig.1. The coefficient of gamma-neutrino correlation a_4^1 for muon capture by ^{15}N.
a) The capture from the F_+ state.
b) The capture from statistical populated hf levels.

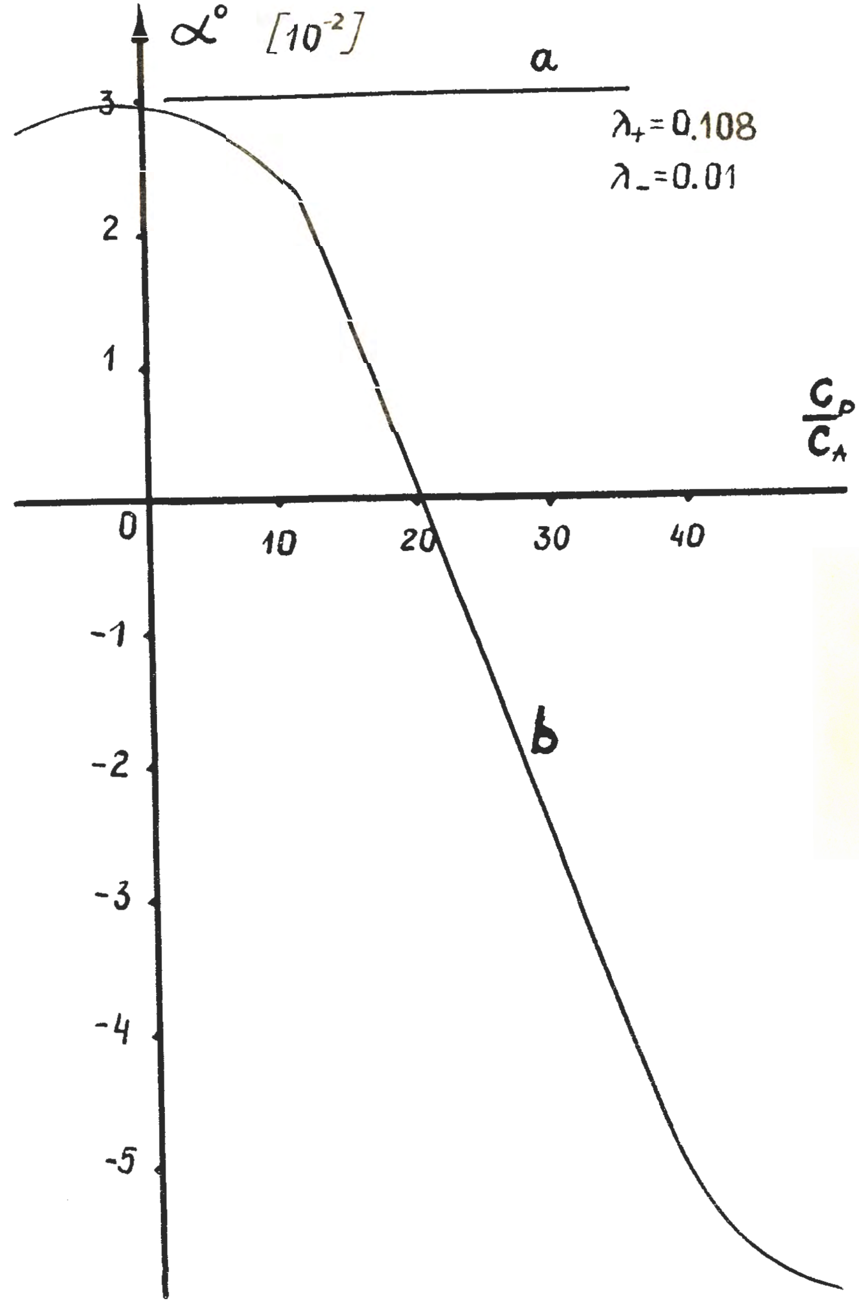

Fig. 2. The coefficient a^0 of the angular distribution of recoil nuclei for muon capture by ^6Li or ^{14}N
a) The capture from the F_- state
b) The capture from statistical populated hf levels.

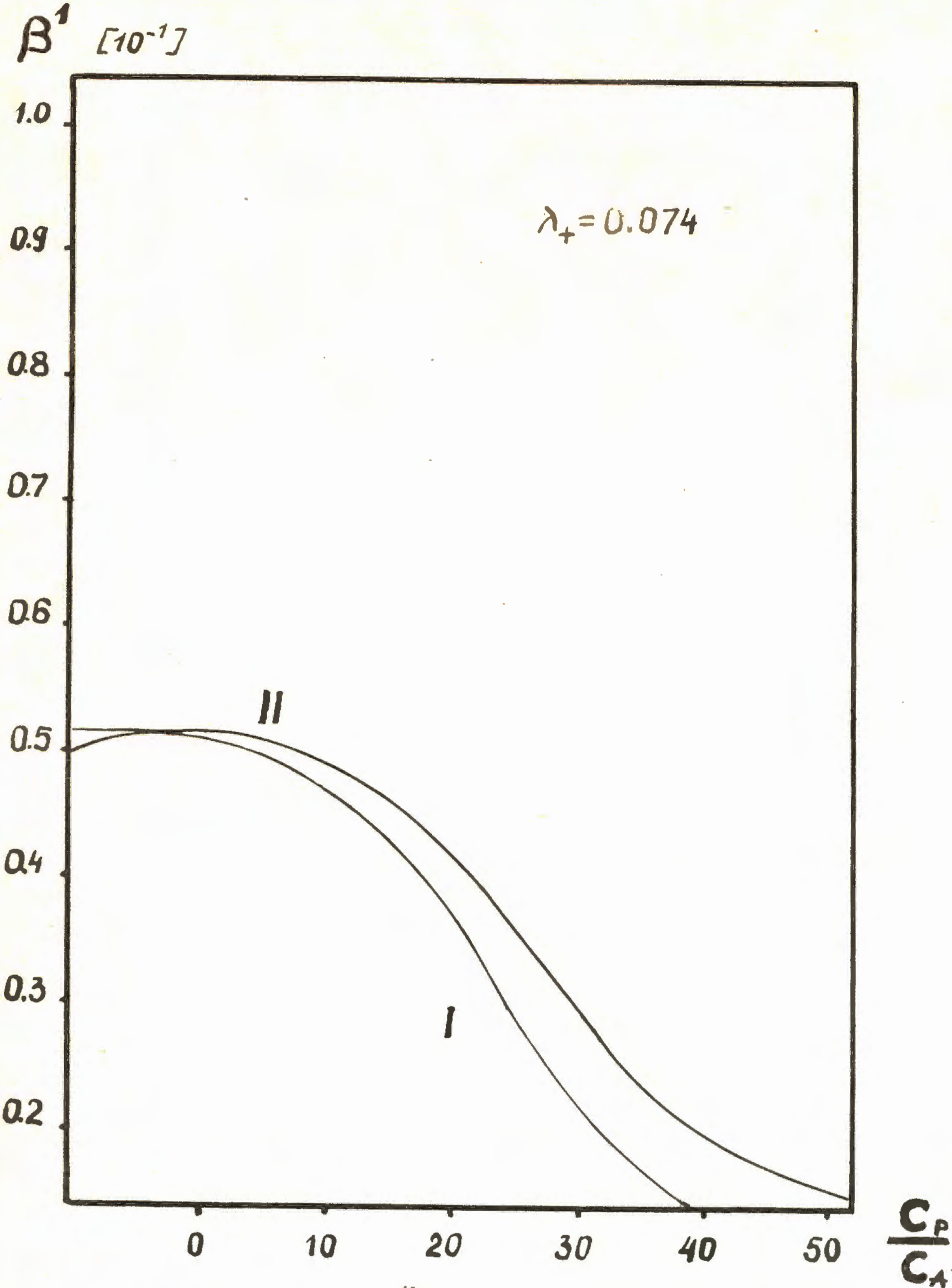

Fig.3. Muon capture by ^{15}N . Coefficient β^1 .
1. The capture from statistical populated hf levels.
II. The capture from the F_+ state.