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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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1. Introduction

Numerous theoretical calculations have attempted to obtain form factors of the elementary particles from current algebra [1] or divergence conditions [2]. Many results are in good agreement with experimental data at least for zero momentum transfer.

Fubini, Furlan and Rossetti [3] pointed out that the commutator of a charge with the divergence of a vector current leads to an equation for mass differences. For $\pi^+ - \pi^0$ mass difference this equation contains a term, of order e^2, which can be transformed in the well known form for electromagnetic mass difference[4]. The agreement with experimental mass difference is very good although only π meson intermediate states are taken into account. This result enables us to expect that the above mentioned commutator now considered for non vanishing momentum transfer may give information about electromagnetic form factor of π meson or about the domain where commutation relations can be saturated by single particle states.

2. Derivation

We start from the identity

$$-ik_\mu \int dy\, e^{iky}\, T(j_\mu^+(y)\,\partial_\nu j_\nu^+(x)) = [Q^+(k,x_0)\partial_\nu j_\nu^+(x)] + \int dy\, e^{iky}\, T(\partial_\mu j_\mu^+(y)\partial_\nu j_\nu^+(x)) \tag{1}$$

obtained by partial integration and neglecting the surface term. The current $j_\mu^+(x)$ belongs to the vector representation of isospin group and

$$Q^+(k, x_0) = \int j_0^+(x)\, e^{ikx}\, d^3x \tag{2}$$

in the limit of vanishing k becomes generator of the isospin group.

Taking the matrix element between two pion states of momenta p^+ and p^- we get from eq.(1) in the limit of $k=0$

$$4(2\pi)^3\,(m_+^2 - m_0^2)\,\frac{f(\Delta^2)}{\sqrt{4p_0^+p_0^-}} = \int dy < p^+ | T(\partial_\mu j_\mu^+(y)\,\partial_\nu j_\nu^+(0)) | p^- >, \tag{3}$$

where $m_+ (m_0)$ represents the mass of $\pi^+ (\pi^0)$ meson, $\Delta = p^+ - p^-$ is the momentum transfer and f is the electric form factor of π -meson.

We restrict ourselves to the lowest power e^2 and consider a partial conservation of vector current of the form [5]

$$\partial_\mu j_\mu^+(x) = i e\, j_\mu^+(x)\, A_\mu(x), \tag{4}$$

where A_μ is the electromagnetic potential vector.

These approximations give for the eq. (3)

$$4(2\pi)^3(m_+^2 - m_0^2)\,\frac{f(\Delta^2)}{\sqrt{4p_0^+p_0^-}} = -\frac{e^2}{(2\pi)^4 i}\int \frac{dq\,dy}{q^2 - i\epsilon}\, e^{iqy} < p^+ | T(j_\mu^+(y)\,j_\mu^+(0)) | p^- >, \tag{5}$$

where a gauge invariant expression must be used for $M_{\mu\nu} = i\int dy\, e^{iqy} < p^+ | T(j_\mu^+(y)\,j_\nu^+(0)) | p^- >$. Dispersion theory, taking into account only π -meson intermediate states, gives [6]

$$M_{\mu\nu} = -\frac{2(2\pi)^3}{\sqrt{4p_0^+p_0^-}} f(q^2) f((q+\Lambda)^2) \left[\frac{(2p^+ + q)_\mu (p^+ + p^- + q)_\nu}{(p + q)^2 - m^2 - i\epsilon} + \frac{(2p^- - q)_\mu (p^+ + p^- - q)_\nu}{(p - q)^2 - m^2 - i\epsilon} - 2g_{\mu\nu} \right]. \quad (6)$$

3. Results

a) π -meson intermediate states.

In the approximation of π -meson intermediate states we obtain from eqs. (5) and (6) an equation for the electromagnetic form factor $f(q^2)$

$$(m_+^2 - m_0^2) f(\Delta^2) = i \frac{e^2}{(2\pi)^4} \int dq \frac{f(q^2) f((q+\Delta)^2)}{q^2 - i\epsilon} \left[4 - \frac{(2p^+ + q)(p^+ + p^- + q)}{(p^+ + q)^2 - m^2 - i\epsilon} \right]. \quad (7)$$

In the limit of vanishing momentum transfer Λ this equation gives the electromagnetic selfenergy of the π- meson [4].

Since we do not know whether or not the eq. (7) is a good approximation for non vanishing momentum transfer, we test it by introducing in its right hand side a form factor which gives a good result for the mass difference. Such a form factor is given by the ρ meson pole approximation

$$f(q^2) = \frac{M^2}{M^2 - q^2}, \quad (8)$$

where M is the mass of the ρ -meson.

Restricting the equation (7) to the first power of Λ^2 we must compare, to see the consistency, the right hand side of eq. (7) with $(m_0^2 - m_0^2)(1 + \frac{\Lambda^2}{M^2})$ of left hand side. By introducing the form factor (8) and integrating we obtain for right hand side

$$\frac{e^2 M^2}{4(2\pi)^2}\left[2 - \frac{1}{\mu^2}\log\mu - \frac{1}{\mu^2}(1+2\mu^2)\sqrt{1-4\mu^2}\log\frac{1+\sqrt{1-4\mu^2}}{2\mu} - \right.$$

$$\left. - \frac{\Delta^2}{M^2}\left(-\frac{1}{2} + \frac{1}{12\mu^2} + \frac{1}{12\mu^4}\log\mu + \frac{1-2\mu^2+16\mu^4-56\mu^6}{12\mu^2\sqrt{1-4\mu^2}}\log\frac{1+\sqrt{1-4\mu^2}}{2\mu}\right)\right] = \quad (9)$$

$$= 0{,}061\ m^2\left(1 - 0{,}45\frac{\Delta^2}{M^2}\right); \qquad \mu = \frac{m}{M}$$

while the left hand side gives

$$0{,}068\ m^2\left(1 + \frac{\Delta^2}{M^2}\right). \qquad (10)$$

The agreement is good only for the part corresponding to the mass difference, a wrong sign and magnitude being obtained for the term corresponding to the mean square radius.

b) ω meson intermediate states.

The contribution of the ω meson to the electromagnetic mass difference $\pi^+ - \pi^0$ [7] is known to be small. This is the reason for which we neglect here the π -meson mass with respect to that of ω meson. Also we neglect $\rho - \omega$ mass difference. The matrix element involving ω meson is defined by

$$\langle p^\pi \mid j_\alpha^+(0) \mid p^\omega \rangle = \frac{g(q^2)}{\sqrt{4 p_0^\pi p_0^\omega}}\, \epsilon_{\alpha\beta\gamma\delta}\, p_\beta^\pi p_\gamma^\omega \epsilon_\delta\ , \qquad (11)$$

where ϵ_δ is the polarization of the ω meson and $q = p^\pi - p^\omega$. The form factor g is written in the pole approximation (8)

$$g(q^2) = 2.3\,\frac{M}{M^2 - q^2} \qquad (12)$$

the factor 2.3 being determined by the width of the $\omega \to \pi^0\gamma$ decay. Taking into account eqs. (11) and (12) the following term must be added to the expression (9)

$$0.0023\ m^2 \left(1 - 15.5 \frac{\Delta^2}{M^2}\right).$$

The first term represents a small contribution to $m_+^2 - m_0^2$ mass difference,. The second term has the same magnitude and sign as that previously obtained (9) when π -meson intermediate state was considered.

4. Conclusion

In spite of the good agreement obtained for the electromagnetic mass difference a wrong magnitude and sign is obtained for mean square radius of π -meson. It is possible that contributions from higher states may give the correct value but then it is difficult to understand why mass difference is very well approximated by π -meson intermediate states whereas the corresponding formula for square radius in not approximated by the same intermediate states.

Also it is possible that the commutator of a charge with divergence of its current is not zero, some unknown singular terms being necessary to add.

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