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ASYMPTOTIC BEHAVIOUR OF THE VERTEX FUNCTION IN THE THEORY OF NONRENORMALIZABLE CP–NONINVARIANT INTERACTION OF PHOTONS WITH SPINOR PARTICLES

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ASYMPTOTIC BEHAVIOUR OF THE VERTEX FUNCTION IN THE THEORY OF NONRENORMALIZABLE CP–NONINVARIANT INTERACTION OF PHOTONS WITH SPINOR PARTICLES

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In papers [1] we have suggested a mechanism of CP-invariance violation in the interactions of spinors particles ψ with photons on a weak electromagnetic level. For transitions without change of the inner quantum numbers of particles (e.g. of strangeness [x)]) the appropriate interaction Lagrangian is of the form

$$L_{int} = i\lambda \left\{ \bar{\psi}\gamma_m (1+\gamma_5) \frac{\partial \psi}{\partial x^n} - \frac{\partial \bar{\psi}}{\partial x^n} \gamma_m (1+\gamma_5) \psi \right\} F^{mn}(x), \quad (1)$$

where λ is the constant of the order eG [xx)] (e is the electron charge, G is the weak interaction constant, $G \approx \frac{10^{-5}}{m_p^2}$) $F^{mn}(x)$ stands for the electromagnetic field tensor.

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x) To explain the decay mode $K_L \to 2\pi$ it is necessary, of course, to consider transitions with change of strangeness [1].

xx) Generally speaking, it may be possible that the interaction of such a type exists already on the electromagnetic level [1], i.e. $\lambda \approx \frac{e}{M^2}$ (where M is a mass of the order of the nucleon mass). In this case transitions must be purely diagonal and the terms containing the γ_5 matrix should be excluded from the Lagrangian (2).

Attempts to apply the hypothetic interaction (1) for calculating some effects in higher orders of perturbation expansion lead to difficulties connected with nonremormalizability of the theory [x)]. In particular, the expressions for physical quantities calculated by perturbation theory contain divergences of increasing powers which can not be removed by the usual renormalization procedure. Therefore it seems advisable to use in nonrenormalizable theories approximations which do not reduced to a simple application of perturbation theory. The choice of such approximations is hinted by quantum-mechanical theory of scattering on strongly singular repulsion potentials $V(r) \geq \frac{g}{r^4}$, $r \to 0$ [3-6]. Indeed, in this case the perturbation series reproduces the characteristic features of nonrenormalizable field theory [xx)] namely: power divergences of increasing order and infinitely increasing powers of the momentum transfer in the asymptotics. Nevertheless, there exists the unique exact solution for Lippman- Schwinger equation for the scattering amplitude (see ref.[3]). This solution decreases at large momentum transfers and has a logarithmic branching point with respect to the coupling constant g for $g = 0$.

We believe that these peculiarities of the scattering on the singular repulsion potential are also valid for nonrenormalizable field theories, at least, for some of them (see lectures [9] where there are references on other papers). Therefore, of especial importance is the inves-

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x) Here we are not dealing with troubles arising in the canonical construction of the S-matrix defined by the Lagrangian (1) which contains the derivative spinor field. As is shown in [2] in constructing the S-matrix in the interaction representation it is also possible to start from L_{int} without recourse to canonical theory.

xx) This circumstance is by no means accidental. Indeed, application of the quasipotential method [7] to nonrenormalizable theories allows to obtain (see [8-9]) potentials having just such a behaviour at $r = 0$.

tigation of the approximate equations which generalize the Lippman-Schwinger equations to the case of relativistic field theory. Such equations for the scattering amplitude (Bethe-Salpeter) and for the vertex function (Edwards) have been investigated in a number of nonrenormalizable theories [10-14]. The results indicate that it is worth-while to continue the study of the abovementioned analogy [x)].

Though we may not expect to construct in such an approach a consistent theory which would allow to calculate any quantities, nevertheless calculations performed with the aid of the equations in the ladder approximation allow to obtain certain interesting results and, what is the main, to obtain the value of the energy, of the momentum transfer and so on, for which the amplitudes begin to decrease.

It should be noted that using these methods one has made the most consistent study of the solutions decreasing over the whole complex domain of the energy variable, which directly corresponds to strong repulsion at small distances of the potential scattering theory. However we can not assert that in all the cases quantum field theory leads to a similar situation. In particular, there may appear functions exponentially increasing in a part of the complex plane of energy variables or even in the whole plane. The consideration made by Jaffe [15] shows that without contradiction with the basic assumptions of quantum field theory the amplitudes can behave at infinity as $\exp\sqrt{S}$ where s is an invariant variable, e.g. the squared total energy in the c.m.s. In a number of nonrenormalizable theories functions increasing at infinity were also obtained [15].

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x) In a number of recent works (see e.g. [10,11]) the latter statement is called in question. The main argument is that when the amplitudes increase exponentially it is impossible to perform the transition to the Euclidean momenta and, strictly speaking, the solution for the Euckidean equation is not the solution for the initial Lorentz-invariant equation. It seems to us that this argument may be removed if one formulate an unambiguous procedure for obtaining solutions also when the quantities increase with respect to the momenta. This problem is, in particular, considered in the present paper.

The problem arises as to how to get such solutions starting from certain approximate solutions. In the present paper we shall deal with one of the aspects of this problem.

Here we consider the Edwards equation for the vertex function in the theory with Lagrangian (1) using the method developed in ref.[14]. The equation for the vertex function is graphically represented in Fig.1, where the circle is the complete vertex function and the point is the vertex corresponding to the Lagrangian (1). This equation is of the form

$$\Gamma^n(p,k) = F^n(p,k) + \frac{1}{(2\pi)^4 i} \int \frac{d^4q}{(p-q)^2} F^\rho(k+q,p-q) \frac{m+\hat{k}+\hat{q}}{m^2-(k+q)^2} \times \\ \times \Gamma^n(q,k) \frac{m+\hat{q}}{m^2-q^2} F_\rho(p,q-p), \qquad (2)$$

where $F^\rho(p,k) = i\lambda\{\gamma^\rho[2(pk)+k^2]-(2p+k)^\rho\hat{k}\}(1+\gamma_5)$ is the free vertex. The notations for the momenta are given in Fig.1. Owing to purely technical difficulties we can sufficiently completely investigate this equation only for small k. For $k=0$, $F(p,k)$ vanishes. Expanding $\Gamma^n(p,k)$ in powers of k_n and retaining only terms of the first order we represent $\Gamma^n(p,k)$ in the form:

$$\Gamma^n(p,k) = 2i\lambda\{[\gamma^n(pk) - p^n\hat{k}]f_1(p^2) + [k^n - \gamma^n\hat{k}]f_2(p^2)\}(1+\gamma_5) \qquad (3)$$

Here we have taken into account also the gauge invariance condition $k_n\Gamma^n = 0$. Simple calculations lead to the following equations for the functions f_1 and f_2 x):

$$f_1(p^2) = 1 - \frac{i\lambda^2}{(pk)} \int \frac{d^4q}{6\pi^4} \frac{q^2 f_1(q^2)\,\Phi(p,q,k)}{(m^2-q^2)^2(q-p)^2} \qquad (4)$$

$$f_2(p^2) = -\frac{i\lambda^2}{(pk)} \int \frac{d^4q}{6\pi^4} \frac{q^2 f_2(q^2)\,\Phi(p,q,k)}{(m^2-q^2)^2(q-p)^2}, \qquad (5)$$

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x) Note that a similar result is obtained also for $V+A$ variant $(1-\gamma_5)$ in the Lagrangian (1)). For a purely vector case (without γ_5) the equation for the most singular part is of the same form (if the coefficient for the integral is divided by 4).

where

$$\Phi(p,q,k) = 2(kq)[q^4 + p^4 - 2(pq)^2] - q^2(pk)(q-p)^2 .$$

For the function $f_1(p^2)$ which is the coefficient for the free vertex structure we have obtained an inhomogeneous equation while the equation for $f_2(q^2)$ is homogeneous and, as we shall see, have only a trivial solution $f_2(p^2) = 0$.

We consider in more details the equation for $f_1(p^2)$. After having rotated, as usually, the contour of integration over q_0 at the angle $\pi/2$ we go over to the Euclidean four-momenta p and q . After the integration over the angles we get the integral equation (see[14]):

$$f(x) = 1 + \frac{4}{3} g^2 \{ \frac{1}{x^2} \int_0^x dy \frac{y^4 (y-2x)}{(y+m^2)^2} f(y) + x \int_x^\infty \frac{dy(x-2y)}{(y+m^2)^2} f(y) \} . \quad (6)$$

Here $f_1(p^2) = f(x)$, $x = -p^2$, $y = -q^2$, $g = \frac{\lambda}{4\pi}$.
This integral equation reduces [14] to the differential one

$$(x\frac{d}{dx} + 2)(x\frac{d}{dx} + 1)(x\frac{d}{dx} - 1)(x\frac{d}{dx} - 2) f(x) + \frac{16 g^2 x^4 f(x)}{(x+m^2)^2} = 4 \quad (7)$$

with the boundary conditions:

1) $x f(x) \to 0$ for $x \to \infty$
2) $|f(x)| < \infty$ for $x \to 0$.

The general solution for eq.(7) is the sum of a particular solution for this equation f_0 and an arbitrary linear combination of the four-linearly independent solutions for the appropriate homogeneous equation. The solutions for the homogeneous equation at $x \to \infty$ have the asymptotics

$$f_{1,2}(x) \approx x^{-3/4} \exp(-4 e^{\pm i\frac{\pi}{4}} \sqrt{gx}) \quad (8)$$

$$f_{3,4}(x) \approx x^{-3/4} \exp\left(4 e^{\pm i \frac{\pi}{4}} \sqrt{gx}\right). \tag{9}$$

Since the functions $f_3(x)$ and $f_4(x)$ do not satisfy the first boundary condition then the general solution for eq.(7) should be found in the form

$$f(x) = f_0(x) + c_1 f_1(x) + c_2 f_2(x). \tag{10}$$

The solution for the inhomogeneous equation (7) has obviously the asymptotics

$$f_0(x) \underset{x \to \infty}{\approx} \frac{1}{4 g^2 x^2}. \tag{11}$$

Let us now consider the behaviour of $f(x)$ at $x \to 0$. It can be shown (see [14]) that the second boundary condition allows to choose in a rather definite manner the coefficients c_1 and c_2 in eq.(10), the obtained solution for eq. (7) being unique.

For the function $f_2(x)$ we have just the same boundary problem but without inhomogeneity in eq.(7), therefore there exists only the trivial solution $f_2(x) = 0$. In the case $m^2 = 0$ the solution for eq.(7) can be found in an explicit form

$$f(x) = \frac{1}{4 g^2 x^2} - \frac{1}{2} G^{30}_{04}\left(g^2 x^2 \mid 1, 1/2, -1, -1/2\right), \quad \mathrm{Re}\, x > 0, \tag{12}$$

where $G^{30}_{04}(y \mid 1, 1/2, -1, -1/2)$ is the Meyer function[18]. Notice that this function has a singularity at $g^2 = 0$, in this case in the expansion there are terms $\sqrt{g^2}$ and $\log g^2$. To demonstrate this we write down several first terms of (12)

$$f(x) = 1 - \frac{2\pi}{3} \sqrt{g^2 x^2} - \frac{2}{3} g^2 x^2 \log(g^2 x^2) + \frac{2}{3} g^2 x^2 \left[\frac{37}{6} - 4\gamma - 4 \log 2\right], \tag{13}$$

where γ is the Euler constant. These singularities remain in the solution in the case $m^2 \neq 0$ too (for the account of $m^2 \neq 0$ see paper[23]). The appearance of the root singularities in g^2 is due to a very strong singularity of the kernel of eq. (6). The solution (12) is determined by the equation only for $\mathrm{Re}\, x > 0$. It cannot be directly applied in the region and the analytic continuation into this region should be performed carefully. To this end we use the series which is obtained after the expansion of the solution

$$f(x) = 1 - \frac{\pi^2}{2}\sqrt{g^2x^2}\sum_{n=0}^{\infty}\frac{(-g^2x^2)^n}{c_1(n)} - \frac{\pi}{2}g^2x^2\log g^2x^2\sum_{n=0}^{\infty}\frac{(-g^2x^2)^n}{c_2(n)} + \qquad (14)$$

$$+\frac{\pi}{2}g^2x^2\sum_{n=0}^{\infty}\frac{(-g^2x^2)^n}{c_2(n)}\chi_n ,$$

where

$$c_1(n) = \Gamma(n+1/2)\,\Gamma(n+1)\,\Gamma(n+2)\,\Gamma(n+5/2);$$

$$c_2(n) = \Gamma(n+1)\,\Gamma(n+3/2)\,\Gamma(n+5/2)\,\Gamma(n+3);$$

$$\chi_n = \psi_n + \psi_{n+1/2} + \psi_{n+3/2} + \psi_{n+2}, \quad \psi_{z-1} = \frac{\Gamma'(z)}{\Gamma(z)} .$$

In order to perform a correct analytic continuation of the solution into the plane x with the cut along the negative real axis we should make the transformation of the series which is identical in the region $x > 0$.

We write $f(x)$ in the form

$$f(x) = 1 - \frac{\pi^2}{2}gx\sum_{n=0}^{\infty}\frac{(-g^2x^2)}{c_1(n)} - \pi g^2x^2\log gx\sum_{n=0}^{\infty}\frac{(-g^2x^2)^n}{c_2(n)} + \qquad (15)$$

$$+\frac{\pi}{2}g^2x^2\sum_{n=0}^{\infty}\frac{(-g^2x^2)^n}{c_2(n)}\chi_n .$$

This function possesses "correct" analytic properties, i.e. it is defined over the whole complex plane with the cut along the negative real axis from zero to $-\infty$. The discontinuity on the cut is

$$\operatorname{Im} f(x) = -2\pi^2 g^2 x^2 \sum_{n=0}^{\infty} \frac{(-g^2x^2)^n}{c_2(n)} = -2\pi^2 g^2 x^2 G^{10}_{04}(g^2x^2 | 0,-1/2,-3/2,-2) \quad (16)$$

and for $x \to \infty$, $\operatorname{Im} f(x)$ exponentially increases according to the law (9). It is interesting to note that initially the function (15) is determined by the series (14) only on that domain ($\operatorname{Re} x > 0$) where it falls off. Our procedure of analytical continuation allows us to pass uniquely also to the region where this function increases. In this case the unambiguity is provided by the condition that the function $f(x)$ have "correct" analytic properties. The fact that the imaginary part of $f(x)$ increases means that for $f(x)$ it is impossible to write dispersion relations with finite number of subtractions. We have not succeeded also in rotating the integration contour back to the real axis and thereby the solution found here is not (generally speaking) the solution of the initial equation. The interpretation of this result may, e.g. consist in the following. We can obtain a receipt for summing the class of diagrams by means of which it is possible to find a vertex function which is not, strictly speaking, the solution of the initial equation (in the pseudoeuclidean space) x). This receipt appears to reduce to the prescription of the rules how to handle the products of strongly singular distributions (see papers [16-17], the advantage of our method is its unambiguity).

Thus, the analysis of the equation for the vertex function in the theory with interaction (1) in the ladder approximation leads to that we can ascribe the meaning to the summation of this class of diagrams, the nonrenormalizability of the theory being, as usually revealed in the nonanalyticity in the coupling constant. We would like to stress especially the modification of the method which has been made here and which consists in the use of the analytical continuation procedure. We believe that some analytical continuation of solutions with respect to energy variables or coupling constant etc. will turn out to be necessary

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x) For more correct formulation of this receipt it seems useful to introduce a subsidiary regularization (see e.g., [15]).

in all problems where we must deal with amplitudes increasing at infinity.

Note that the example chosen here is, of course, interesting for studying the properties of the hypothetic interaction (1) which may turn out to be responsible for CP-invariance violation. In addition, it gave us the possibility of fdemonstrating the modified method for solving the singular Edwards-type equations.

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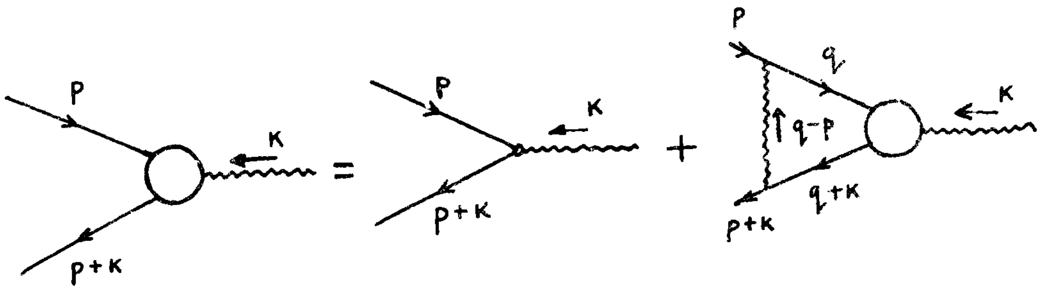

Fig.1