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ISOBARIC ANALOGUE RESONANCES IN NUCLEI OF CLOSED NEUTRON SHELLS

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Изобарические аналоговые резонансы в ядрах, имеющих замкнутые оболочки нейтронов

В работе дается микроскопическая теория аналоговых резонансов. На основе предложенной теории проведены вычисления дифференциальных сечений возбужденных аналоговых резонансов в функции энергии. Результаты вычислений согласуются с экспериментальными данными.

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ISOBARIC ANALOGUE RESONANCES IN NUCLEI OF CLOSED NEUTRON SHELLS

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A b s t r a c t

The scattering amplitude of the elastic proton scattering is investigated in the energy range of the isobaric analogue resonances. The scattering amplitude is expressed as $\mathcal{T} = \mathcal{T}^{opt} + \mathcal{T}^{res}$, where the background term $\mathcal{T}^{opt}$ is calculated by means of the standard optical model and the resonance term is derived by the help of Feshbach's projection operator method. The "ideal" isobaric analogue states are introduced by applying the stepdown isospin operator T_- on the wave functions of the low lying states of the target + neutron system. Afterwards the "target ground state + proton" component of the "ideal" IAS is projected out. Such a doorway type state, the so called "projected" IAS is assumed to play the role of the intermediate state of the scattering process. The method is applied for the calculation of the cross section and polarization of elastic proton scattering on ^{138}Ba, ^{140}Ce and ^{142}Nd. The low lying states of the target are calculated in the framework of the quasiparticle random phase approximation and the wave functions for the target + neutron system are obtained by diagonalizing the residual interaction in the quasiboson + single neutron basis.

1. Introduction

It has been observed many years ago that the energy spectra of light nuclei show remarkable similarities which can be explained by the charge independence of the nuclear forces. The energy levels of the isobaric nuclei can be organized into charge multiplets characterized by a given isospin. If the charge dependent interactions are completely neglected then the isospin is a good quantum number, the elements of a given charge multiplet are exactly degenerate and the states of the neighbouring isobars are connected to each other by the isospin step-up and step-down operators:

$$T_{\pm} \, | \Phi_{\alpha}^{T M_T} > = \sqrt{T(T+1) - M_T (M_T \pm 1)} \, | \Phi_{\alpha}^{T M_T \pm 1} > . \qquad (1)$$

In reality the charge dependent interactions do exist and they can cause a mixing among different isospin states, on the other hand the degenerate charge multiplets split up. In the case of light nuclei the mixing and the shifts are not too large therefore the isospin can be used as an appro-

ximately good quantum number. Proceeding along the periodic table the Coulomb shift is ever increasing and sooner or later it may happen that some elements of a given charge multiplet are in the discrete spectrum, the rest of them, however, are already in the continuum. During many years it was expected that the importance of the off diagonal matrix elements of the charge dependent interactions, giving rise to the isospin mixing, are increasing together with the diagonal elements and it was assumed that the isospin looses its meaning in the medium and heavy nuclei.

This state of affairs has changed remarkably since the beginning of the sixties and the isospin has started to regain its credit. In 1961 Anderson and Wong [1] have discovered some narrow peaks in the yield of direct (p, n) reactions. It turned out that the observed peaks correspond to highly excited states of the residual nucleus which are in analogy with the low lying states of the target nucleus being shifted by the Coulomb energy ΔE_c.

The theoretical intepretation of these results was first given by Lane and Soper [2]. It was pointed out that in a direct (p, n) process among others the proton can jumps into the same single particle state from where the neutron was knocked out. In this case the final state has the same symmetries, the same isospin as the ground state of the target nucleus. This state, the so called isobaric analogue state (IAS) of the target was identified by Ikeda, Fujii and Fujita [3] as a coherent superposition of monopole type (proton-particle) (neutron-hole) excitations. In 1964 Fox, Moore and Robson [4] have observed some narrow resonances in (p, p) elastic scattering. As far as the quantum numbers and the energies are concerned a surprisingly good correspondence was observed between these resonances and the low lying states of the target + neutron system except for an overall energy shift which can be identified with the Coulomb energy ΔE_c. Since that time a large number of IAS have been observed in the proton induced reactions [5] and also a great deal of theoretical activity was devoted to the interpretation of the IAS and to the study [2,6,7] of the "isospin impurity". To analyze the proton scattering problem a reach variety of weapons were used. In Robson's approach [7] the R-matrix formalism was employed. Weiden-

muller and Mahaux [8] studied the different aspects of the IAS using the tools of the extended shell model. Many investigation were based upon the Lane's equations[9]. Feshbach's projection operator technics[10], which will be applied in this paper too, was used by Kerman and Toledo de Pisa [11] and also by Stephen [12].

The starting point of the approach developed by Kerman and Toledo de Pisa is the definition of the IAS $|A>$ of the target + proton system:

$$|A> = \frac{T_- |\Phi>}{[<\Phi| T_+ T_- |\Phi>]^{1/2}} \quad , \qquad (2)$$

where $|\Phi>$ is some low lying state of the target +neutron system. Comparing the equations (1) and (2) it is obvious that thi definition is based upon the hypothesis of the existence of an approximate charge multiplet structure. Of course, $|A>$ in itself can not be the eigenstate of the system since it is embedded into the continuum and surrounded by the multitude of compound states, but it is expected that $|A>$ is the dominant component of the scattering wave function $|\Psi>$ in some energy range. In order to analyse the structure of the scattering wave function in a systematic way, it is convenient to introduce the projection operators A, p and q by the following definitions:

(i) $p + q + A = 1$

(ii) $pq = pA = qA = 0$

(iii) $A = |A><A|$

(iv) p projects out the "target ground state +proton" component of $(1 - A)|\Psi>$.

This kind of decomposition of the wave function is very advantageous because the coupling between the components $A|\Psi>$ and $(p + q)|\Psi>$ can be produced only by the charge dependent part of the Hamiltonian. Thus, in principle, we are able to calculate exactly the coupling , giving rise to the width of the IAS, since the exact form of the Coulomb inte-

raction is known. There is some drawback, however, in this decomposition , namely the "target ground state+proton" component of the wave function is splitted up and one part is contained already by $A|\Psi>$ and only the remaining part is represented by $p|\bar{\Psi}>$. It is not obvious at all how to get an explicit representation for $p|\bar{\Psi}>$ since in $A|\Psi>$ all nucleons including also the last proton are in bound states, on the other hand in $p|\Psi>$ the proton is in a continuum state. Because of this difficulty in this paper we choose another view point and apply a somewhat different procedure the main points of which can be summarized as follows:

a) The projection operators P , Q and R are introduced by the following definitions:

(i) $P + Q + R = 1$

(ii) $PQ = PR = QR = 0$

(iii) P projects out the full target ground state+proton" component of $|\Psi>$.

(iv) Q selects out the components of $|\Psi>$ corresponding to such configurations in which all nucleons are in bound states. Because of the required orthogonality of the P and Q subspaces the target ground state must be excluded from the subspace Q .

(v) $R|\Psi>$ is the remaining part of $|\Psi>$ which corresponds assymptotically to inelastic scattering and different fragmentation of the nuclear system.

b) The elements of the Q subspace characterized predominantly by the isospin eigenvalue $T = T_0 + \frac{1}{2}$ are picked out. Here T_0 stands for the isospin of the target ground state $(T_0 = M_{T_0})$.

c) An equation is derived for the component $P|\Psi>$ containing all the information on elastic scattering. In this equation the coupling between $P|\Psi>$ and the special components of $Q|\Psi>$, labelled by T , is expressed explicitly by one term of the effective Hamiltonian.

d) The special components of $Q|\Psi>$ the so called "projected"

IAS's are expressed in terms of the wave functions of the target+neutron system.

Using this method we will calculate the scattering amplitude, the cross section and the polarization of the elastic proton scattering in the energy range of the isobaric analogue resonances.

We conclude this introduction with a brief description of the contents of the following sections. In Section 2 we give a formal derivation of the scattering amplitude in terms of effective Hamiltonians obtained by means of projection operators. Involving some heuristic arguments we replace these projected Hamiltonians by model operators. In Section 3 we formulate a simple model for the target + one particle system and we calculate the low lying states of the target + neutron system. Using these model wave functions we construct the "projected" IAS and the scattering amplitude is expressed in terms of these states. In Section 4 we outline the main steps of an application of the method for the calculation of elastic proton scattering cross section and polarization on the $N = 82$ isotones.

2. Formal Derivation of the Scattering Amplitude

Multiplying the Schrödinger equation by the projection operators P, Q and R we get the coupled system of equations:

$$
\begin{aligned}
(E - PHP)\,P\,|\Psi\rangle &= (PHQ)\,Q\,|\Psi\rangle + (PHR)\,R\,|\Psi\rangle \\
(E - QHQ)\,Q\,|\Psi\rangle &= (QHP)\,P\,|\Psi\rangle + (QHR)\,R\,|\Psi\rangle \\
(E - RHR)\,R\,|\Psi\rangle &= (RHP)\,P\,|\Psi\rangle + (RHQ)\,Q\,|\Psi\rangle\,.
\end{aligned}
\tag{3}
$$

Eliminating the components $Q\,|\Psi\rangle$ and $R\,|\Psi\rangle$, the equation for $P\,|\Psi\rangle$ can be written as follows:

$$(E - H_P - H_Q - H_R)P|\Psi> = 0, \qquad (4)$$

where

$$H_P = PHP$$

$$H_Q = PHQ \frac{1}{E - QHQ} QHP \qquad (5)$$

$$H_R = [PHR + PHQ \frac{1}{E - QHQ} QHR] \cdot$$

$$[\frac{1}{E - RHR - RHQ \frac{1}{E-QHQ} QHR + i\epsilon}][RHQ \frac{1}{E - QHQ} QHP + RHP]$$

Introducing the projection operator t which projects out the subspace characterized by the isospin $T = T_0 + \frac{1}{2}$ we decompose the second term of the Hamiltonian into two components:

$$H_Q = PHQt \frac{1}{E - QHQ} tQHP + \hat{H}, \qquad (6)$$

where $\hat{H}_Q$ is given by :

$$\hat{H}_Q = PHQ(1-t) \frac{1}{E - QHQ} tQHP +$$

$$+ PHQ(1-t) \frac{1}{E - QHQ} (1-t)QHP + \qquad (7)$$

$$+ PHQt \frac{1}{E - QHQ} (1-t)QHP .$$

Now we can write the equation (4) in the following form:

$$(E - \hat{PHP})P|\Psi> = (PHQ\, t \frac{1}{E - QHQ} tQHP)P|\Psi>, \qquad (8)$$

where $\hat{PHP}$ is defined by

$$\hat{PHP} = H_P + \hat{H}_Q + H_R . \qquad (9)$$

If we wanted to analyse the scattering problem in full details, then, of course, equation (8) is completely useless because of the complexity of $\hat{PHP}$. On the other hand, as far as practically applicable approximations are concerned, the form of the equation (8) seems to be rather convenient.

The formal solution of (8) can be expressed as

$$P|\Psi\rangle = |\Psi_p^{(+)}\rangle + \hat{G}^{(+)} PHQ\, t\, [E - QHQ - tQHP\hat{G}^{(+)}PHQt]^{-1} tQHP |\Psi_p^{(+)}\rangle , \qquad (10)$$

where $|\Psi_p^{(+)}\rangle$ is defined as the solution of the homogeneous equation:

$$(E - \hat{PHP}) |\Psi_p^{(+)}\rangle = 0 \qquad (11)$$

and the Green operator $(E - \hat{PHP} + i\epsilon)^{-1}$ is denoted by $\hat{G}^{(+)}$.

By the help of this formal solution the amplitude of the elastic scattering can be written as the sum of two terms:

$$\mathcal{T} = \hat{\mathcal{T}} + \langle\Psi^{(-)}| PHQt\, [E - QHQ - tQHP\hat{G}^{(+)}PHQt]^{-1} tQHP |\Psi^{(+)}\rangle , \qquad (12)$$

where the first term $\hat{\mathcal{T}}$ is the amplitude of the elastic scattering caused by $\hat{PHP}$ alone.

We expand the second term of the scattering amplitude using the complete set of wave functions defined by the equation

$$(E_r - QHQ)|\Theta_r\rangle = 0 . \qquad (13)$$

Then

$$\mathcal{T} = \hat{\mathcal{T}} + \sum_{rs} \langle\Psi^{(-)}|PHQt|\Theta_r\rangle (A^{-1})_{rs} \langle\Theta_s|tQHP|\Psi^{(+)}\rangle , \qquad (14)$$

where the matrix elements $(A^{-1})_{rs}$ can be obtained by matrix inversion of the matrix A_{rs} defined by

$$A_{rs} = \langle\Theta_r| E - QHQ - tQHP\hat{G}^{(+)}PHQt|\Theta_s\rangle . \qquad (15)$$

To start with a given form of the Hamiltonian H and to work out in details all the quantities in the scattering amplitude is rather hopeless therefore, we are forced to introduce some heuristic assumptions based upon some physical considerations in order to get a more practical form for the scattering amplitude. Doing so, we shall replace the "exact" projected operators by model operators. At the same time we reduce our program and we are not going to analyse the "exact" scattering amplitude in its total complexity, instead of that we shall approximate it by an "average" scattering amplitude $\langle \hat{\mathcal{T}} \rangle$. It can be seen from the definition of $\hat{P}H\hat{P}$ that in addition to the so-called potential scattering (H_p), the effect of the inelastic channels (H_k) and that of some kind of intermediate states ($\hat{H}_Q$) are incorporated into the amplitude $\hat{\mathcal{T}}$.

In most cases the number of open channels and that of the intermediate states is rather high and usually their effect can be treated properly by a complex optical potential. We will adopt this procedure and we define the average scattering amplitude $\langle \hat{\mathcal{T}} \rangle$ replacing $\hat{P}H\hat{P}$ by the optical model Hamiltonian H^{opt} of the target+ proton system. In addition to this we must introduce some assumptions for the projected operators $\hat{Q}H\hat{P}$ and $\hat{P}H\hat{Q}$ they will be identified with the two-body residual interaction. For the sake of simplicity we shall assume that the residual Coulomb interaction is negligible, that is the residual interaction V commutes with the isospin operators.

Adopting these assumptions the average scattering amplitude will have the following form:

$$\langle \hat{\mathcal{T}} \rangle = \mathcal{T}^{opt} + \sum_{rs} \langle \tilde{\Psi}_p^{(-)} | Vt | \Theta_r \rangle (A^{-1})_{rs} \langle \Theta_s | tV | \Psi_p^{(+)} \rangle . \qquad (16)$$

The quantities involved in this expression are reinterpreted by the following equations:

$$(E - H^{opt}) | \Psi_p^{(+)} \rangle = 0 \qquad (17)$$

$$(E - (H^{opt})^*) | \tilde{\Psi}_p^{(+)} \rangle = 0 \qquad (18)$$

$$G_p^{(+)} = (E - H^{opt} + i\epsilon)^{-1} \qquad (19)$$

$$A_{rs} = \langle \Theta_r \mid E - QHQ - tVG_p^{(+)} Vt \mid \Theta_s \rangle . \qquad (20)$$

In the equation (18) $(H^{opt})^*$ stands for the complex conjugate of the optical model Hamiltonian.

We must emphasize that this kind of decomposition of the scattering amplitude into two terms is meaningfull only if the coupling of states having different isospin is weak enough. On the other hand this decomposition is useful only if the number of the intermediate states characterized by the isospin T is small enough compared to that of other states. If these requirements are not fulfilled then we are forced to treat all the intermediate states on equal footing and we have no right to performe such a separation of H_Q what we did in (6). A great deal of experimental evidences, however, shows that the isospin of bound states is an approximately good quantum number, that is, the intermediate states dominated by different isospin values are weakly coupled. On the other hand in the interesting energy range the density of states characterized by isospin $T = T_0 + \frac{1}{2}$ is very low compared to that of the "normal" isospin states $(T = T_0 - \frac{1}{2})$. Consequently, we can accept the expression under (16) as a good approximation for the average scattering amplitude at least in a limited energy range.

It is expected that the cross section derived from (16) will show intermediate structure, in addition to the broad single particle resonances, we shall get narrow peaks due to some quasistationary states characterized by the isospin $T = T_0 + \frac{1}{2}$.

Our next task is to establish a relationship between the states $|\Theta_r\rangle$ of the target + proton system and the bound states $|\Phi_r\rangle$ of the target+ neutron system. The wave functions of the low lying states of the target-neutron system, satisfying the equation:

$$H \mid \Phi_r^{TT} \rangle = \mathcal{E}_r \mid \Phi_r^{TT} \rangle \qquad (21)$$

are labeled by the isospin quantum numbers T and $M_T (T = M_T = T_0 + \frac{1}{2})$.

By means of the isospin step-down operator we get:

$$H\left(\frac{T_- |\Phi_r^{TT}>}{\sqrt{2T_0+1}}\right) = \left(\mathcal{E}_r + \frac{[H, T_-]T_+}{2T_0+1}\right)\left(\frac{T_- |\Phi_r^{TT}>}{\sqrt{2T_0+1}}\right), \qquad (22)$$

or in the usual approximation

$$H |\Phi_r^{TT-1}> = (\mathcal{E}_r + \Delta E_c) |\Phi_r^{TT-1}>, \qquad (23)$$

where the Coulomb shift is denoted by ΔE_c. Manipulating with the projection operators we get:

$$\begin{aligned}(\mathcal{E}_r + \Delta E_c - QHQ)Q |\Phi_r^{TT-1}> = \\ = (QHP)P |\Phi_r^{TT-1}> + (QHR)R |\Phi_r^{TT-1}>.\end{aligned} \qquad (24)$$

On the right hand side the second term is zero by the definition of the projection operator R and the first term is rather small if the neutron excess is large enough; neglecting this term we have:

$$(\mathcal{E}_r + \Delta E_c - QHQ)Q |\Phi_r^{TT-1}> = 0. \qquad (25)$$

Comparing the equations (13) and (25) we see that

$$\begin{aligned}E_r &= \mathcal{E}_r + \Delta E_c \\ |\Theta_r> &= N_r Q |\Phi_r^{TT-1}>,\end{aligned} \qquad (26)$$

where N_r stands for a normalization constant.

According to these relations the approximate eigenstates of QHQ can be constructed from the "ideal" IAS's $|\Phi_r^{TT-1}>$ by the help of the projection operator Q.

3. A Simple Model

In this section we formulate a simple model for the description of the target-neutron system, and in the frame-work of this model we calculate the energy eigenvalues $\mathcal{E}_r$ and eigenfunctions $|\Phi_r^{TT}>$ for the low lying states, furthermore using these wave functions we calculate the matrix elements involved in the scattering amplitude.

For the sake of simplicity we restrict ourselves to such cases when the even-even target contains only closed neutron shells. The model Hamiltonian is written as:

$$H = H_0 + h + V , \tag{27}$$

where H_0 and h stand for the Hamiltonians of the target and the last odd particle, respectively, and the residual interaction is denoted by V. The low lying eigenstates of the target+ neutron system take the form:

$$|\Phi_r^{TT}> = \sum_{Ij} C_{Ij}^{r} \; |\phi_I \; \chi_{nj}> , \tag{28}$$

where ϕ_I is the eigenfunction of H_0 and the single particle wave function for the last neutron outside of the closed shells is denoted by χ_{nj}. The amplitudes C_{Ij}^{r} are obtained by diagonalization in a restricted space of the basis functions $|\phi_I \chi_{nj}>$. Now making use of these model wave functions we compute the matrix elements involved in the scattering amplitude. Let us consider at first the wave function $|\Theta_r>$ given by

$$|\Theta_r> = \frac{N_r}{\sqrt{2T_0+1}} \; \mathcal{T}_- \; |\Phi_r^{TT}> . \tag{29}$$

The "ideal" IAS as we pointed out previously has a "target ground state + proton" component. In order to get the "projected" IAS we must eliminate this component. It is easy to see that in our model the "target ground state + proton" component of the ideal IAS has the form

$$\frac{C^{r}_{oj}}{\sqrt{2T_o+1}} |\phi_o \chi_{pj}> . \qquad (30)$$

where ϕ_0 is the ground state wave function of the target and χ_{pj} is a bound neutron state occupied by a proton, in other words it is a neutron orbital multiplied by a proton isospin factor. Substracting this component from "ideal" IAS we get the "projected" one:

$$|\Theta_r> = N_r Q |\Phi_r^{TT-1}> = N_r \{ |\Phi_r^{TT-1}> - \frac{C^{r}_{oj}}{\sqrt{2T_0+1}} |\phi_0 \chi_{pj}> \} , \qquad (31)$$

where the normalization factor N_r is given by

$$N_r = (1 - \frac{(C^{r}_{oj})^2}{2T_0+1})^{-\frac{1}{2}} . \qquad (32)$$

In order to evaluate the effect of the isospin projection operator t on $|\Theta_r>$ we must manipulate with the second term of $|\Theta_r>$:

$$t |\phi_o \chi_{pj}> = t \frac{T_- T_+}{2T_0+1} t |\phi_o \chi_{pj}> = \frac{t T_-}{2T_0+1} |\phi_o \chi_{nj}> , \qquad (33)$$

where the relation

$$T_+ t |\phi_0 \chi_{pj}> = |\phi_0 \chi_{nj}> \qquad (34)$$

was used. Introducing the notation

$$|\Lambda_r^{TT}> = N_r \{ |\Phi_r^{TT}> - \frac{C^{r}_{oj}}{2T_0+1} |\phi_0 \chi_{nj}> \} , \qquad (35)$$

we get

$$t |\Theta_r> = \frac{t T_-}{\sqrt{2T_0+1}} |\Lambda_r^{TT}> . \qquad (36)$$

The wave function $|\Psi_p^{(+)}>$ involved in the scattering amplitude can be expressed in terms of our model wave functions as

$$| \Psi_p^{(+)} > = | \phi_0 \, \eta_p^{(+)} > \, . \qquad (37)$$

where $\eta_p^{(+)}$ stands for the optical model wave function of the proton satisfying the "Coulomb distorted plane waves +outgoing waves" boundary condition . Now the relevant matrix elements can be written as follows

$$< \Theta_r \mid tV \mid \Psi_p^{(+)} > = \frac{1}{\sqrt{2\,T_0+1}} < \Lambda_r^{TT} \mid V T_+ t \mid \Psi_p^{(+)} > = \qquad (38)$$

$$= \frac{1}{\sqrt{2\,T_0+1}} < \Lambda_r^{TT} \mid V \mid \Psi_n^{(+)} > \, ,$$

where

$$| \Psi_n^{(+)} > = | \phi_0 \, \eta_n^{(+)} > \qquad (39)$$

and $\eta_n^{(+)}$ stands for a scattering proton state occupied by a neutron. By the help of similar manipulations we get for A_{rs} the following expression :

$$A_{rs} = (E - \mathcal{E}_r - \Delta E_0) \delta_{rs} - \Delta_{rs} + \frac{i}{2} \Gamma_{rs} \, , \qquad (40)$$

where

$$\Delta_{rs} = \frac{1}{2T_0+1} \, \mathrm{Re} < \Lambda_r^{TT} \mid V G_n^{(+)} V \mid \Lambda_s^{TT} > \qquad (41)$$

$$\Gamma_{rs} = - \frac{2}{2T_0+1} \, \mathrm{Im} < \Lambda_r^{TT} \mid V G_n^{(+)} V \mid \Lambda_s^{TT} > \qquad (42)$$

and $G_n^{(+)}$ is defined by

$$G_p^{(+)} = T_+ \, t \, G_p^{(+)} t \, T_- \, . \qquad (43)$$

To see the meaning of $G_n^{(+)}$ it is enough to look at the spectral representation of the Green operator. The operator $G_p^{(+)}$ is constructed from scattering proton states occupied by proton, the operator $G_n^{(+)}$ is constructed from the same scattering proton states occupied by neutron.

The matrix elements involving the Green operator $G_n^{(+)}$ can be written in the following form:

$$< \Lambda_r^{TT} \mid V G_n^{(+)} V \mid \Lambda_s^{TT} > = \tag{44}$$

$$= \int dx\, dx' < \Lambda_r^{TT} \mid V \mid \phi_0 , x > G^{(+)}(x, x') < \phi_0 , x' \mid V \mid \Lambda_s^{TT} > .$$

The Green function $G^{(+)}(x, x')$ is expressed by the regular and irregular solutions of the Schrödinger equation:

$$G^{(+)}(x, x') = -\pi \sum_{j\ell m} \mathcal{Y}_{j\ell m}(\theta, \phi, \sigma)\, \mathcal{Y}_{j\ell m}(\theta', \phi', \sigma')(-1)^{j-m} \tag{45}$$

$$\frac{1}{r r'} \begin{cases} v_{j\ell}(r)[\, w_{j\ell}(r') + i v_{j\ell}(r')\,] & r \leq r' \\ v_{j\ell}(r')[\, w_{j\ell}(r) + i v_{j\ell}(r)\,] & r > r' . \end{cases}$$

The normalization and the assymptotic form of the regular $v_{j\ell}(r)$ and irregular $w_{j\ell}(r)$ solutions are given by

$$v_{j\ell}(r) \approx \sqrt{\frac{k}{\pi E}} \sin\left(kr - \frac{\pi \ell}{2} + \delta_{j\ell} + \eta \ell n\, kr + \sigma_\ell\right) \tag{46}$$

$$w_{j\ell}(r) \approx \sqrt{\frac{k}{\pi E}} \cos\left(kr - \frac{\pi \ell}{2} + \delta_{j\ell} + \eta\, \ell n\, kr + \sigma_\ell\right), \tag{47}$$

where the phase shifts $\delta_{j\ell}$ are complex numbers. It is worth while to point out that in our case the level width Γ_{rs} in addition to the escape width, due to the coupling of the "projected IAS to the continuum states, contains the damping width too since our Hamiltonian has an imaginary part giving rise to a coupling among the doorway type "projected IAS's and the more complicated states, or more honestly speaking, the imaginary part of the potential imitates such a coupling.

Because of the factor $\frac{1}{2T_0 + 1}$ the level shift is quite small, consequently the spacing of the isobaric analogue resonances are quite similar to the spacing of the low lying states of the neighbouring isobar. From experimental point of view this factor is very important, because it

recognize the analogy among the various states would be rather difficult if the shifts of the individual levels were too big. The explanation for the surprisingly small widths of the observed isobaric analogue resonances can be found also in the factor $\frac{1}{2T_0 + 1}$ in front of the level width formula (42).

4. The Cross Section and Polarization of the Elastic Proton Scattering on ^{142}Nd

In this section we summarize the main points and some results of a detailed calculation for the cross section and the polarization of elastic proton scattering on ^{138}Ba, ^{140}Ce, and ^{142}Nd. In the case of such nuclei the model outlined in the previous section is directly applicable because the target is of even-even type and contains only closed neutron shells. We shall compare our results with the experimental data obtained by von Brentano et al. in Heidelberg [13]. We discuss in detail only the case of the ^{142}Nd because the most detailed experimental information was available for this case, on the other hand, our results obtained for the elastic proton scattering on ^{138}Ba and ^{140}Ce are qualitatively the same as in the case of ^{142}Nd.

Our first task is to work out an appropriate description for the low lying states of the target (^{142}Nd) and the target + neutron system (^{143}Nd).

It seems to be justified to treat the low lying states of the target in the framework of the microscopic collective model, because the energy of the first excited state ($E^{2+} = 1.57$ MeV) is appreciably less than the energy gap in this region and the observed E2 transition probability [14] ($B(E2) = 0{,}47\ e^2\ 10^{-48}\ cm^4$) is considerably higher then the single particle estimate.

Assuming a "pairing + quadrupole" type interaction among the protons being in the last open shall the Hamiltonians h and H_0 will have the following form:

$$h = \sum_{jm} \epsilon^{n}_{j}\, b^{+}_{jm}\, b_{jm} \qquad (48)$$

$$H_0 = \sum_{jm} \epsilon^{p}_{j}\, a^{+}_{jm}\, a_{jm} - \frac{1}{2} G \sum_{jmj'm'} a^{+}_{j'm'}\, a^{+}_{j'-m'}\, a_{j-m}\, a_{jm} -$$

$$- \frac{1}{2} \kappa \sum_{\substack{j_1 m_1 j_2 m_2 \\ j'_1 m'_1 j'_2 m'_2}} \langle j'_1 m'_1 j'_2 m'_2 | r_1^2 r_2^2 \sum Y_{2\mu}(\theta_1, \phi_1)\, Y^{*}_{2\mu}(\theta_2, \phi_2) | j_1 m_1 j_2 m_2 \rangle \qquad (49)$$

$$a^{+}_{j'_1 m'_1}\, a^{+}_{j'_2 m'_2}\, a_{j_2 m_2}\, a_{j_1 m_1} .$$

In ^{142}Nd 10 protons are distributed in the single particle states $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ outside of the double magic core containing 50 protons and 82 neutrons. For the last neutron of ^{143}Nd the single particle states $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, and $3p_{1/2}$ are available. In order to have correct assymptotic form for the single particle wave functions both the neutron operators b^{+}_{jm}, b_{jm} and the proton operators a^{+}_{jm}, a_{jm} are defined on a Saxon-Woods basis. The potential parameters are given in the Table I.

Table I. The parameters of the Saxon-Woods potential.

	V MeV	V_{so} MeV	r_0 fm	a fm
Proton	-57	-7	1.25	0.65
Neutron	-47	-7	1.25	0.65

Having no reliable information about the location of the relevant single particle states we have treated the single particle energies ϵ_j and

ϵ_j^p as adjustable parameters instead of worrying too much about the "best choice" of the Saxon-Woods potential.

The effect of the pairing correlations of the proton system was taken into account by the Valatin-Bogolubov transformation.

The assumed values of the single particle energies ϵ_j^p and the strength of the pairing force (G) together with the calculated values of the quasiparticle energies (E_j), the transformation coefficients (u_j, v_j), the energy gap 2Δ and the Fermi level λ are given in the Table II.

Table II. The parameters of the quasiparticle transfromation.

G = 28/A = 0.197 MeV		2Δ = 2.21 MeV	λ = 0.625 MeV	
j	ϵ_j^p MeV	E_j MeV	u_j	v_j
$1g_{7/2}$	0.00	1.27	0.504	0.864
$2d_{5/2}$	0.76	1.11	0.749	0.663
$1h_{11/2}$	2.18	1.91	0.953	0.304
$2d_{3/2}$	2.88	2.51	0.974	0.226
$3s_{1/2}$	3.20	2.80	0.980	0.201

The long range term of the target Hamiltonian was treated by the method of the quasiparticle random phase approximation.

Introducing the quadrupole phonon creation operator as the linear

combination of the quasiproton pair creation and annihilation operators $A^{+}_{2\mu}(j_1, j_2)$ and $A_{2\mu}(j_1, j_2)$:

$$Q^{+}_{2\mu} = \sum_{j_1 j_2} X_{j_1 j_2} A_{2\mu}(j_1, j_2) - Y_{j_1 j_2} A_{2-\mu}(j_1 j_2)(-1)^{\mu} \qquad (50)$$

the Hamiltonian is transformed into the form

$$H_0 = E_0 + \omega \sum_{\mu} Q^{+}_{2\mu} Q_{2\mu} , \qquad (51)$$

where ω stands for the enrgy of the one-phonon excitation and the ground state energy is denoted by E_0 .

The coefficients $X_{j_1 j_2}$, $Y_{j_1 j_2}$ and the excitation energy ω are determined by the secular equation of the QRPA . We have chosen the value $\kappa = 4.8 \cdot 10^{-3}$ MeV fm^{-4} for the strength of the quadrupole force in order to fit the one phonon energy ω to the energy of the first excited state:

$$\omega = E^{2+} = 1.57 \text{ MeV} .$$

The calculated values of the coefficients $X_{j_1 j_2}$ and $Y_{j_1 j_2}$ are tabulated in the Table III.

The eigenfunctions of H_0 denoted by $\phi_I(N)$ are labelled by the angular momentum I and the number of the quadrupole phonons N . The basis for the matrix representation and diagonalization of the Hamiltonian can be constructed by vector coupling the target states $\phi_I(N)$ to the single neutron states χ_{nj} . In this basis the matrix of the Hamiltonian has the following form:

$$\langle [\phi_{I'}(N')\chi_{nj'}]^{J} | H | [\phi_I(N)\chi_{nj}]^{J} \rangle =$$

$$= \delta_{II'}\delta_{NN'}\delta_{jj'}(N\omega + \epsilon^{n}_{j}) + \langle [\phi_{I'}(N')\chi_{nj'}]^{J} | V | [\phi_I(N)\chi_{nj}]^{J} \rangle . \qquad (52)$$

Table III. The coefficients of the quasiparticle pair creation and annihilation operators in the quadrupole phonon creation operator.

j_1	j_2	$X_{j_1 j_2}$	$Y_{j_1 j_2}$
1/2	1/2	0	0
	3/2	-0.0275	-0.0150
	5/2	-0.1052	-0.0450
	7/2	0	0
	11/2	0	0
3/2	1/2	0.0275	0.0150
	3/2	-0.0338	-0.0177
	5/2	0.0690	0.0273
	7/2	-0.1204	-0.0498
	11/2	0	0
5/2	1/2	-0.1052	-0.0450
	3/2	-0.0690	-0.0273
	5/2	-0.5286	-0.0915
	7/2	-0.1049	-0.0216
	11/2	0	0
7/2	11/2	0	0
	3/2	-0.1204	-0.0498
	5/2	0.1049	0.0216
	7/2	-0.3774	-0.0890
	11/2	0	0
11/2	1/2	0	0
	3/2	0	0
	5/2	0	0
	7/2	0	0
	11/2	-0.1607	-0.0670

In the actual diagonalization we restrict our basis allowing only the zero and one phonon states ($N = 0,1$) .

Let us see now the problem of the residual interaction V . In general V can be written as the sum of two-body interactions:

$$V = \sum_i V(x, x_i) + (r r_i) W(x, x_i). \tag{53}$$

For the sake of simplicity we take into account only the lowest multipole components of $V(x, x_i)$ and $W(x, x_i)$. Adopting the well known ideology, we can say that the monopole component of $V(x, x_i)$ is already included into the Saxon-Woods type single particle potential of h therefore we approximate $V(x, x_i)$ by a quadrupole term assuming the same form and the same strength as in the Hamiltonian H_0 :

$$V(x, x_i) = -\frac{1}{2} \kappa r^2 r_i^2 \sum_\mu Y_{2\mu}(\theta, \phi) Y^*_{2\mu}(\theta_i, \phi_i). \tag{54}$$

The monopole component of $W(x, x_i)$ cannot be included completely into the single particle potential therefore we keep this term and neglect all the higher multipole components.

Thus the residual interaction in our schematic model can be expressed as the sum of a quadrupole and a monopole term:

$$V = V^{(2)} + V^{(0)}. \tag{55}$$

The relevant matrix elements of the quadrupole component can be computed easily and are given by [15,16]

$$<[\phi_1(N)\chi_{nj}]^J \| V^{(2)} \| [\phi_{1'}(N')\chi_{nj'}]^J> = \qquad (56)$$

$$= <\chi_{nj} \| r^2 Y_2(\theta,\phi) \| \chi_{nj'}> R^{N(1j)J}_{N'(1'j')J} \ ,$$

where

$$R^{0(0j)J}_{0(0j)J} = 0 \ ,$$

$$R^{1(2j)J}_{0(0j')J} = -\frac{\kappa \delta_{j'J}}{\sqrt{5(2J+1)}} \sum_{j_1 j_2} <\chi_{pj_1} \| r^2 Y_2(\theta,\phi) \| \chi_{pj_2}>$$

$$(u_{j_1} v_{j_2} Y_{j_1 j_2} + u_{j_2} v_{j_1} X_{j_1 j_2}) \qquad (57)$$

$$R^{1(2j)J}_{1(2j')J} = (-1)^{J+j'+1} 10\kappa \begin{Bmatrix} 2 & 2 & 2 \\ j & j' & J \end{Bmatrix}$$

$$\sum_{j_1 j_2 j_3} (-1)^{j_1+j_3} <\chi_{pj_1} \| r^2 Y_2(\theta,\phi) \| \chi_{pj_2}>$$

$$\begin{Bmatrix} 2 & 2 & 2 \\ j_1 & j_2 & j_3 \end{Bmatrix} (u_{j_1} u_{j_2} - v_{j_1} v_{j_2})(X_{j_1 j_3} X_{j_2 j_3} + Y_{j_1 j_3} Y_{j_2 j_3}) .$$

Here the notation $<\| \ \|>$ is used for the reduced matrix elements of the single particle quadrupole operator and the $6j$ symbols are denoted by $\{\ \}$.

The monopole component $V^{(o)}$ has only diagonal matrix elements in our restricted basis. These diagonal elements can be absorbed into the single particle energies ϵ^n_j if the same value is assumed for the zero - and the one-phonon states. As we see the monopole term is rather irrelevant from the point of view of the target + neutron problem, it will play an important role, however, in the case of the

matrix elements between bound and continuum states of the target + proton system.

We have performed the numerical diagonalization of the Hamiltonian for a few sets of the single particle energies in order to get the best agreement between the energy eigenvalues ε_i and the experimental spectrum of ^{143}Nd. In this way we have obtained the eigenfunctions for the low lying states of the target + neutron system :

$$| \Phi^{TT}_{Jr} > = \sum_{IjN} C^{Jr}_{IjN} \; | [\phi_I (N) \chi_{nj}]^J > . \qquad (58)$$

The results of the diagonalization, that is, the energy eigenvalues and eigenvectors together - with the experimental energies [17] of the low lying states of the ^{143}Nd are tabulated in the Table IV.

Our next task is to specify the optical model parameters and to solve the optical model problem, that is, to calculate the phase shifts, the regular and irregular solutions as the function of the energy E. First of all we have calculated the average of the experimental cross section and the optical model parameters were obtained by fitting to this smooth cross section. The form and the parameters of the optical model potential are given in obvious notation by the following formulas:

$$V^{opt}(r) = V_{coul}(r) + V_c f(r) + iW_c g(r) + V_{so} h(r) (\ell \cdot \sigma) ,$$

$$V_{coul}(r) = \begin{cases} \dfrac{Z e^2}{2R_c} \left(3 - \dfrac{r^2}{R_c^2} \right) & r \leq R_c \\ \dfrac{Z e^2}{r} & r > R_c \end{cases} \qquad (59)$$

$$f(r) = [1 + \exp((r - R_v)/a_v)]^{-1}$$

$$g(r) = -4a_w \frac{d}{dr} ([1 + \exp((r - R_w)/a_w)]^{-1})$$

$$h(r) = -\frac{1}{r} \frac{d}{dr} ([1 + \exp((r - R_v)/a_v)]^{-1}) \left(\frac{\hbar}{\mu c} \right)^2$$

Table IV. The results of the diagonalization.

J	E_{exp} [17] (MeV)	$E_{cal.}$ (MeV)	C^{J}_{IJN} I = 0 N = 0				I = 2 N = I			
			$^3p_{1/2}$	$^3p_{3/2}$	$^2f_{5/2}$	$^2f_{7/2}$	$^3p_{1/2}$	$^3p_{3/2}$	$^2f_{5/2}$	$^2f_{7/2}$
7/2	0	0	0	0	0	0.9606	0	-0.1451	-0.0419	-0.2334
3/2	0.743	0.743	0	0.8131	0	0	-0.1556	-0.1849	-0.0802	-0.5236
1/2	1.311	1.284	0.8745	0	0	0	0	0.3872	-0.2922	0
5/2	1.560	1.531	0	0	0.7233	0	-0.1659	0.1323	-0.1623	0.6388
5/2	(1.916)	1.867	0	0	0.5860	0	-0.1651	0.0831	-0.1725	-0.7699
7/2	(2.016)	1.877	0	0	0	0.2296	0	-0.0392	-0.0193	0.9723
3/2	(2.131)	2.014	0	0.4527	0	0	-0.1734	-0.2211	-0.0702	0.8433

$$V_o = -55\ \text{MeV}\ , \qquad R_v = 1{,}25\,A^{1/3}\ \text{fm}\ , \qquad a_v = 0{,}65\ \text{fm}$$

$$W_o = -11\ \text{MeV}\ , \qquad R_w = 1{,}25\,A^{1/3}\ \text{fm}\ , \qquad a_w = 0{,}47\ \text{fm} \qquad (60)$$

$$V_{so} = -7{,}5\ \text{MeV}\ , \qquad R_o = 1{,}25\,A^{1/3}\ \text{fm}\ .$$

Using this optical potential we have calculated the phase shifts $\delta_{j\ell}(E)$, $\sigma_\ell(E)$ the regular $v_{j\ell}(r,E)$ and irregular solutions in the 9.0 - 11.5 MeV energy range.

Now all the necessary wave functions being at our disposal we calculate the elments of the T-matrix which in angular momentum representation can be written as follows:

$$< \mathcal{T}_J > = \mathcal{T}_J^{opt} + \frac{1}{2T_0+1} \sum < \tilde{\Psi}_{nJ}^{(-)} \mid V \mid \Lambda_{Jr}^{TT} > (A^{-1})_{rs} < \Lambda_{Js}^{TT} \mid V \mid \Psi_{nJ}^{(+)} > . \qquad (61)$$

The amplitude $\mathcal{T}_J^{opt}$ can be expressed by the phase shifts and it was computed including all the partial waves up to $\ell_{max} = 8$. The second term of $< \mathcal{T}_J >$ must be computed only for the $\ell = 1$ and $\ell = 3$ partial waves because in our model the contribution of other partial waves vanishes.

In terms of our model wave function the matrix elements involved in the second term of $< \mathcal{T}_J >$ are expressed as follows:

$$< \Lambda_{Js}^{TT} \mid V \mid \Psi_{nJ}^{(+)} > = N_{Js} \{ (1 - \frac{1}{2T_0+1}) C_{oJo}^{Js}$$

$$< \phi_0(0) \chi_{nJ} \mid V^{(0)} \mid \phi_0(0)\, \eta_{nJ}^{(+)} > + \qquad (62)$$

$$+ \sum_j C_{2j1}^{Js} < [\phi_2(1) \chi_{nj}]^J \mid V^{(2)} \mid [\phi_0(0)\, \eta_{nJ}^{(+)}]^J > ,$$

where the normalization constant is given by

$$N_{Js} = (1 - \frac{(C_{oJo}^{Js})^2}{2T_0+1})^{-1/2} \qquad (63)$$

Instead of assuming a specific expression for the monopole term $V^{(0)}$, it is enough to introduce an approximation for its "expectation value" $\langle \phi_0(0) | V^{(0)} | \phi_0(0) \rangle$. In order to have the same radial dependence as in the case of the quadrupole component we approximate $\langle \phi_0(0) | V^{(0)} | \phi_0(0) \rangle$ by a simple quadratic expression :

$$\langle \phi_0(0) | V^{(0)} | \phi_0(0) \rangle = K r^2 \quad , \tag{64}$$

where the strength K is treated as a free parameter. Using the expression given by (56) we get:

$$\langle \Lambda^{TT}_{Js} | V | \Psi^{(+)}_{nJ} \rangle = N_{Js} \{ (1 - \frac{1}{2T_0 + 1}) C^{Js}_{0Jo} K \langle \chi_{nJ} || r^2 || \eta^{(+)}_{nJ} \rangle + \sum_j c^{Js}_{2j1} R^{1(2j)1}_{0(0J)J} \langle \chi_{nJ} || r^2 Y_2(\theta, \phi) || \eta^{(+)}_{nJ} \rangle \tag{65}$$

The calculation of the matrix elements A_{sr} defined by the equation (40) is quite straightforward and it is easy to see that both the level shifts Λ_{sr} and level width Γ_{sr} can be obtained as a quadratic expression of the monopole strength K .

Substituting the partial scattering amplitudes $\langle \mathcal{T}_J \rangle$ into standard formulas we have computed the differential cross reaction and the polarization of the elastic proton scattering. We have repeated the computation a few times in order to select out the best value for the monopole strength K .

The calculated cross sections together with the experimental points measured in Heidelberg are plotted on Fig.1. The energy and angle dependence of the polarization is shown on the Fig.2.

It seems to be useless to describe in details the calculations for the nuclei ^{138}Ba and ^{140}Ce. It is enough to refer as an illustration to the Fig.3 which demonstrates the qualitative identity of the cross sections for ^{138}Ba and ^{142}Nd.

5. Discussion

In addition to the calculations described in the previous section we have made a few tests in order to see what we can learn about the IAS. The first problem what we have examined is the importance of the "target ground state + proton" component of the "ideal" IAS. We have repeated the relevant part of the calculation using the wave functions of the "ideal" IAS's instead of the "projected" ones. Comparing the results it turned out that there is no appreciable deviation. Of course, this is not surprising at all, because the weight of the "target ground state + proton" component in the "ideal" IAS's is less the 5%. It is expected, however, that in the case of lighter nuclei where the neutron excess is smaller the situation is quite different and this component is much more important.

The aim of the second test was to examine the coupling of the IAS's to each other via the continuum states. To see the importance of such a coupling we have repeated the calculations neglecting the non-diagonal elements of the A_{sr} matrix, that is replacing the inverse matrix $(A^{-1})_{sr}$ by the simple diagonal matrix:

$$(A^{-1})_{sr} = \delta_{sr} \frac{1}{A_{rr}} . \qquad (66)$$

We have compared the results with the previous ones and only insignificant differences were found. This shows that the coupling among the IAS's completely negligible, they act independently and the scattering amplitude can be written in the usual Breit-Wigner form:

$$\langle \mathcal{T}_J \rangle = \mathcal{T}_J^{opt} + \frac{1}{2T_0 + 1} \sum_r \frac{\langle \tilde{\Psi}_{nJ}^{(-)} | V | \Lambda_{Jr}^{TT} \rangle \langle \Lambda_{Jr}^{TT} | V | \Psi_{nJ}^{(+)} \rangle}{E - (\mathcal{E}_r + \Delta E_0 + \Delta_{rr}) + \frac{1}{2} \Gamma_{rr}} . \qquad (67)$$

Von Brentano et al.[13] have performed the phenomenological analysis of the cross sections for the proton elastic scattering on the isotopes N=82 using such a representation for the scattering amplitude.

We have summarized the results of this analysis together with our results for the ^{143}Nd in the Table V.

Table V. The energies , the total widths and the spins of the isobaric analogue resonances.

E_{res} [MeV]		Γ_{res} [KeV]		J^{π}_{res}	
exp	th	exp	th	exp	th
9.50	9.50	54	30	7/2-	7/2-
10.23	10.27	76	86	3/2-	3/2-
10.80	10.82	75	92	3/2-	1/2-
11.05	11.04	59	44	5/2-	5/2-

Comparing the spin values of the resonances obtained by this fitting procedure with our results a contradiction can be found in the case of the third resonance. Our model gives the spin 1/2 instead of 3/2.

Looking for the possible reason of this contradiction we have increased the dimension of our basis system, namely in addition to the functions $|[\phi_0(0)\chi_{nJ}]^J>$ and $|[\phi_2(1)\chi_{nj}]^J>$ we have included also the functions $|[\phi_0(2)\chi_{nJ}]^J>$ $|[\phi_2(2)\chi_{nj}]^J>$ and $|[\phi_4(2)\chi_{nj}]^J>$, which correspond to the two-phonon states of the target. Varying the parameters within reasonable limits we have diagonalized the Hamiltonian. It turned out that our model is unable to produce a 7/2, 3/2, 3/2 spin sequence.

In favour of our model perhaps it is worth while to mention that the angular distributions of inelastically scattered protons measured at the second and third resonances are quite different. In the case of the third resonance the distribution is almost isotropic which is expected if the spin is 1/2 [18]. On the other hand it is obvious that our model is oversimplified and a more sophisticated model would be able to explain the spin requence suggested by the phenomenological analysis of the cross section. In any case it would be highly desirable to perform a direct experiment in order to get an unambiguous spin assignment for

the third resonance. It seems to us that the best tool for this purpose is the proton polarization measurement. As it can be seen on the Fig.2 the sign of the polarization is opposite for $J = \ell + 1/2$ and $J = \ell - 1/2$ and it is enough to measure only the relative sign of the polarization at the different resonances.

This possibility of the unambiguous spin assignment was pointed out recently by Adams et al. [19]

Finally we want to emphasize that the method discussed in this paper can be applied for more realistic residual interactions and for more sophisticated models without difficulty. Of course, much more computer time is required and to perform such an extensive calculation seems to be reasonable only in such case where we have more detailed and more reliable information about the circumstances e.g. about the single particle energies, about the character of the excitations of the target and target + neutron system.

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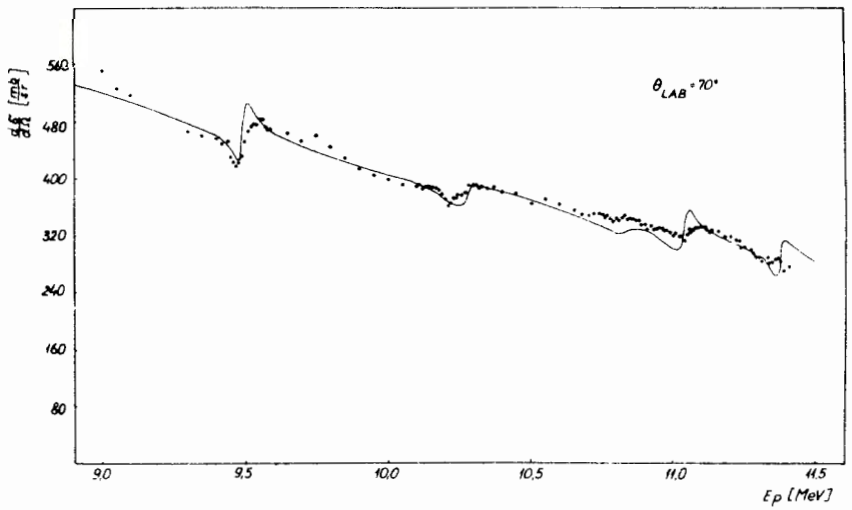
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9,5
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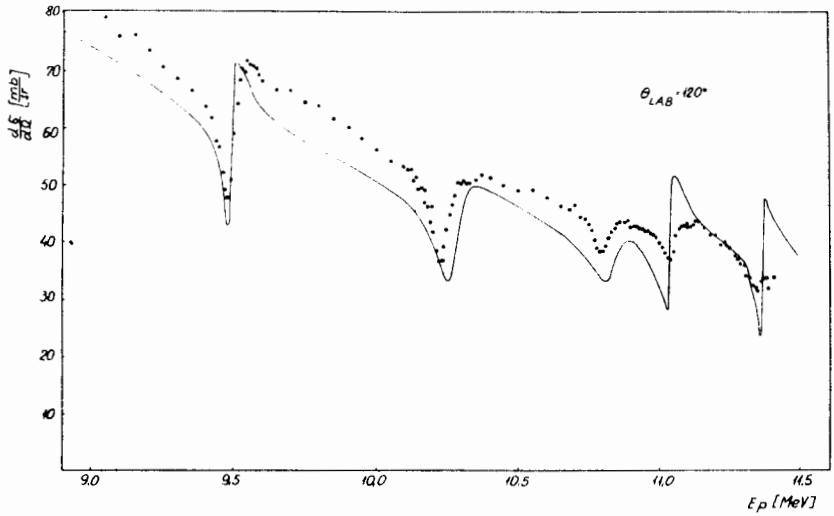
10,5
11,0
11,5
Ep [MeV]

a)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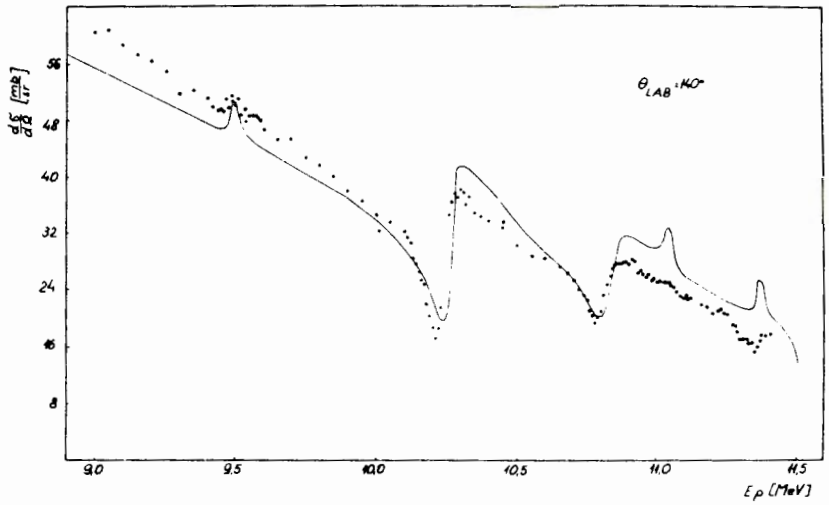
Ep [MeV]

b)



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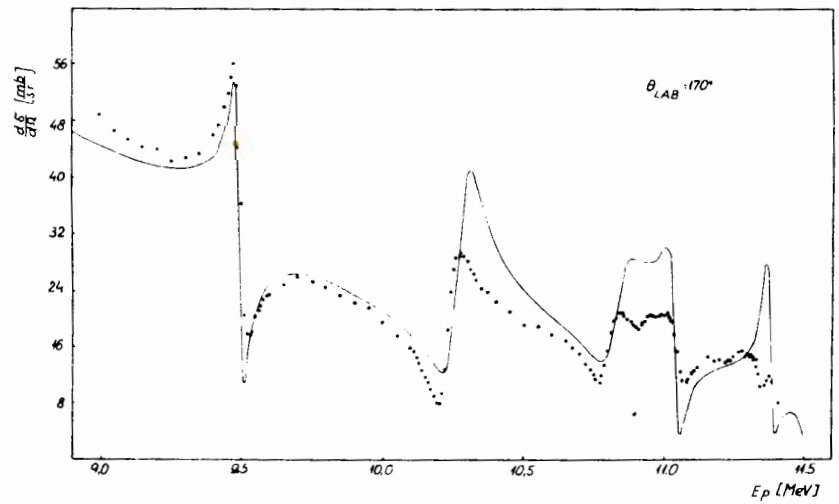
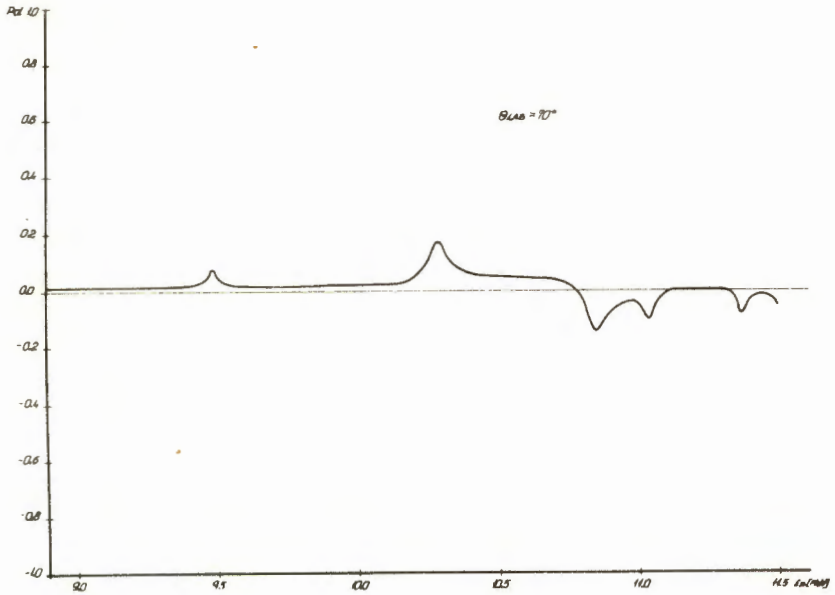


d)

Fig.1. The energy dependence of the cross section of the elastic proton scattering on ^{142}Nd at different angles. The experimental points were measured by the Heidelberg group [13].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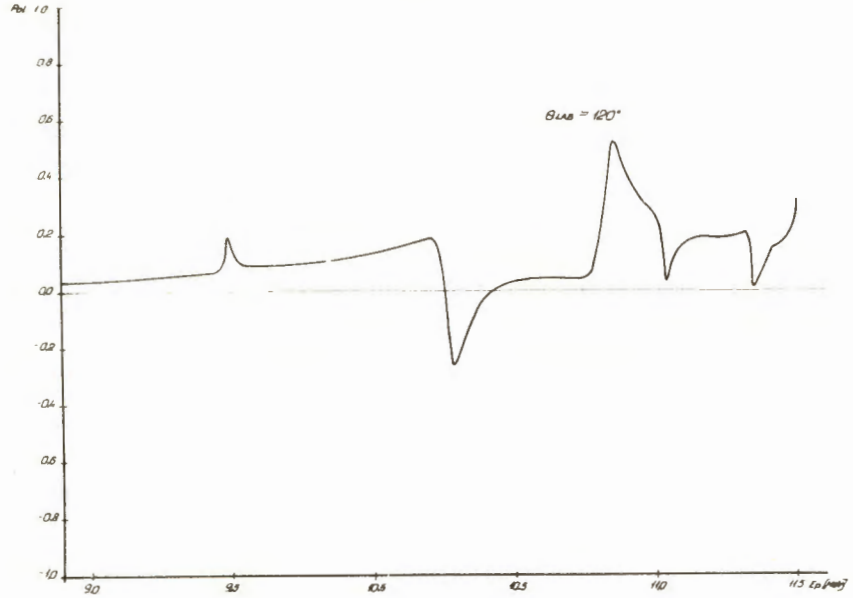
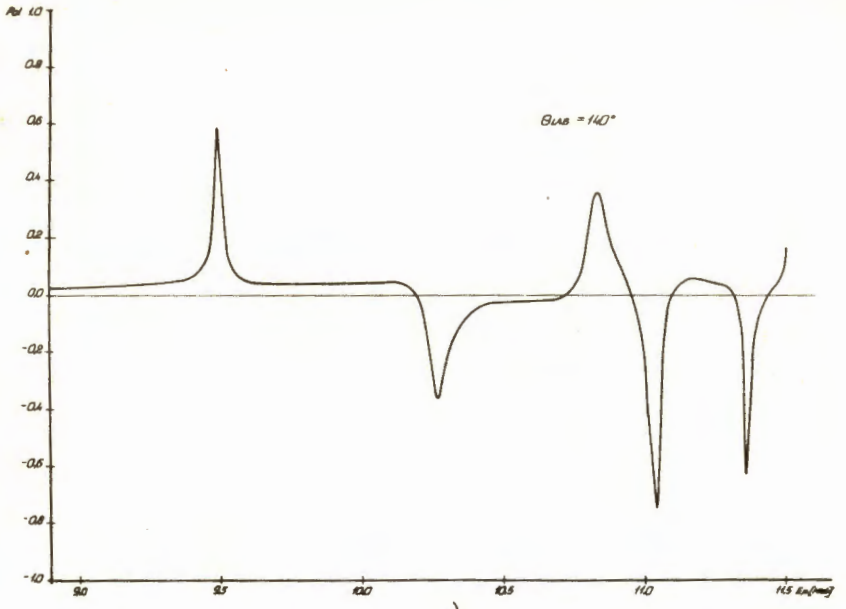
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
$\theta_{LAB} = 90°$
9.0
9.5
10.0
10.5
11.0
11.5 Ex[MeV]

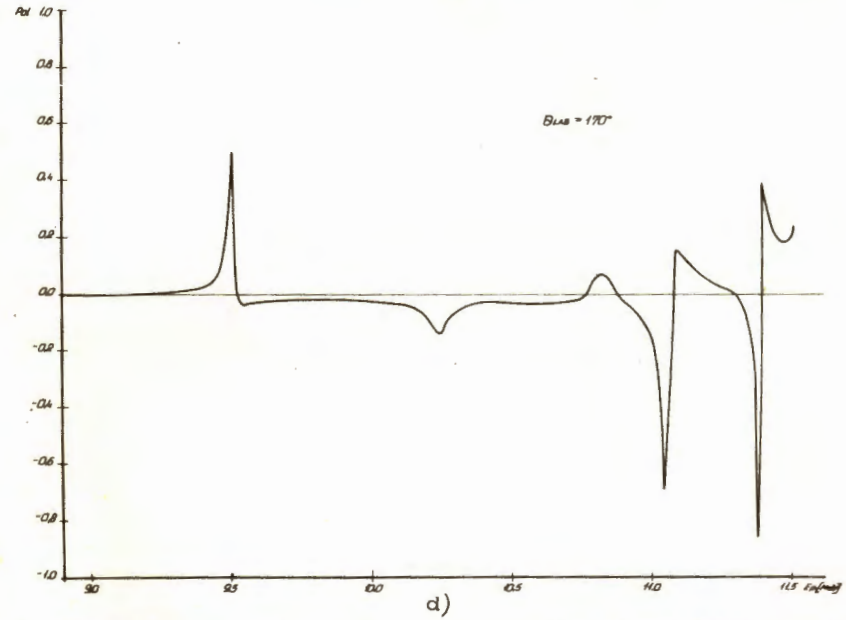
a)


Pol 1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
$\theta_{LAB} = 120°$
9.0
9.5
10.0
10.5
11.0
11.5 Ex[MeV]

b)

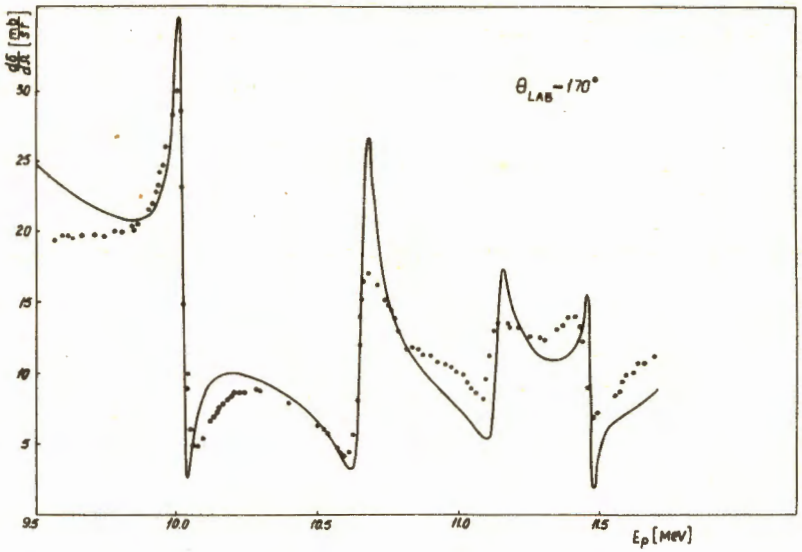


c)



d)

Fig.2. The energy dependence of the polarization of the elastic proton scattering on ^{142}Nd at different angles.



Fig,3. The energy dependence of the cross section of the elastic proton scattering on ^{138}Ba at 170°. The experimental points were measured by the Heidelberg group [13].