ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2 - 3511

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ON THE DISPERSION SUM RULES IN THE THEORY OF STRONG INTERACTIONS

(Talk at the International Theoretical Physics Conference
on Particles and Fields
Rochester, August 28 - September 1, 1967)

1967.

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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A great deal of attention has been paid recently to the dispersion sum rules which are based on superconvergence /10-13/ and on the Regge pole hypothesis /15-16/. We discuss here some problems concerning applications of these sum rules.

Derivation of Dispersion Sum Rules

The derivation of these rules is very simple. For the amplitude $f(\nu)$ which is analytic in the ν-plane with a cut along the real axis, we can apply the Cauchy theorem with the contour going in the upper half-plane along the real axis and along the semi-circle C_A of a big radius A (Fig. 1)

$$\int_{-A}^{A} f(\nu)\, d\nu + \int_{C_A} f(\nu)\, d\nu = 0 \tag{1}$$

If for large ν the amplitude $f(\nu)$ decreases rapidly enough (for instance, as $\nu^{-1} \ln^{a} \nu$, $a < -1$, or quicker) so that the integral over C_A tends to zero as $A \to \infty$, then we obtain a superconvergence sum rule, the imaginary part of which (more convenient for applications) is of the form

$$\int_{-\infty}^{\infty} \mathrm{Im}\, f(\nu)\, d\nu = 0. \tag{2}$$

If the amplitude does not decrease, then to obtain a sum rile it is necessary to calculate the integral over C_A. That can be done if we know the behaviour of $f(\nu)$ at large ν. Since at energies above 5 GeV the amplitude can be represented, within experimental accuracy, as a sum over Regge poles[55] $f(\nu) = \sum_i b_i \nu^{\alpha_i}$, then, substituting this expression into the integral over C_A we get a generalization of expression (2)

$$\int_{-A}^{A} \mathrm{Im}\, f(\nu)\, d\nu - \sum \frac{A^{\alpha_i+1}}{\alpha_i+1} \mathrm{Im}\, b_i (1+e^{i\pi\alpha_i}) = 0, \quad A = 5 \text{ GeV}. \tag{3}$$

In this derivation, we followed refs.[17,18]. The sum rules (3) are sometimes called finite[20], sometimes - divergent[19]. We shall call them the Regge sum rules.

Generalizations

The superconvergence relations have different generalizations. Following papers of Faustov, Pisarenko and Kallosh[21,22] consider, for expample, an invariant amplitude $f(\nu, q^2)$ describing scattering of virtual isovector photons with mass q^2 and assume that the superconvergence condition is safisfied by the difference $f(\nu, q^2) - f(\nu, 0)$. The superconvergence relation for this difference

$$\int_{-\infty}^{\infty} [\mathrm{Im}\, f(\nu, q^2) - \mathrm{Im}\, f(\nu, 0)]\, d\nu = 0 \tag{4}$$

is the Cabibbo-Radicati[23] relation for form-factors. This method was used by the authors of paper[14] for the real Compton scattering on mesons with isospin 2 in the t -channel. It yielded the following sum rule[14]

$$\frac{1}{m_\pi^2} = \frac{1}{2\pi^2\alpha} \int_{\nu_0}^{\infty} [\sigma(\gamma\pi^0) - \sigma(\gamma\pi^+)\, d\nu\,. \tag{5}$$

Another derivation of this and similar relations was proposed by Pagels[24] and Harari[25]. It would be of interest to clear up how well these relations are fulfilled.

A number of interesting papers are devoted to the discussion of these relations[26-34], in particular, their connection with perturbation theory[32]. I only note that the superconvergence relations are in general incompatible with finite order perturbation theory.

The superconvergence sum rules allow generalizations not only on the account of the information about the high-energy behaviour of the amplitude as it is in the Regge sum rules (3). If we know in a certain region the real part of the amplitude, then by multiplying the amplitude $f(\nu)$ by properly chosen function $\psi(\nu)$ which is analytic in the ν-plane with cuts along the real axis and which decreases at infinity we obtain a sum rule[16]

$$\int_{-\infty} \mathrm{Im}(f\,\psi)\, d\nu = 0 \qquad (6)$$

which relates the real and imaginary parts of $f(\nu)$ or a sum rule only for the real part. The sum rules of such a type are treated in a number of papers[35-39,18].

If the amplitude is known at a certain point, e.g., the Compton scattering amplitude at threshold[3], then one can take $\psi(\nu)=\nu^{-1}$ and obtain sum rules for the Compton scattering[1,4,5,40]. Relations of this type include the sum rules for the scattering lengths, which are known from the very beginning of the dispersion relation theory[1,2], Adler-Weisberger[7] relations and other sum rules obtained by algebra of currents[6], quark model[41] or assumptions about scattering lengths[42,130] and other relations[124]. Mention should also be made of the sum rules for amplitudes at a fixed angle[14] and for partial amplitudes[43].

Superconvergence Relations

The superconvergence sum rules have been known long ago and were first used in a paper by Logunov and Soloviev[9] to demonstrate a co-existence of two sets of dispersion relations for virtual photoproduction which correspond to two different expansions of the amplitude into invariants[8,9]. Note that recently De Alfaro, Fubini, Furlan and Rosetti[44] made use of superconvergence relation in a similar way to show the co-existence of the linear and quadratic mass formulae for bayons following from current algebra.

In the summer of 1965 N.N.Bogolubov, when analysing interesting results for the magnetic moments obtained by Fubini, Furlan and Rosetti[6] by means of the algebra of currents, noticed that they can be obtained from ordinary one-dimensional dispersion relations without any commutators. In particular, he pointed to a possibility of using for this purpose the sum rules for the Compton scattering. The superconvergence sum rules for photoproduction[10] were also used for this purpose. After that the sum rules for meson-baryon scattering were considered[11]. The Italian physicists - Fubini and Segre[12], and De Alfaro, Fubini, Furlan and Rosetti[13] obtained the superconvergence sum rules starting from the algebra of currents. The latter authors[13] proposed to use the Regge pole hypothesis as a criterion for choosing superconvergent amplitudes.

The interest to superconvergence relations is due to the fact that, in the resonance approximation, they give relations between the coupling constants of particles and resonances which are similar to higher symmetry relations. It is obvious that the superconvergence relations by themselves yield not much information (they are present both in the Lee model[45] and in the static theory[47,46]). They are a simple auxiliary tool allowing to make dynamical assumptions which, on the one hand, lead to symmetry relations, and, on the other - permit, generally speaking, a straightforward experimental check. Thus, they make it possible to relate different facts of the strong interaction dynamics, and if combined with experiment, as well as with different models, these relations may shed some light on the problems of dynamical symmetries.

The first approximation in superconvergence relations consists in neglecting the non-resonance background. We known that the contributions of higher resonances decrease when their masses grow because their elasticity exponentially goes to zero. Therefore the saturation of the sum rules by baryon resonances can be checked by a direct calculation. Note that for meson resonances it is still impossible to do even that. But besides resonances there is also a non-resonance background. (Fig. 2). Is it so small for superconvergent amplitudes as to give a negligible contribution after integrating over a large interval, say, from I GeV to infinity? Unfortunately, we do not know a direct answer to this question for any superconvergence amplitude and can judge of it only indirectly, for instance, by consistency of the obtained predictions with a symmetry in which we believe. The point is that there is no superconvergent amplitude for the pion nucleon forward scattering, the only process for which such an estimate of the background can be done at present from experiment. Such an estimate could be done, in principle, for photoproduction, and partially, - for nucleon-nucleon and nucleon-antinucleon scattering, for pion-nucleon backward scattering and for nucleon-antinucleon annihilation into two mesons.

The next question concerns the magnitude of the momentum transfer t for which an approximate saturation of superconvergence relations is considered. The problem of exact saturation of the sum rules for all t is very complicated; it wad discussed in a number of papers [44,60-66,111] and will not be considered here. Very often t is chosen to be zero. However, there seems to be no reason for asserting that in this case the background or the contributions of far resonances are especially small. As far as the sum rules for derivatives in t at $t = 0$ are concerned, the role of the higher states in them is enhanced and they are, as a rule, less reliable [46,67-69]. In the spin-flip amplitudes the contribution of a Regge pole is proportional to the Regge trajectory $\alpha(t)$ and if $\alpha(t)$ vanishes at a certain t, then at this value of t the contribution of the non-resonance background is suppressed. For instance, if the Regge pole contribution to an amplitude is proportional to $\alpha_{\rho}(t)$, then the background for this amplitude should be suppressed for $t = -0.6$ GeV^2 [70]. Dolen, Horn and Schmidt [71] have shown that the background of amplitudes which are not superconvergent can be also strongly suppressed in this case. It is clear that the choice of $t = 0$ is not the only one. At any rate the choice of the value of t is a part of the assumptions used in the analysis of the sum rules and deserves a further study.

It should be also borne in mind that the coupling constants of resonances are usually determined from the sum rules in the approximation of infinitely narrow resonance. Other definitions of these constants, especially for nearest and wide resonances, could change their numerical values by 30%.

Consider some results obtained by means of the superconvergence sum rules.

Relations between Meson-Baryon Constants

These relations follow from superconvergence sum rules for meson-baryon scattering.

For the fixed $t = 0$ the only superconvergent amplitude of the meson-baryon scattering could be the amplitude $B^{(2)}$ (in the usual notations of ref.[48]) corresponding to the exchange of the isospin 2 in the t -channel, since it is possible to assume that the trajectory of the corresponding reggeon $\alpha(0) < 0$.

However, as Musinich[75] and Phillips[76] pointed out the exchange of two ρ -reggeons could give $\alpha(0) = 2\alpha_\rho(0) - 1 = 0.15 > 0$. A study of the reaction $\pi^- p \to \pi^+ N^{*-}$ could be a direct check of this circumstance. In the $SU(3)$ - symmetry approximation this can be done by studying reactions involving the exchange of the 27-plet. The available meagre data indicate that probably $\alpha_{27}(0) = -0.6 < 0$[77]. Assuming that the amplitude $B^{(2)}$ of the process $\pi^+\Sigma^- \to \pi^-\Sigma^+$ is superconvergent and taking into account all the known resonances Babu, Gilman and Suzuki[74] have obtained a relationship between the constants $g^2_{\pi\Lambda\Sigma}$ and $g^2_{\pi\Sigma\Sigma}$ corresponding to $d/f = 1.3 - 1.7$ i.e., to $SU(6)$ symmetry[79]. The same result has been obtained, in the $SU(3)$ - symmetry approximation, by Sakita and Wali[60], and by Altarelli, Bucella and Gatto[67]. The authors of the latter paper estimated this ratio also for the trajectory N_γ: $d/f = 2$ by assuming that d/f is the same along the Regge trajectory and considering the sum rule for the derivative in t. A general result of this consideration is that it is possible to obtain some symmetry relations for lower resonances from superconvergence. However, higher resonances are not indifferent to this symmetry: many of them give appreciable contributions, but all these contributions cancel each other.

It is also instructive to consider the original derivation of the sum rules for meson-baryon scattering which was done in refs.[11,72]. The superconvergent amplitudes were chosen not according to the exchange of reggeons in the t -channel but according to some property of the amplitudes in the direct channel. Namely, it was assumed that all the amplitudes B, into which the unitary singlets in the direct channel give no contribution, could be superconvergent. The simplest octet-decuplet approximation in the sum rules then led to relations between meson-baryon coupling constants corresponding to $d/f = 3$ and to a rather accurate relation between the pion-nucleon coupling constant and the width of the 33-resonance. The earlier determinations of the $KN\Lambda$ and $KN\Sigma$ coupling constants[78] did not contradict the ratio $d/f = 3$ (by the way, this ratio corresponds to a quark model in which the axial current is not additive in quarks and has the structure of the anomalous magnetic moment[73]). But the recent analysis of Kim[78,126] seems to give $g^2_{Kp\Lambda} = 16.0 \pm 2.5$ and $g^2_{Kp\Sigma^0} = 0.3 \pm 0.5$. These numbers agree quite well with the SU (6) value $d/f = 3/2$ and definitely contradict $d/f = 3$. This result, if confirmed, means that the direct channel criterion for superconvergence is in general not correct. Nevertheless, it gives a good result for pion nucleon scattering. It is easy to see[81,11,50-54] that the reason for this good result is the following (i) the contributions of the nucleon and the 33 resonance in the sum rule compensate each other, (ii) the contributions from all known higer resonances enter the sum rule with alternating signs and compensate one another, (iii) consequently, the contributions of the Regge asymptotics in eq. (3) and the middle-energy background also compensate each other (see Fig. 3 for a schematic behaviour of $\mathrm{Im}\, B^{(+)}$ for pion-nucleon scattering). We see, that a symmetry of low-lying states can correspond to the mutual compensation of the contributions of higher states not only in the superconvergence sum rules but also in the Regge sum rules for non-superconvergent amplitudes.

If for the Regge term in eq. (3) use is made of the available data[55] then it turns out that for pion-nucleon scattering the background contribution up to 5 GeV (or less) is of the same order as the contribution of the nucleon or the 33 resonance. This means that taking into account the Regge term in eq. (3) for the amplitude $B^{(+)}$ of pion-nucleon scattering we cannot restrict ourselves to the resonance approximation in the integral of this sum rule as it has been demonstrated in refs.[56,57].

On the other hand, the compensation of the background and Regge asymptotics contributions for this amplitude has led some authors[50-53] to the suggestion that such a compensation always takes place, i.e. the sum of the resonance contributions is always equal to zero. One can check the mutual compensation of higher resonance contributions (partly, because not all partial widths are known now) for $\pi\Lambda$ and $\pi\Sigma$ scattering[11,81], but as we have seen above the total compensation of all one-particle and resonance contributions in both amplitudes for $\pi\Sigma$ scattering cannot take place simultaneously. Moreover, this compensation strongly depends on the spin properties of amplitudes. For instance, it cannot appear in the sum rules for the amplitude $\nu A^{(+)} + \nu^2 B^{(+)}$ or $\nu^{-1} A^{(+)} + B^{(+)}$ of pion nucleon scattering[57].

Thus, the compensation of higher state contributions depends on spin and isospin properties of the amplitudes. It would be interesting to understand whether this compensation is accidental or not.

Relations between Meson-Baryon and Meson-Meson Constants

Relations between these constants can be obtained from the sum rules for baryon-antibaryon annihilation into two mesons. However, we know very little about higher meson resonances. Besides, the coupling constants of meson resonances, say ρ with nucleon, are known only indirectly and may have noticeable errors which are essential in the sum rules since the ρ-meson gives a large contribution to the sum rule for the amplitude B. This is true to a greater extent for the coupling constants of mesons with other baryons.

The sum rules for annihilation processes were first considered by Matveev[58] who obtained a d/f ratio for vector meson-baryon coupling constants and a relationship for the coupling constants of ρ meson with nucleon and pion which agrees with available data.

The asymptotics of these processes is determined by the baryon Regge poles. Beder and Finkelstein[59] have noticed that although the amplitude of nucleon-antinucleon annihilation which corresponds to the exchange of the nucleon reggeon is not superconvergent, nonetheless, in the Regge sum rule (3) the Regge term for it vanishes due to the positive signature of this reggeon and we have a superconvergence sum rule with the Cauchy principal value of the integral at infinity. This is a particular case of the asymptotic crossing symmetry for annihilation processes. The integral term of this sum rule is not identically zero since the crossing symmetry is only asymptotic. It is possible to admit that the contribution of non-resonance background affected by the Regge asymptotics, is also considerably weaken in this integral. This makes it possible to consider the integral in the resonance approximation. The sum rule for the amplitude B gives a good relation between ρ meson coupling constants. The sum rule for the amplitude A points to an appreciable contribution of the $\pi\pi$ -interaction in the S wave. The sum rules for the annihilation amplitude B of different baryons were treated in detail by Dass and Michael[51] with the account of all known baryon resonances. The sum rules for nucleon are well fulfilled and allow to estimate the ρ -nucleon coupling constant.

For the Σ -hyperon, however, more accurate estimates seem to be necessary. The authors interpret the obtained result as due to a large contribution from $\pi\Sigma$ -scattering with isospin 2.

Electromagnetic Constants of Baryons

Relations between the magnetic moments and radiative decay constants of baryons are obtained from the sum rules for meson photoproduction on baryons if the meson-baryon constants are known, or - directly, from the sum rules for Compton scattering.

The photoproduction processes were treated in the above-mentioned papers by the Dubna authors[11,72] and in a paper by Pisarenko[82]. In these papers, those sum rules were chosen which are known from the low-energy dispersion theory and correspond to the unsubtracted dispersion relations for virtual photoproduction[9].

It should be noted that if small longitudinal multipoles and the meson mass are neglected then these sum rules go over into the sum rules of Fubini, Furlan and Rosetti[6] obtained with the help of the current algebra.

Considering only those channels to which the unitary singlets do not contribute and leaving the baryon octet and decuplet in the intermediate states, the authors of ref.[11,72] have obtained relations between the anomalous magnetic moments of baryons and the magnetic moments of the radiative decays of resonances, which are close numerically to the SU (6) results[82], and a relation between anomalous magnetic moments of baryons $\mu'_d / \mu'_f = 3$ (or $\mu'_p + \mu_n = 0$). An account of the next nucleon resonance does not change these results[6,83]. They agree with the available experimental data on nucleon-33-resonance constants[11,80]. In the derivation of these results use was made of the meson-baryon constants corresponding to $d/f = 3$. However, as Aznauryan[49] has shown, these results are practically independent of the meson-baryon constants. This follows from the consideration (in the same resonance approximation) of the sum rules for the Compton scattering[4,5,40] (sometimes called the Gerasimov-Drell-Hern sum rules)

$$\frac{4\pi^2 \mu'^2}{S} = \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} (\sigma_P - \sigma_A), \qquad (7)$$

where μ' and S are the anomalous magnetic moment and spin of the particle, and $\sigma_{P(A)}$ is the total cross section for the interaction of this particle and the photon with parallel (antiparallel) spins. In doing this, as earlier, the channel with the unitary singlets was not considered. If this channel is taken into account in the Compton scattering or photoproduction, then, as Pais[84] and Cini, De Maria and Taglienti[85] noticed, we can obtain a non-trivial solution for the constants, only if we take into consideration the unitary singlet resonances. These authors introduced the notion of the minimum(in the unitary spin) set of states which should be taken into account in the sum rules to get a non-trivial solu-solution. In this case this set consists of an octet, decuplet and a singlet.

Beg and Pais[86] have analysed this notion in application to the generalization of the sum rules (7) to the isovector and isoscalar photons, obtained by Beg[87], and to the sum rules for photoproduction. We'll come back to the sum rules (7) and try to clear up whether it is possible in them to restrict ourselves to the singlet, lowest octet and lowest decuplet, neglecting higher octets and decuplets. We have seen that the mutual compensation of higher resonances may be essential. At present we know very little about electromagnetic constants of higher resonances. However, in the given case it is possible to establish the sign of the contribution of each resonance to the sum rule (7), assuming that in the radiative decays of the baryon resonances the photons with mimimum possible momentum are predominant. The available data on three nucleon resonances do not contradict this assumption [88].

Then it turns out that both the octet and decuplet higher resonances enter the sum rules (7) with alternating sings and may mutually weaken each other. In this case the considered minimum set may prove to be good. Since the account of one lowest singlet Λ (1405) does not suffice (it gives a negative contribution to the sum rule (7), while in the sum rule for photoproduction it would correspond to a great width of the decay $\Lambda(1405) \to \Lambda\gamma$) we take into account the next candidate for the siglet assignement Λ (1520). Higher singlets are probably negligible due to elasticity. This allows to estimate the decay widths $\Gamma(\Lambda(1405) \to Y\gamma) \approx 1\%$ and $\Gamma(\Lambda(1520) \to Y\gamma) \approx 10\%$ (in per cent to the total widht). The second width turns out to be rather appreciable. Obviously, this result is essentially a working hypothesis, because we are not at all sure that Λ (1520) is a pure singlet.

The superconvergence relations for photoproduction based on the Regge pole model have been treated in many papers [61,68,69,89-93]. The process $N\bar{N} \to \pi\gamma$ has been considered in ref. [94]. There are much more relations for photoproduction than for scattering and the situation here is at present rather complicated. A number of relations in the octet-decuplet approximation yields results in agreement with symmetry and experiment. At the same time 6 out of 7 relations corresponding to the exchange by $10, \overline{10}$ and 27 multiplets in the t channel turn out to be contradictory. It would be interesting to consider unitary singlets in these relations. On the other hand, these results as well as those of Pisarenko [95], who treated the saturation of the sum rules by the 33-resonance for all the amplitudes of the virtual photoproduction, are likely to point out that in this approach not only the superconvergence but also higher resonances affect noticeably the symmetry of lower resonances.

Other Relations

The superconvergence relations based on the Regge pole hypothesis were treated for many other processes. In some cases it was simply shown that the sum rules can be fulfilled, in other cases estimates were given for unknown decay constants, or a comparison was made with predictions of different models. The following scattering processes were considered: $\pi\rho$, πK^*, $K\rho$, [13,46,70, 96–99] πA_1 [100], ρN [101], $\pi N \to \rho N$ [102] πN^* [103,104], NN [105–108].

In a paper by Ademollo et al.[70] it has been obtained that the decay $A_1 \to \rho\pi$ should be the s -wave one. This result comes from the sum rules at $t = -0.6$ GeV^2, for which the Regge pole contribution vanishes, and under certain additional assumptions.

If should be also pointed to an application of the superconvergence relations to obtain mass formulae without using the algebra of currents. which was done by Faustov[113]. This result confirms the general statement that all the results of the algebra of currents obtained by means of zero commutators [ ] = 0 can be got with the help of superconvergence relations.

In papers by Oehme et al.[109–111] it was shown how the collinear group U (6) and higher symmetry relations can be obtained from superconvergence. The algebraic structure of the superconvergence relations is also treated in refs.[112,62] under different assumptions.

Use of Asymptotic Symmetries

The superconvergence relations make it possible to use symmetries, which are fulfilled only at high energies. If at high energies the amplitudes of different processes are related through some symmetry relations, one can form combinations which would satisfy superconvergence relations. In the resonance approximation they lead, as usual, to relations between the low-energy constants. Such a possibility of transportation of symmetry was first considered in a paper by Matveyev, Struminsky and Tavkhelidze[72]. Costa and Zimerman[114] considered a simple example of $\pi\pi$ and πK scattering with isospin 1 in the t -channel. If the ρ -reggeon vertex describing these processes at high energies obeys the SU (3) symmetry, then at high energies $f^{(1)}_{\pi\pi} = 2 f^{(1)}_{\pi K}$ and therefore the difference $f^{(1)}_{\pi\pi} - 2 f^{(1)}_{\pi K}$ satisfies the superconvergence relation. Its saturation by ρ and K^* -mesons yields a relation between their widths which takes into account their masse difference and agrees with experiment. The quark model or the Regge pole model with a symmetry give many relations between the amplitudes of different processes at high energies which are given e.g. in refs. [115-116]. This approach is developed in a paper of Kadyshevsky, Mir-Kasimov and Tavkhelidze [17]. It seems most attractive to apply the asymptotic symmetries to the Green functions and form-factors, because they have only one variable what makes the matters much simpler. Dass, Mathur and Okubo [117] applied this approach to the propagater functions (i.e. to the Fourier - transforms of $\langle 0 \mid T(j^A j^A) \mid 0 \rangle$) and $\langle 0 \mid T(j^V j^V) \mid 0 \rangle$ for the vector and axial currents, assuming that at infinity these functions obey the symmetry $SU(2) \times SU(2)$, and obtained the Weinberg sum rules for the spectral densities. Owing to the conservation laws only the states with unit spin and isospin give contribution to the intermediate states in these sum rules. One can expect, therefore, that these sum rules are well saturated by lower resonances ρ and A_1.

The use of the ρ -meson approximation in the formfactor of the $\pi_{\ell 3}$ decay along with the data on the widths of the $\rho \to \pi\pi$ and $\pi_{\ell 3}$ decays leads to the well-known Weinberg relation [118] for the A_1 -meson mass $m_{A_1} = \sqrt{2}\, m_\rho$. In the same way the asymptotic symmetry $SU(3) \times SU(3)$ leads to an estimate [117] for the mass of the strange axial-vector K_A - meson to be 1311 MeV, what is close to the mass 1313 + MeV of the observed $K\pi\pi$ resonance. Knowing only the masses of K_A, A_1 and ρ - mesons it is possible to obtain [119] a ratio of the constants for the $K_{\ell 2}$ and $\pi_{\ell 2}$ decay to be $F_K/F_\pi = 1.17$, in agreement with experiment. Finally, as Pandit [120] has shown, the application of the asymptotic $SU(3)$-symmetry to the form-factors of the $\pi_{\ell 3}$ and $K_{\ell 3}$ decays gives a superconvergence relation, whose saturation by ρ and K^* -meson gives for the $K_{\ell 3}$ decay constant $F_+(0) = - m_\rho^2 / \sqrt{2}\, m^2_{K^*}$.

However, the assumption about saturation here seems to be not so fortunate.

Applications of Regge sum rules

Until now we discussed a possibility to use sum rules for obtaining some information about low-energy constants starting from superconvergence high energy behaviour. We'll see now whether it is possible to use the information concerning the low-energy processes in the analysis of the processes at high energies by means of the Regge sum rules. This question was first considered in a paper by Restignoli, Sertorio and Toller[15], and by Logunov, Soloviev and Tavkhelidze[16], as well as in papers of other authors[121,20,53]. In refs.[122-125,71,127-129] some further applications of the Regge sum rules have been considered for the analysis of $\pi N, KN$ and NN -scattering. The Regge sum rules are the simplest consequence of the dispersion relations and the Regge pole hypothesis. If we believe that the Regge pole model works starting from a certain energy A, then, in principle, using only the data at lower energies we can calculate all the Regge parameters with the aid of dispersion relations. In the language of the sum rules this implies the using of the sum rules for $\nu^n f(\nu)$ with different n's. It is clear, however, that for large n the key role would belong to the data in the vicinity of A, and their accuracy would be a determinant one. The simplest sum rules with small n allow to use low- and medium-energy data where the accuracy is better and where the complete phase-shift analysis has been done. The different energy regions give the following contributions[122] to the integral up to 5 GeV in the sum rule for the forward πN amplitude $A^{(-)}+\nu B^{(-)}$ $(\nu A^{(+)}+\nu^2 B^{(+)})$.

up to 2 GeV	up to 3 GeV	up to 5 GeV
13%,(8%)	40%(20%)	100%(100%)

It is interesting to clear up how the role of these contributions changes at $t \neq 0$. In any case, good data in the middle energy region 3-4 GeV would be very useful in the analysis of the Regge parameters by means of the sum rules. This analysis is likely, even now, to allow a discrimination of some models[121] or a separate determination of some Regge residua[71], what cannot be done by using the available data at high energies. Finally, these rules are helpful for a qualitative consideration of the high-energy parameters and for relating their properties with those of the low-energy baryon resonances. So, if we take $A = 1$ GeV, as it was done by Dolen, Horn and Schmidt /71/, then the properties of the Regge parameters and those of high-energy scattering, say a vanishing of the ρ trajectory, may be associated with mutual cancellation of the nucleon and the 33-resonance contributions.

In conclusion, I am in a position to say that the sum rules are a useful auxiliary tool. Their consideration is to stimulate a study of higher resonances, as well as scattering in the middle-energy region. I think that it would be interesting to proceed with considering the superconvergence relations for photoproduction, as well as with analysing more throughly the choice of the t -value in the sum rules.

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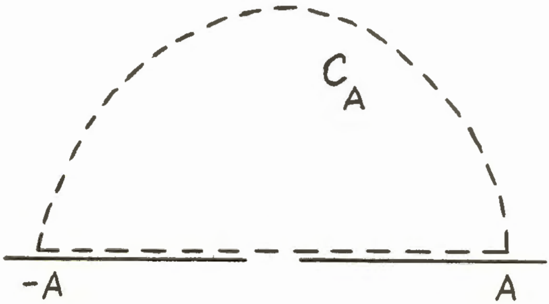
Received by Publishing Department
on September 14, 1967.



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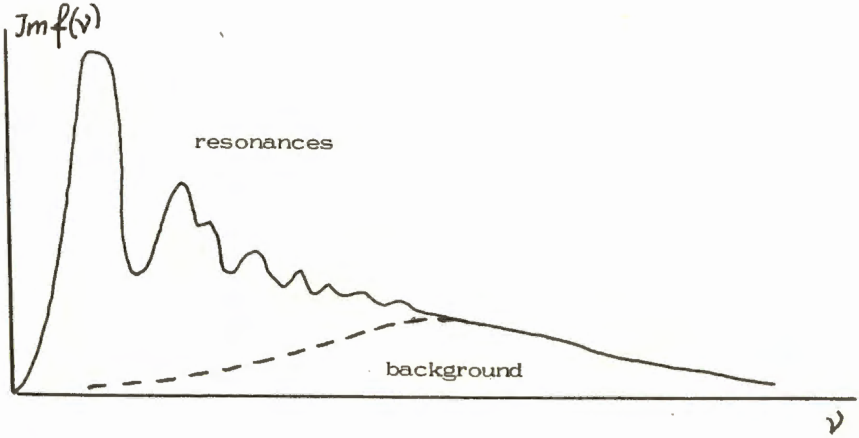

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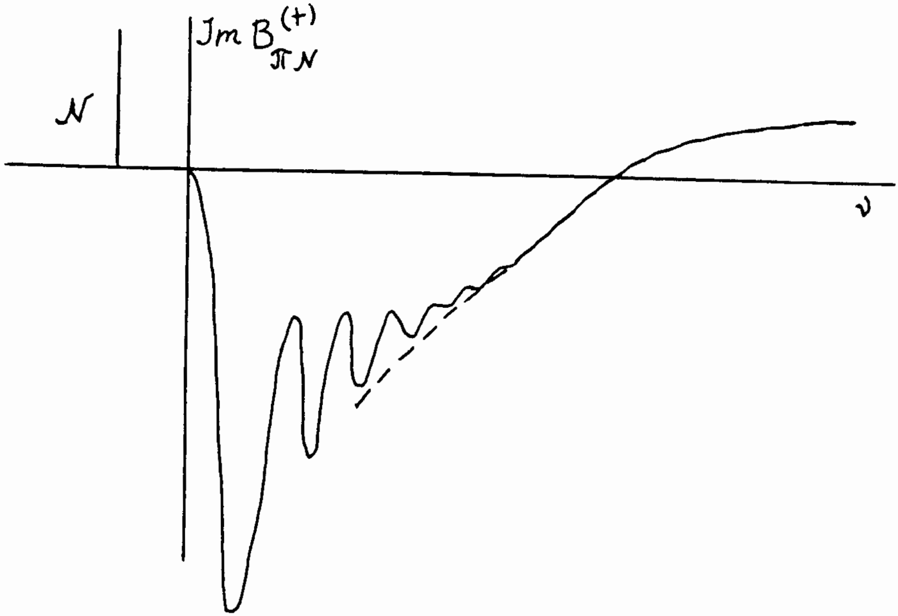

Fig. 3.