ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2 - 3469

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REMARKS ON CURRENT ALGEBRA AT INFINITE MOMENTUM

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1967.

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To be submitted to JETP

I. Introduction

The method of "infinite momentum frame of reference"[1,2] is not only useful for a derivation of exact sum rules from the current algebras, but allows a purely algebraic discussion of it in terms of (perhaps infinite) multiplets of one-particle states with infinite momentum.[2,3] These considerations are of great interest promising a deeper understanding of the origin of broken symmetries.

The main dynamical assumption included in such a calculations is the possibility of exchanging the limit $p \to \infty$ with the sum over intermediate states (SIS). Following the analogy of the dispersion[4] and "$p \to \infty$" approaches one usually expects such exchange to be allowed at least for the case of the algebras of "good operators"[2] like the chiral and (or colinear) $SU(3) \times SU(3)$. Sum rules obtained from these algebras are then manifestly covariant and moreover, when saturated by low-lying one-particle states (at infinite momentum) they contain results arising from the bad algebras like $SU(6)$ considered at rest.[5,6]. Because of this the restriction to the chiral algebra $SU(3) \times SU(3)$ of good charges (or its infinite-dimesional extension in the case of current densities) is considered to be sufficient and without any contradiction with relativity.

However from a more general algebraic point of view the full incorporating of relativity means an inclusion of the whole Poincaré algebra[7] and considering it together with the algebras generated by weak and electromagnetic currents. Once the Poincaré algebra is included, the consideration of algebras generated by bad components of currents (like $SU(6) \times SU(6)$) cannot be avoided and moreover the representation space can hardly be restricted to the states of infinite momentum.

In this paper we want to make two remarks which, we hope, will be useful for an algebraic treatement of the symmetries generated by local currents and the Poincaré algebra, in spite of the difficulties with the bad operators well-known in the dispersion approach to the algebra of current densities /8/.

We first show that a covariant limiting procedure exists, which is formally equivalent to $\vec{p} \to \infty$ but need not a restriction to the states with infinite momentum. From this point of view we clarify the meaning of the possibility of exchanging the limit with the SIS (section 2 and 3). Then, using the covariant formalism of the section 2, we consider the saturation of an algebra containing bad operators by the same set of states at any momentum. We try to interpret the impossibility of exchanging the limit with the SIS as a non-vanishing contribution of Z -graphs, arising from disconnected intermediate states. We find Z -graphs to be not only relevant for the problem but even to possess a very good features: they give rise to the results which are fully covariant and represent a natural extension of the static case.

Though we restrict ourselves to the very simplified situation (forward commutators, saturation by finite number of states etc.) the same can be in principle done for the general cases.

2. The Covariant Analogue of the Limit $\vec{p} \to \infty$

Here we shall show that the " $\vec{p} \to \infty$ method" is formally equivalent to the following covariant procedure: One first considers the commutator of two currents on the space-like hyperplane $(nx) = 0$ (n positive, time-like, but not unite) and then provides the limit $n^2 = n_\alpha n^\alpha \to 0$ i.e. to a hyperplane touching the light-cone.

Let us consider e.g. the commutator of vector densities on the hyperplane $(nx) = 0$ $(n^2 > 0,\ n_0 > 0)$

$$[V^i_\alpha(x/2), V^j_\beta(-x/2)]\,\delta(nx) = \frac{i}{n^2} f^{ijk} [n_\alpha V^k_\beta + n_\beta V^k_\alpha - g_{\alpha\beta}(nV^k)]\,\delta^4(x) + S^{ij}_{\alpha\beta} \,. \tag{1}$$

$S^{ij}_{\alpha\beta}$ contains terms antisymmetric in $\alpha\beta$ and the Schwinger terms. The factor $\frac{1}{n^2}$ in the right-hand side assures the invariance of equation (1) under the transformation $n \to \lambda n$. For our purpose it is sufficient to sandwich equation (1) by the states with the same momenta p and integrate over x. This procedure corresponds to considering the forward commutator of integrated charges and eliminates any effect of the Schwinger terms /9/.

Writting $\delta(nx) = \frac{1}{2\pi} \int d\lambda\, e^{i\lambda(nx)}$ and using on the left-hand side of (1) the convolution rule we have /10/

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} d\lambda \; t^{ij}_{\alpha\beta}(p, \lambda n) = \frac{i}{n^2} [n_\alpha p_\beta + p_\alpha n_\beta - g_{\alpha\beta}(np)]\, f^{ijk} F^k \,, \tag{2}$$

where $t^{ij}_{\alpha\beta}(p,k)$ is the symmetric part of

$$\frac{1}{2} \int d^4x\, e^{ikx} \langle p \,|\, [V^i_\alpha(x/2), V^j_\beta(-x/2)] \,|\, p \rangle \,. \tag{3}$$

For simplicity the state $|p\rangle$ is considered to be spinless and

$$\langle p \,|\, V^k_\alpha(0) \,|\, p \rangle = p_\alpha F^k \,.$$

Our main task is to use equation (2) in any Lorentz frame that is to use it for an arbitrary time-like vector n. Let us introduce the usual expansion

$$t_{\alpha\beta}(p,k) = a\, p_\alpha p_\beta + b\,(p_\alpha k_\beta + k_\alpha p_\beta) + c_1 g_{\alpha\beta} + c_2 k_\alpha k_\beta \,, \tag{4}$$

where the scalar structures a , b , c depend on (p , k) and k^2. Putting (4) into equation (2) and considering the vector n to be arbitrary we get the set of sum rules

$$\frac{1}{\pi}\int d\lambda\, a^{ij}(P\lambda, n^2\lambda^2) = 0$$

$$\frac{1}{\pi}\int \lambda\, d\lambda\, b^{ij}(P\lambda, n^2\lambda^2) = i\frac{1}{n^2} f^{ijk} F^k \qquad (5)$$

$$\frac{1}{\pi}\int \lambda^2\, d\lambda\, c_2^{ij}(P\lambda, n^2\lambda^2) = 0$$

$$\frac{1}{\pi}\int d\lambda\, c_1^{ij}(P\lambda, n^2\lambda^2) = -i\frac{P}{n^2} f^{ijk} F^k ,$$

where we have denoted $P = (p,n)$. Introducing a new integration variable $\nu = P\lambda$ and a parameter

$$\epsilon = \frac{n}{P}, \quad n = (n_\alpha n^\alpha)^{1/2} \qquad (6)$$

relations (5) can be written as

$$\frac{1}{\pi}\int d\nu\, a^{ij}(\nu, \epsilon^2\nu^2) = 0 \qquad (7a)$$

$$\frac{\epsilon^2}{\pi}\int \nu\, d\nu\, b^{ij}(\nu, \epsilon^2\nu^2) = i f^{ijk} F^k \qquad (7b)$$

$$\frac{1}{\pi}\int \nu^2\, d\nu\, c_1^{ij}(\nu, \epsilon^2\nu^2) = 0 \qquad (7c)$$

$$\frac{\epsilon^2}{\pi}\int d\nu\, c_2^{ij}(\nu, \epsilon^2\nu^2) = -i f^{ijk} F^k . \qquad (7d)$$

The dependence of the sum rules (7a-d) on the Lorentz frame (i.e. on the four-vector n) enters only through the parameter ϵ which has a meaning of the inverse power of energy of the state $|p\rangle$ in the frame where the commutator (1) is considered at equal times.

The method $p \to \infty$ chooses from the whole family of a sum rules like (7a-d) those containing integration over the energy variable with the mass variable fixed. This corresponds to taking the limit $\epsilon \to 0$ which can be reached either by limiting $P \to \infty$ or by $n^2 \to 0_+$ for an arbitrary fixed P. The equivalence of both is seen from the relations (6) and (7) but their interpretation is not exactly the same: The limit $P \to \infty$

taken for a fixed n (say $n = (1,0,0,0)$) means a special choice of the reference frame with e.g. $p_z \to \infty$. This corresponds to what is usually done. On the other hand setting $n^2 \to 0_+$ we let the frame with the commutator (1) taken at equal times to be a "photon-like" frame. In such a frame any state with non-zero mass has an infinite energy. Thus limiting $n^2 \to 0_+$ allows to sandwich relation (1) by the states with any finite momenta. In this sence the procedure $n^2 \to 0_+$ is in contrary to the $p_z \to \infty$ one, fully covariant, although both are formally equivalent.

In neither of relations (7a-d) the limit $\epsilon \to 0$ can be exchanged with the integral over ν. Allowing such an exchange in the relation (7a) we would obtain a superconvergence relation

$$\int d\nu\, a^{ij}(\nu, 0) = 0$$

which, though consistent with the reggeized dynamics[11], contradicts the Fubini sum rule[4]

$$\frac{1}{\pi}\int d\nu\, a^{ij}(\nu, 0) = i f^{ijk} F^k . \qquad (8)$$

In the case of relations (7b,d) the exchange of the limit and the integral will contradict the non-zero right-hand side and the integral (7c) is for $\epsilon = 0$ probably divergent.

This is not strange if we take into account that the set of relations (7a-d) is a part of what we would have obtained from the algebra of charges

$$Q^i_\alpha = \int V^i_\alpha(x)\,\delta(nx)\, d^4x \qquad (9)$$

using Lorentz covariance of the starting commutator. Since (9) includes bad operators the impossibility of using the same dynamical ansatz for the whole set of relations (7a-d) is an analogue of what has been found in the reference[8] within the dispersive approach.

On the other hand if we restrict ourselves to the algebra of good charges

$$Q^i = \int n^\alpha V^i_\alpha(x)\,\delta(nx)\, d^4x \qquad (10)$$

we get, taking the $n_\alpha n_\beta$ projection of relation (2), one sum rule

$$\frac{1}{\pi} \int d\nu \, [a^{ij}(\nu, \epsilon^2\nu^2) + 2\epsilon^2 \nu \, b^{ij}(\nu, \epsilon^2\nu^2) + \epsilon^4 \nu^2 c_1^{ij}(\nu, \epsilon^2\nu^2) + \epsilon^2 c_2^{ij}(\nu, \epsilon^2\nu^2)] = i f^{ijk} F^k \tag{11}$$

instead of the whole set (7a-d). The dynamical ansatz needed to obtain the Fubini sum rule (8) is the possibility of exchanging the limit $\epsilon \to 0$ and the integral in the relation (11) which is a linear combination of the relations (7a-d).

In the next section we shall, using the $n^2 \to 0_+$ interpretaion of the limit $\epsilon \to 0$, discuss the meaning of exchanging this limit with the integral over ν. In particular we shall see why one can expect such an exchange to be possible for the special combination (11) of the relations (7a-d) though it is impossible for each of them separately.

3. Charges on the Light Cone

The properties of the limit $\epsilon \to 0$ of the relations like (7a-d) or (11) are closely related to the behaviour of the charges

$$Q_\alpha = \int J_\alpha(x) \, \delta(nx) \, d^4x \tag{12}$$

in the limit $n^2 \to 0_+$ ($J_\alpha(x)$ is a local vector or axial-vector current). First of all let us discuss matrix elements of (12) using the possibility of an explicit comparison of the limiting case $n^2 = 0$ with the limit $n^2 \to 0_+$. (Note that such a comparison is impossible within the standart $p \to \infty$. method). Let us begin by considering the operators (12) for a light-like n. For any connected matrix element we have

$$< p \,|\, Q_\alpha \,|\, q > = (2\pi)^3 < p \,|\, J_\alpha(0) \,|\, q > \delta_n(p-q) \,, \tag{13}$$

where (for $n^2 = 0$)

$$\delta_n(p-q) = \frac{1}{(2\pi)^3} \int e^{i(p-q)x} \delta(nx) \, dx = \frac{E_p}{(pn)} \delta^3 [\vec{p} - \vec{q} + \frac{m_1^2 - m_2^2}{2(qn)} \vec{n}] \tag{14}$$

and m_1, m_2 are the masses of the states $|q>$, $|p>$, respectively. From (14) we see that (13) vanishes unless $(p-q)^2 = 0$. Thus in any frame of reference sum rules obtained from an algebra of the charges (12) for $n^2 = 0$ will include matrix elements of currents with a constant (here zero) momentum transfer. Similarly one can find that (12) with $n^2 = 0$ can never create pairs from the vacuum. This is because for $n^2 = 0$ we have

$$\delta_n(p) = \delta_n(-p) \equiv 0 \tag{15}$$

for any time-like p. In spite of the Coleman theorem /12/ we thus have

$$Q_\alpha(n^2 = 0) \,|\, 0 > = 0 \tag{16}$$

provided the theory is free of massless particles. Relation (16) in particular implies vanishing of all Z-graph contributions to any commutator of the charges (12) taken for $n^2 = 0$.

This completes the discussion of connections between the procedures $p \to \infty$ and $n^2 \to 0_+$ contained in the previous section. We see that the matrix elements of the "charges on the light-cone" (i.e. (12) for $n^2 = 0$) have all essential properties generally attributed to the limit $p \to \infty$ /2/. Here however these properties are relativized since they are no more consequences of a choice of the Lorentz frame.

For a comparison let us consider matrix elements of (12) for time-like n and take the limit $n^2 \to 0_+$. It can be easily verified that for a connected matrix elements like (13) the limit $n^2 \to 0_+$ leads back to the $n^2 = 0$ case. This is however not the case for the disconnected matrix elements containing creation of pairs from the vacuum. Let us consider e.g. the connected part

$$< p,q \,|\, Q_\alpha \,|\, 0 > = (2\pi)^3 < p,q \,|\, J_\alpha(0) \,|\, 0 > \delta_n (p+q) \,, \tag{17}$$

where the state $|\, p,q >$ contains a pair of particles, for simplicity with the same masses. For a time-like n we have

$$\delta_n (p+q) = \frac{E}{(q\,n)} \, \delta^3 [\, \vec{q} + \vec{p} - 2 \, \frac{(p\,n)}{n^2} \, \vec{n} \,] \,, \tag{18}$$

while for light-like n $\delta_n(p+q)$ vanishes identically. Since the limit $n^2 \to 0_+$ is not for the expression (18) well-defined we see that vanishing of the Z-graph contributions in the limit $n^2 \to 0_+$ will depend on the more detailed properties of the matrix elements of currents. From (18) we have for the invariant momentum transfer

$$(p+q)^2 = \frac{4}{\epsilon^2} \to \infty \,, \tag{19}$$

where ϵ is given by (6). Consequently the result of the limit will depend on the asymptotic behaviour of the form-factors included in (17). This in principle need not to be sufficiently good for the case of the axial-vector form-factors.

This constitutes a particular examples of the problems involved in comparing the $n^2 \to 0_+$ limit with the limiting case $n^2 = 0$ and therefore concerns the limit $\epsilon \to 0$ of the relations like (7a-d). The possibility of exchanging the limit with the integral will, in general, depend on the algebraic properties of the charges (12) for a light-like n. If they will close into the same algebra as (12) do for the case $n^2 > 0$, we can start all calculations with $n^2 = 0$ and according to (14) and (16) no limit need to be performed. In contrary any extra terms appearing in the commutator of the "charges on the light cone" (comparing to the $n^2 > 0$ case) will reflect the difference between the limit $\epsilon \to 0$ inside and outside the integrals like (7a-d).

The algebraic properties of (12) for $n^2 = 0$ cannot be discussed beyond a dynamical models since they involve a knowledge of the commutators of the underlying fields on the whole light-cone. As an illustration let us considere the charges

$$Q^i(\Gamma) = \int : \bar{\Psi}(x) \, \Gamma \, \sigma^i \, \Psi(x) : \delta(n\,x) \, d^4x \,, \tag{20}$$

where $\Psi(x)$ contains two free fields with different masses. Taking e.g. the commutator of the +, − isotopic components we find for $n^2 = 0$

$$[Q^{(+)}(\Gamma_I), Q^{(-)}(\Gamma_{II})] = \int d^3\vec{q} \, d^3\vec{p} \, [\frac{m_1^2}{E_1(p)\,E_1(q)}]^{1/2} \sum_{r,s} \delta_n (p-q) \times$$

$$\times \frac{1}{2(p,n)} \{ \bar{b}^1_r(\vec{q}) \, b^1_s(\vec{p}) \, \bar{u}^1_r(\vec{q}) \, \Gamma_I \, [\, \hat{p} + m_2 - \frac{m_1^2 - m_2^2}{2(pn)} \hat{n} \,] \, \Gamma_{II} \, u^1_s(\vec{p}) - \tag{21}$$

$$- \bar{d}^1_s(\vec{p}) \, d^1_r(\vec{q}) \, \bar{v}^1_r(\vec{q}) \, \Gamma_I \, [\, \hat{p} - m_2 - \frac{m_1^2 - m_2^2}{2(pn)} \hat{n} \,] \, \Gamma_{II} \, v^1_s(\vec{p}) \} - (1, I \to 2, II) \,,$$

where b^i, d^i are annihilation operators of the i-the particle and anti-particle, respectively. Concerning $\Gamma = \gamma_\alpha$, or $\gamma_\alpha \gamma_5$ we find that only for

$$\Gamma_{I,II} = \hat{n} \quad \text{or} \quad \hat{n}\gamma_5 \tag{22}$$

the algebra of operators (20) closes into the chiral $SU(2) \times SU(2)$. (The same is true for the case of $SU(3)$) Since the remaining commutation relations of the chiral $SU(4) \times SU(4)$ algebra are not maintained in the case $n^2 = 0$ even for the free fields, one can hardly expect them to hold for the case of interacting currents. This can be considered as an explanation why in the whole set of sum rules obtained from the chiral $SU(4) \times SU(4)$ algebra (like (7a-d)) the limit $n^2 \to 0_+$ (or generally $\epsilon \to 0$) cannot be interchanged with the integration over ν. On the other hand since the chiral $SU(2) \times SU(2)$ algebra is maintained for a light-like n one may expect this exchange to be possible for the corresponding sum rule like (11).

4. Saturation and Z-graphs

The previous discussion suggests that once we saturate the whole set of relations like (7a-d) by the same set of states, we must take into account the impossibility of exchanging the limit $\epsilon \to 0$ and the integral over ν. In this section we examine the simple possibility that this fact is due to a non-vanishing contribution of Z-graphs. This assumption seems to be reasonable because of the analogy between the saturation of current algebra sum rules and the old-fashioned (non-covariant) perturbation theory/13/. Like here we will consider together with any connected contribution of an intermediate state (Figure 1a) its Z-graph counterpart (Figure 1b).

We shall restrict ourselves to considering the simplest case: $SU(4)$ algebra in the limit of $SU(2)$ symmetry, sandwiched by nucleons and saturated according to the Figures 1a, 1b, with $|n\rangle$ being again a one-nucleon state. Without Z-graph contribution the answer is well known: for the states at rest predictions of the static $SU(4)$ are reproduced,/14/ while no consistent result exists for an arbitrary momenta/15/.

Because of the $SU(2)$ symmetry only commutators between the axial charges are relevant. Following the line of the section 2 we can write $(n^2 > 0)$

$$[A_\mu^{(+)}(x/2), A_\nu^{(-)}(-x/2)]\,\delta(nx) = \delta^4(x)\frac{2}{n^2}\{n_\mu V_\nu^{(3)} + n_\nu V_\mu^{(3)} - \tag{23a}$$

$$- g_{\mu\nu}(n V^{(3)}) - i\,\epsilon_{\mu\nu\alpha\beta}\, n_\alpha A_\beta^{(0)}\}$$

$$[A_\mu^{(+)}(x/2), A_\nu^{(0)}(-x/2)]\,\delta(nx) = -\delta^4(x)\frac{2i}{n^2}\epsilon_{\mu\nu\alpha\beta}\, n_\alpha A_\beta^{(+)} . \tag{23b}$$

We shall treat the relation (23a), the remaining one can be considered analogously. Sandwiching (23a) by the one-proton states with momentum p and integrating over x we have

$$\frac{1}{\pi}\int R_{\mu\nu}(p,\lambda n)\, d\lambda = \frac{1}{n^2}\{\frac{1}{m}(n_\mu p_\nu + p_\mu n_\nu - P\, g_{\mu\nu}) - \tag{24}$$

$$- i\, g(0)\,\epsilon_{\mu\nu\alpha\beta}\, n_\alpha \gamma_\beta \gamma_5\},$$

where $P = (pn)$, $g(0)$ is the isoscalar axial-vector form-factor and $R_{\mu\nu}(p,k)$ is defined by

$$\frac{1}{2}\int \langle p,f|[A_\mu^{(+)}(x/2), A_\nu^{(-)}(-x/2)]|p,i\rangle\, e^{ikx} d^4x = \tag{25}$$

$$= \frac{1}{(2\pi)^3}\frac{m}{E}\, \bar{u}_f(p) R_{\mu\nu}(p,k) u_i(p) .$$

Considering the $SU(4)$ algebra means to take the $e_\mu^i e_\nu^j$ -projection of relation (24) where e_μ^i ($i = 1,2,3$) are arbitrary vectors satisfying

$$(n e^i) = 0, \quad (e^i, e^j) = -\delta_{ij} .$$

Let us introduce the most general expansion of $R_{\mu\nu}(p,k)$

$$R_{\mu\nu}(p,k) = a\, p_\mu p_\nu + b_1 p_\mu k_\nu + b_2 k_\mu p_\nu + c_1 k_\mu k_\nu + c_2 g_{\mu\nu} +$$

$$+ \epsilon_{\mu\nu\lambda\rho}[\alpha\, p_\lambda k_\rho \hat{k} + (\beta_1 p_\rho + \beta_2 k_\rho)\gamma_\lambda]\gamma_5 + \tag{26}$$

$$+ [\epsilon_{\mu\lambda\rho\sigma}(\delta_1 p_\nu + \delta_2 k_\nu) + \epsilon_{\nu\lambda\rho\sigma}(\delta_3 p_\mu + \delta_4 k_\mu)]\, p_\rho k_\sigma \gamma_\lambda \gamma_5 ,$$

where the scalar structures a, b, c, α, β, δ depend on variables (pk) and k^2, respectively. Taking the $e_\mu^i e_\nu^j$ -projection of relation (24), comparing the spin non-flip and the spin-flip parts and using arbitrariness of the vectors n and p we get the set of five sum rules

$$\frac{1}{\pi}\int a(\nu, \epsilon^2\nu^2)\, d\nu = 0$$

$$\frac{\epsilon^2}{\pi}\int c_2(\nu, \epsilon^2\nu^2)\, d\nu = -\frac{1}{m} \tag{27}$$

$$\frac{1}{\pi}\int \{\epsilon[\epsilon^2\nu^2\alpha(\nu,\epsilon^2\nu^2) - \beta_1(\nu,\epsilon^2\nu^2)] - \nu\,\delta_3(\nu,\epsilon^2\nu^2)\}\, d\nu = 0$$

$$\frac{1}{\pi}\int \nu[\delta_1(\nu,\epsilon^2\nu^2) + \delta_3(\nu,\epsilon^2\nu^2)]\, d\nu = 0$$

$$\frac{\epsilon^2}{\pi}\int [m^2\beta_1(\nu,\epsilon^2\nu^2) + \nu\beta_2(\nu,\epsilon^2\nu^2) + (1-\epsilon^2 m^2)\nu^2\alpha(\nu,\epsilon^2\nu^2)]\, d\nu = i\, g .$$

Relations (27) represent an analogue of the sum rules (7a-d) discussed roughly in the section 2. As in that case the whole dependence on the Lorentz frame is contained in the parameter ϵ given by (6).

The contribution of the two graphs indicated by Figures 1a and 1b ($|n\rangle$ being a one-nucleon state) to the scalar structures introduced by (26) is

$$a(\nu,\epsilon^2\nu^2) = 2b_1(\nu,\epsilon^2\nu^2) = 2b_2(\nu,\epsilon^2\nu^2) = \frac{1}{im}\beta_1(\nu,\epsilon^2\nu^2) =$$

$$= \frac{2}{im}\beta_2(\nu,\epsilon^2\nu^2) = -\frac{c_2(\nu,\epsilon^2\nu^2)}{\nu/2+m^2} = \frac{\pi}{m}|G_1(\epsilon^2\nu^2)|^2\Delta(\epsilon,\nu) \qquad (28)$$

$$c_1(\nu,\epsilon^2\nu^2) = \frac{\pi}{4m}\left\{\frac{\nu}{2m^2}|G_2(\epsilon^2\nu^2)|^2 - 2\,\mathrm{Re}\,[G_1(\nu^2\epsilon^2)G^*_2(\nu^2\epsilon^2)]\right\}\Delta(\epsilon,\nu)$$

$$\alpha(\nu,\epsilon^2\nu^2) = \delta_1(\nu,\epsilon^2\nu^2) = \delta_3(\nu,\epsilon^2\nu^2) = 0,$$

where

$$\Delta(\epsilon,\nu) = \delta(\nu) - \delta\left(\nu + \frac{2}{\epsilon^2}\right) \qquad (29)$$

and G_1, G_2 are the isovector axial-vector form-factors. The contribution of the Z-graph corresponds to the second δ-function term of relation (29). In the case this term is not present we find the system of relations (27) to be overdefined. This corresponds to the contradiction found in the reference /15/, where the saturation at any momentum by means of the diagramm 1a only was considered. Including Z-graphs, i.e. putting (28) into (27), we get the following three relations

$$G_1^2(0) = \left|G_1\left(\frac{4}{\epsilon^2}\right)\right|^2$$

$$\left|G_1\left(\frac{4}{\epsilon^2}\right)\right|^2 = 1 \qquad (30)$$

$$\left|G_1\left(\frac{4}{\epsilon^2}\right)\right|^2 = g.$$

Thus the inclusion of Z-graphs not only removes the mentioned contradiction, but leads to the results of the static $SU(4)$ reproduced for any momentum p. Moreover in the limit $\epsilon \to 0$ relations (30) imply the non-vanishing asymptotics for the axial-vector formfactors, reflecting the relevance of the Z-graphs for the problem /16/.

For a comparison it is interesting to treat in the same manner the chiral algebra $SU(2) \times SU(2)$. Taking the $n_\mu n_\nu$-projection of the relation (24) we get

$$\frac{1}{\pi}\int d\nu\,\{a + \epsilon^2\nu(b_1+b_2) + \epsilon^4\nu^2c_1 + \epsilon^2c_1\} = \frac{1}{m}. \qquad (31)$$

Putting (28) into the sum rule (31 we have.

$$G_1^2(0)(1-\epsilon^2m^2) + \epsilon^2m^2\left|G_1(4/\epsilon^2) + \frac{1}{\epsilon^2m^2}G_2(4/\epsilon^2)\right|^2 = 1 \qquad (32)$$

We see that even in the case of the chiral algebra of the good charges the vanishing of the Z-graph contributions in the limit $\epsilon \to 0$ still depends on the asymptotic behaviour of the axial-vector form-factors. The predictions (32) of the chiral algebra $SU(2) \times SU(2)$ are in the limit $n^2 \to 0_+$ consistent with the more detailed predictions of the algebra $SU(4)$ only if the form-factor G_2 fulfils certain asymptotic constraint.

5. Conclusion

Let us recapitulate the results contained in the previous sections.

1. We have shown that a covariant limiting procedure exists, which has all good properties of the limit $p \to \infty$ but need not to restrict the algebra of currents to the space of states with infinite momentum.

2. Since the limiting procedure considered here in principle allows an explicit comparison of the limiting case with the result of the limit, it is appropriate for discussing the problems concerning the possibility of exchanging the limit with the sum over intermediate states. It thus appears that such a possibility hinges on the algebraic properties of the "charges on the light-cone" (discussed in the section 3) which can generally differ from the usual algebras of currents.

3. This was demonstrated for the currents built up from a free fields:

only the chiral algebra $SU(3) \times SU(3)$ is reproduced by the charges on the light-cone and consequently only in the sum rules corresponding to this case one may expect an exchange of the limit and the integral to be allowed.

4. Concerning the saturation of the algebras containing a bad operators we have tried to interpret the impossibility of exchanging the limit with the sum over intermediate states as a contribution of Z -graphs. On the simplest example of the $SU(4)$ algebra saturated by nucleons in the case of $SU(2)$ symmetry we have shown that inclusion of Z -graphs a) removes the well-known contradiction of saturating algebras of this type at any momentum by the same set of intermediate states and b) leads to the results of static $SU(4)$ plus conditions on the asymptotics of axial-vector form-factors. This indicates the relevant role of the Z -graphs, which can have a good influence on the result whithout introducing new form-factors or parameters.

Though nothing unexpected is contained in these results we would like to point out the relevance of the limit $n^2 \to 0_+$ considered in this paper, for a simultaneous treatment of the Poincaré algebra and the algebra of currents. The fact that operators (20) (for $\Gamma = \hat{n}, \hat{n}\gamma_5$), generated by local (non-conseved) currents, form for $n^2 = 0$ an algebra and are exactly represented in the space of one-particle states with different masses and arbitrary finite momenta, is a simple example of a non-trivial avoiding of the Coleman theorem,[17] whithout losing the possibility of incorporating the full space-time symmetries. From this point of view a non-trivial binding of the Poincaré symmetry with a internal symmetry generated by a local currents still seems to be possible. This would represent a natural way out the ambiguity of constructing the infinite-dimensional relativistic algebraic structures generating (besides other) the spectrum of hadron masses[18]. We hope to return to this question elsewhere.

In conclusion we would like to acknowledge the stimulating discussions with J.Jersák and the critical remarks of Professors J.A.Smorodinsky and Nguyen Van Hieu.

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Received by Publishing Department on August 1, 1967.

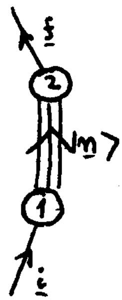

Fig. 1a.

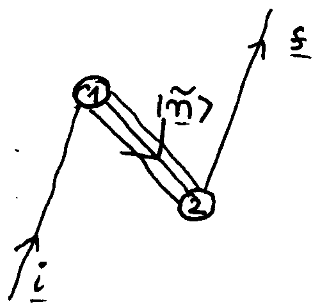

Fig. 1b.

Diagramms considered in the section 4. | ñ > is the anti-particle state, corresponding to a state | n >. 1 and 2 reffer to the two (different) currents appearing in the commutator.