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K.S.Wohlrab

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Summary

In paper I of this series the physical particles and resonances were described as bound states (by the help of the quasiparticle approximation applied to a 4-fermion interaction). In the present paper II we study the resonance scattering of the physical particles of I one another.

The first chapter defines field operators in the x-space describing th free motion of the physical particles and the interior motion within them. Simple interactions between the physical particles are written down, and a simple finite nonperturbational renormalisation of certain 4-fermion interactions results.

The second chapter gives the calculation of the K(reactance) matrix and the S-matrix for resonance scattering, using either a Lippmann-Schwinger or Weidenmüllers recent formulation. This leads to a Breit-Wigner formula with resonances just at the masses of that physical particles (resonances) which are possible intermediate states of the scattering process. The corresponding graph scheme describes the resonance scattering of physical (compound) particles, with formfactor at the vertices, as in dispersion relation frames.

In the approximation performed our field theory of compound systems contains no divergences and so disentangles the problem of resonances from the divergency problem. It shows interesting connections with the Newton-Wigner localisation problem.

1. Introduction

In the present situation of particle physics there is a strong need for a simple quantum field theory of the interior region (core) of the physical particles, where the main mass is concentrated – even if it would be oversimplified in many respects. The present author made such an attempt in I [x], using the qausiparticle method, in considering the interior region with its high mass concentration as a system of many (virtual) particles. This leads to an independent particle model plus additional pairing interactions of all physical particles and resonances. Paper I contains the first half of the full problem , the calculation of the masses of the physical particles in a bound state approximation. It resulted in the determination of the field operation Ω , M etc. of each physical particle or resonance in its rest system. Further it gave the right type of symmetry breaking in the strong coupling limit.

Now we attack the second half, the scattering of these physical particles one another, with creation of a resonance during the process. In this way the (idealized) stable resonance states of paper I are now embedded into the continuum of scattering states. We proceed in two steps: Chapter II treats in the main the free motion of the physical particles, chapter III the calculation of the K- and S-matrix.

It is important to remark already now, that a large part of the present paper is not bound to the simple quasiparticle approximation of I, because often we shall not use, how the field operator of a physical particle was constructed in paper I, but only that he was constructed. So far the methods of the present paper II meet essentially all field theoretical compoind system treatments of particles. Especially both our derivations of the resonance formula are completely independent of the quasiparticle approximation.

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x) Dubna preprint E-2714, Nuclear Physics to be published.

II.x- Space Properties

a) Fields

We shall work always in the Schrödinger picture, and write

$$x = \vec{x}\,, \quad p = \vec{p}\,.$$

From paper I we have destruction operators N_ν, $\vec{N}_\nu$ of independent ("bare") barions and antibarions or quarks and antiquarks of mass m on the orbits of a central potential,

$$\nu = E_{\bar{\nu}}\ j_\nu\ m_\nu\ n_\nu\ i_\nu\ i_{s\nu}\ y_\nu\ \pi_\nu\ .$$

From them we get destruction operators of free (first order) quasiparticles at rest

$$\Omega_\nu = u_\nu N_\nu + v_\nu \bar{N}^*_{-\nu} \qquad u_\nu^2 + v_\nu^2 = 1\,,$$
$$\bar{\Omega}_\nu = u_\nu \bar{N}_\nu - v_\nu N^*_{-\nu}$$

the corresponding vacuum being defined by

$$\Omega_\nu\,|0> = \bar{\Omega}_\nu\,|0> = 0\ .$$

The states of one quasiparticle at rest are

$$|1_\nu> = \Omega^*_\nu\,|0> \qquad (|\bar{1}_\nu) = \bar{\Omega}^*_\nu\,|0)$$

with mass ω_ν. We consider them as our approximation of the physical states. Similar but somewhat more complicated formulas we have written down for second order quasiparticles (measons) etc.

Free Motion of Physical Particles

According to our program we consider now the operators Ω_ν, $\bar{\Omega}_\nu$ (M_α etc.), destroying one physical particle in its rest system, as given. How to construct from them the field operators in the x-space? Certainly we can write down

$$\Omega_\nu(X) = \sum_{m_\nu} \sum_{p} \sqrt{\frac{\omega_p}{E_{\nu p}}}\ (\chi_\nu(p)\, e^{ipX}\, \Omega_{\nu p} + \bar{\chi}_\nu(p)\, e^{-ipX}\, \bar{\Omega}^*_{\nu p})\,, \quad E^2_{\nu p} = p^2 + \omega^2_\nu\,, \quad (1)$$

Here for $j_\nu = \frac{1}{2}$ the χ_ν, $\bar{\chi}_\nu$ are free Dirac spinors, for $j_\nu = 3/2$ free Rarita-Schwinger spinors etc. The problem is only, how to calculate the destruction operators $\Omega_{\nu p}$ of a physical particle with momentum p from that of the same particle at rest: Ω_ν, . For this we have to apply a Lorentz transformation Λ_β corresponding to p ,

i.e. $$\beta = \beta_{\nu p} = \frac{p}{E_{\nu p}} \qquad \Lambda(\beta_{\nu p}) = \Lambda_{\nu p} \qquad U(\Lambda_{\nu p}) = U_{\nu p}$$

$$\Omega_{\nu p} = U(\Lambda_{\nu p})\, \Omega_\nu\, U^{-1}(\Lambda_{\nu p}). \tag{2a}$$

In this paper we always confine to special Lorentz transformations along the x-axis (and therefore also to such momenta p, k). Using plane wave Dirac solutions k, $r = \frac{(\sigma k)}{|k|} = r(k)$ (with the same SU_3 quantum numbers as ν)

$$U(\Lambda)\, N_{kr}\, U(\Lambda)^{-1} = N_{\Lambda k, r} \qquad (r(\Lambda k) = r(k) : \Lambda k \| k)$$

$$N_\nu = \sum_k^0 \phi(k,r)\, N_{kr} \qquad N_{kr} = \sum_\nu^0 \phi^*_\nu(k,r)\, N_\nu \quad x),$$

and $\Omega_\nu = u_\nu N_\nu + v_\nu N^*_{-\nu}$, one gets easily

$$\Omega_{\nu p} = u_\nu N_{\nu p} + v_\nu \bar{N}^*_{-\nu p} \qquad N_{\nu p} = U_{\nu p}\, N_\nu\, U^{-1}_{\nu p}$$

$$= \sum_\lambda^0 \left(u^\nu_\lambda(p)\, N_\lambda + v^\nu_\lambda(p)\, \bar{N}^*_\lambda \right) \tag{2b}$$

$$= \sum_\lambda^0 \left(x^\nu_\lambda(p)\, \Omega_\lambda + y^\nu_\lambda(p)\, \bar{\Omega}^*_\lambda \right)$$

$$u^\nu_\lambda(p) = u_\nu \gamma^\nu_\lambda(p) \qquad v^\nu_\lambda(p) = v_\nu \gamma^\nu_\lambda(p)$$

$$x^\nu_\lambda(p) = u_\nu u_\lambda \gamma^\nu_\lambda(p) + v_\nu v_\lambda \gamma^{-\nu}_{-\lambda}{}^*(p) \qquad y^\nu_\lambda(p) = -u_\nu v_\lambda \gamma^\nu_{-\lambda}(p) + v_\nu u_\lambda \gamma^{-\nu}_\lambda{}^*(p)$$

x) The point above the sum means, that it runs only about the energies and angular momenta in $\nu = E_\nu\, j_\nu\, m_\nu\, i_\nu\, i_{3\nu}\, y_\nu\, \pi_\nu$. We neglect the fact, that strictly the set of states ν is not complete.

correspondingly

$$|0)_{\nu p} = U_{\nu p}|0)$$

$$|1_{\nu p}) = U_{\nu p}|1_{\nu}) = \Omega^*_{\nu p}|0)_{\nu p} \qquad (2c)$$

$$= Z_{\nu} N^*_{\nu p}\,(1 + \sum_{\nu'} y_{\nu\nu'} N^*_{\nu' p} \bar{N}^*_{-\nu' p} + \ldots.)|0>.$$

The $\sum_{\lambda}^{0}$ in (2b) mean, that our Lorentz transformation mixes into the state or particle ν other states λ with the same SU_3 properties, but with other masses and "spins". The degree of admixture is given (comp. (2b)) essentially by

$$\gamma_{\lambda}^{\nu}(p) = \sum_{k,r} \phi_{\nu}(k,r)\,\phi^*_{\lambda}(\Lambda_{\nu p} k, r). \qquad (2d)$$

This remarcable formula shows, that our Lorentz transformation causes a momentum transfer $(\Delta k)^2 = (\Lambda_{\nu p} k - k)^2$ from state ν to λ accompanying the admixture, which determines the deviation from the case of no admixture: $\gamma_{\lambda}^{\nu}(p) = \delta_{\nu\lambda}$.
One gets

$$(\Delta k)^2 = -2\,\frac{E^{kin}_{\nu p}}{\omega_{\nu}}\,(k_0^2 - k_x^2),$$

pointing out the different factors determining the admixture. According to (2d) the $\gamma_{\lambda}^{\nu}(p)$ can be considered as known and indeed can be calculated numerically (though not easily), because the Fourier coefficients $\phi_{\nu}(k)$ are known.

Intrinsic Motion Within Physical Particles

Besides the exterior motion of the physical particles as a whole we consider the x-space properties of the intrinsic motion of our states (particles). From I, ch.III d:

$$|1_\nu) = \Omega^*_\nu |0) = Z_\nu N^*_\nu \left(1 + \sum_{\nu'} y_{\nu\nu'} N^*_{\nu'} \bar{N}^*_{-\nu'} + \dots\right)|0>$$

one gets the x-space representation of this state:

$$W_\nu(xx'x''\dots) = Z_\nu A\, w_\nu(x)\left(|1>_\nu + \sum_{\nu'} y_{\nu\nu'} w_{\nu'}(x') w_{-\nu'}(x'')|3_{\nu\nu'}> + \dots\right),$$

where $w_\nu(x)(\bar{w}_\nu(x))$ is a solution of the central field Dirac equation corresponding to $N_\nu(\bar{N}_\nu)$ and A the antisymmetrizer. w_ν and W_ν are normalized. So we have first a polylocal field

$$\Omega(xx'\dots) = \sum_\nu \left(W_\nu(xx'\dots)\Omega_\nu + \bar{W}_\nu(xx'\dots)\bar{\Omega}^*_\nu\right) \equiv \Omega^+(xx'\dots) + \Omega^-(xx'\dots).$$

Sometimes it is useful to derive from this a local field by averaging about the pairs, where $w_\nu(x)$ is not included into the antisymmetrization in (3):

$$W_\nu(xx'\dots) = Z_\nu w_\nu(x)|P_\nu> \qquad |P_\nu> = P_\nu(x'x''\dots) = A\left(|1_\nu> + \sum_{\nu'} y_{\nu\nu'} w_{\nu'}(x') w_{-\nu'}(x'')|3_{\nu\nu'}> + \dots\right)$$

$$\Omega(x) = \sum_\nu \left(\tilde{Z}_\nu w_\nu(x)\Omega_\nu + \tilde{\bar{Z}}_\nu \bar{w}_\nu(x)\bar{\Omega}^*_\nu\right) \equiv \Omega^-(x) + \Omega^+(x) \qquad (4)$$

$$\tilde{Z}_\nu = Z_\nu <P_\nu|P_\nu> \equiv Z_\nu \int P^*_\nu P_\nu\, dx'dx''\dots \equiv Z_\nu a_\nu = Z_\nu^{-1}$$

$$a_\nu = Z_\nu^{-2} = \sum_{\nu'} \left(1 + |y_{\nu\nu'}|^2 + \dots\right).$$

$\Omega(x)$ can be called average quasiparticle field, because it describes (in the c.m.s.) the intrisic motion as that of one bare particle within the average "pair medium" (whereby it is neglected that the Pauli principle acts between the one particle and identical ones in the medium).

It is interesting to ask for the connection between $\Omega(x)$ and the bare field

$$\psi(x) = \sum_\nu^0 \left(w_\nu(x)N_\nu + \bar{w}_\nu(x)\bar{N}^*_\nu\right) \equiv \psi^-(x) + \psi^+(x)). \qquad (5)$$

Using

$$\Omega_\nu = Z_\nu \int (\tilde{w}_\nu(x)\, \Omega^-(x)\, dx \qquad \text{(pair medium approximation)} \tag{6}$$

$$N_\nu = \int (\tilde{w}_\nu(x)\, \psi^-(x))\, dx, \qquad \text{(exact)},$$

one finds easily the x-form of the Bogolubov transformation connecting Ω_ν and N_ν :

$$\Omega(x) = \int F^+(x,x')\psi^+(x')\,dx' + \int F^-(x,x')\psi^-(x')\,dx', \tag{7}$$

where the 4 x 4 matrices F consist out of expressions like $U^-(x,x') = \sum_\nu u_\nu\, w_\nu(x')\, w_\nu(x)$. This shows, that the Bogolubov transformation is a non-local (unitary, canonical) transformation between the bare and the average quasiparticle field. The extension of the non-locatily is given by the extension of the correlated barion-antibarion pairs (correlation length!), which decisively determine the intrinsic motion within our quasiparticles.

Rejecting the (partial) average about the pairs, the full x-space form of the Bogolubov transformation can be written down in a similar way, connecting now $\Omega(xx'\ldots)$ with $\psi(x)$ in a nonlocal way.

Combined Free and Intrinsic Motion

If we introduce (6) into (1), (2) we get the field operator

$$\Omega_\nu(Xx) = \sum_{m_\nu} \sum_p \sqrt{\frac{w_\nu}{E_{\nu p}}} \left(\chi_\nu(p)\, e^{ipX} \cdot (\tilde{w}_\nu(x)\, \Omega^-_{\nu p}(x)) + \ldots . \right)$$

$$\Omega_\nu(X) = \int \Omega_\nu(Xx)\, dx, \qquad \Omega^-_{\nu p}(x) = U_{\nu p}\, \Omega^-(x)\, U^{-1}_{\nu p},$$

containing the combined exterior and intrinsic motion. That we were led in this way to a bilocal field operator $\Omega_\nu(Xx)$, is only for the first moment an astronishing fact. For such field operators are clearly typical and unavoidable in a field theory of compound i.e. extended, systems: Already in standard quantum electrodynamics one gets them, if

one considers the scattering of two H-atoms, both treated in Furry picture. Kinematically our problem is completely analogous to this.

Comparing out bilocal fields with that of Yukawa [x], besides some minor deviations in the detailed form there is one main difference: Our bilocal fields are derived from the original (bare) local field (5) and not postulated from the beginning.

Another important property of $\Omega(Xx)$ is, that it is strictly separated in X and x, so that exterior and average intrinsic motion are independent (up to kinematical effects like Lorentz contraction etc.). This is the direct consequence of our bound state representation and means, that "Einschwingvorgänge" or transient processes in the creation and destruction of our particles (resonances) connecting both variables dynamically are neglected.

In general - without the pair average - we have not only a bilocal, but a polylocal field $\Omega^{-}_{\nu p}(xx'\dots) = U_{\nu p}\,\Omega^{-}(xx'\dots)\,U^{-1}_{\nu p}$

$$\Omega_{\nu}(X\,xx'\dots) = Z_{\nu}^{-\frac{1}{2}} \sum_{m_\nu p} \sqrt{\frac{w_\nu}{E_{\nu p}}} \left(X_\nu(p)\, e^{ipX} \cdot \left(W_\nu(xx'\dots)\,\Omega^{-}_{\nu p}(xx'\dots)\right) + \dots\right).$$

Finally we remark, that similar considerations can be made for higher order quasiparticles. So for a (pseudo) scalar meson field we have

$$M(X) = \sum_p \sqrt{\frac{1}{2E_p}} \left(e^{ipX} M_p + e^{-ipX} \bar{M}^*_p \right)$$

$$M_p = \sum_k \sum_{\nu\nu'} \left(\mu_{\nu\nu'}\, \Omega_{\nu k}\, \bar{\Omega}_{\nu' p-k} + \nu_{\nu\nu'}\, \Omega^*_{\nu k}\, \bar{\Omega}^*_{\nu' p-k} \right).$$

b) Commutation Relations

Let us consider first the intrinsic motion. We based our formulation

x) Yukawa, Phys.Rev., 77, 219, 849 (50).

upon one distinguished system of coordinates, the rest system of each particle, by starting with the levels of some central potential $V(r)$. So we work in a bound state representation (Furry) and have necessary no translation invariance:

$$\{\psi(x),\ \psi^*(x')\} = S_\nu(x,x') \qquad \{\Omega(x),\Omega^*(x')\} = \tilde{S}_\nu(x,x').$$

For the motion of the particles as a whole, one gets easily from (2b)

$$\{\Omega_{\nu p},\ \Omega^*_{\nu' p'},\} = 0,$$

so that $\Omega^2_{\nu p} = 0$ and therefore our particles behave like ordinary Fermions occupying each state only once: $\Omega^{*2}_{\nu p}\,|0\rangle_{\nu p} = 0$.

On the other hand

$$\{\Omega_{\nu p},\ \Omega^*_{\nu' p'},\} = 1\, B_{\nu\nu'}(q-q') \equiv$$

$$\equiv 1\,(u_\nu u_{\nu'} A_{\nu\nu'}(q-q') + v_\nu v_{\nu'} A^*_{-\nu-\nu'}(q-q'))$$

$$q = \frac{p}{\mu} \qquad \mu = \mu_{\nu\nu'} = \frac{\omega_\nu}{\omega_\nu + \omega_{\nu'}}$$

$$q' = \frac{p'}{\mu'} \qquad \mu' = \mu'_{\nu\nu'} = \frac{\omega_{\nu'}}{\omega_\nu + \omega_{\nu'}}.$$

This anticommutator is a number, but not the usual $\delta_{\nu\nu'}\delta(p-p')$. Instead we have the non-δ-functions $A_{\nu\nu'}$ depending on the difference of the "reduced" momenta $q - q'$:

$$A_{\nu\nu'}(p,p') = \sum_{k,r} \phi_\nu(\Lambda^{-1}_{\nu p}\, k, r)\,\phi^*_{\nu'}(\Lambda^{-1}_{\nu' p'}, k, r) = \sum_{k,r} \phi_\nu(k,r)\,\phi^*_{\nu'}(\Lambda^{-1}_{\nu' p'}\,\Lambda_{\nu p}\, k, r) =$$

$$= \sum_{k,r} \phi_\nu(k,r)\,\phi^*_{\nu'}(\Lambda_{\nu', p'_\nu - p'} k, r) \qquad p'_\nu = p\,\frac{\omega_{\nu'}}{\omega_\nu}$$

$$= A_{\nu\nu'}(p'_\nu - p') = A_{\nu\nu'}(q-q').$$

Here we used, that Λ depends only on β, so that

$$\Lambda_{\nu p} = \Lambda(\beta_{\nu p}) = \Lambda(\beta_{\nu' p'_\nu}) = \Lambda_{\nu' p'_\nu} \quad (\beta_{\nu p} = \frac{p}{E_{\nu p}} = \beta_{\nu' p'_\nu} = \frac{p'_\nu}{E_{\nu' p'_\nu}}),$$

and the group properties of the Λ's. The deviation from $\delta_{\nu\nu'}\delta(p-p')$ produces a nonorthogonality

$$(1_{\nu p} \mid 1_{\nu' p'}) = (1_\nu \mid U^{-1}_{\nu p} U_{\nu' p'} \mid 1_{\nu' p'}) = (1_\nu \mid U_{\nu p_{\nu'} - p} \mid 1_{\nu'}) =$$

$$= f_{\nu\nu'}(q-q') \neq \delta_{\nu\nu'}\delta(p-p').$$

With the help of Schmidt's orthogonalisation procedure one easily gets orthogonal states, but they mix different momenta, i.e. they are wave pakets. What are the commutation relations in the X-space? For the moment let us suppress the spinor properties and work with

$$\Omega_\nu(X) = \sum_p \sum_{m_\nu} \sqrt{\frac{1}{E_{\nu p}}} \left(e^{ipX} \Omega_{\nu p} + e^{-ipX} \bar{\Omega}^*_{\nu p}\right).$$

This leads to

$$\{\Omega_\nu(X), \Omega^*_{\nu'}(X')\} = 1\, W_{\nu\nu'}\left(\frac{Y-Y'}{2}\right) \cdot$$

$$\cdot \left(u_\nu u_{\nu'} A_{\nu\nu'}\left(\frac{R}{2}\right) + v_\nu v_{\nu'} A^*_{-\nu-\nu'}\left(\frac{R}{2}\right) + \ldots\ldots\right)$$

$$W_{\nu\nu'}\left(\frac{Y}{2}\right) = \int \frac{e^{\frac{i}{2}QY}}{\sqrt[4]{M^2_{\nu\nu'} + Q^2}}\, dQ \qquad A_{\nu\nu'}\left(\frac{R}{2}\right) = \int \frac{e^{\frac{1}{2}QR}}{\sqrt[4]{M^2_{\nu\nu'} + Q^2}} \sum_{m_\nu} \sum_{m_{\nu'}} A_{\nu\nu'}(Q)\, dQ$$

$$Y = \mu X, \quad Y' = \mu' X', \quad R = Y + Y' = \frac{\omega_\nu X + \omega_{\nu'} X'}{\omega_\nu + \omega_{\nu'}}, \quad M_{\nu\nu'} = \omega_\nu + \omega_{\nu'},$$

if we approximate ($M = \omega_{\nu} + \omega_{\nu'}$).

$$\frac{1}{\sqrt{E_{\nu_p} E_{\nu' p'}}} = \frac{1}{\mu\mu' \sqrt[4]{(M^2+q^2)(M^2+q'^2)}} \approx \frac{1}{\mu\mu' \sqrt[4]{(M^2+(q+q')^2)(M^2+(q-q')^2)}}$$

in order to get separability in $q+q'$ and $q-q'$.+) With these simplifications we get a simlpe interpretable result. The first factor is the Newton-Wigner function ++), expressing the fact that relativistic particles have localized states of a certain spread. Clearly this produces a spread in the localization of X and X', expressed by $W(\frac{Y-Y'}{2})$. Remarkably the difference of the "reduced" coordinates Y appears. Further we have the functions $A(\frac{R}{2})$ giving a similar spread in the total center of mass coordinate R. That means, that we are not exactly in the total c.m.s. with $\delta(R)$, but instead the total c.m. itself has a Newton-Wigner spread: Indeed the behaviour of A for large R is determined by $A_{\nu\nu'}(0) = \delta_{\nu\nu'}$, so that $A \to W$ for $R \to \infty$. In this connection two remarks are necessary. First it is well known from nuclear physics, that in a shell model approach the center of mass cannot be fixed completely, because shell model states are not exact eigenstates of the total momentum,. The same is true for our "shell model of elementary particles". Secondly in the resonance scattering of particles ν and ν' the total center of mass is identical with the center of mass of the intermediate resonance state. So considering the resonance state as a particle too - as we did - again a Newton-Wigner spread of the localized state has to appear for this particle, i.e. in R. In this sense the present formulation is most convenient for our purpose. Without both simplifications one gets

+) The difference between the exact and the approximate denominator is $\approx q^2$ and q'^2 . So it vanishes for $q, q' \to 0$, i.e. $X, X' \to \infty$

++) Newton, Wigner, Rev.Mod.Ph., 21, 400 (49).

$$\{\Omega_{\nu}(X), \Omega^{*}_{\nu'}(X')\} = 1\, F_{\nu\nu'}(X,X')$$

$$F_{\nu\nu'}(X,X') = \sum_{pp'} \sum_{m_\nu m_{\nu'}} \sqrt{\frac{w_p w_{p'}}{E_{\nu p} E_{\nu' p'}}} \left(\chi_{\nu p} \chi^{*}_{\nu' p'} e^{i(pX - p'X')} B_{\nu\nu'}(p,p') + \chi_{\nu p} \chi^{*}_{\nu' p'} \cdot \right.$$

$$\left. \cdot e^{-i(pX - p'X')} B_{\nu\nu'}(p,p') + \chi_{\nu p} \chi^{*}_{\nu' p'} e^{i(pX + p'X')} D_{\nu\nu'}(p,p') + \chi_{\nu p} \chi^{*}_{\nu' p'} e^{-i(pX + p'X')} D_{\nu'\nu}(p',p) \right)$$

$$D_{\nu\nu'}(p,p') = -u_{\nu} v_{\nu'} C_{\nu-\nu'}(p,p') + v_{\nu} u_{\nu'} C_{-\nu\nu'}(p,p') \qquad C_{\nu-\nu'}(p,p') = \sum_{\lambda}^{\Omega} \gamma^{\nu}_{\lambda}(p) \bar{\gamma}^{-\nu'}_{\lambda}(p') .$$

Now the spreads in $Y - y'$ and $Y + Y'$ are not separated, because they are not independent. The result is a dependency on X and X', which nontheless is mainly a dependency on $Y - Y'$, because R is "nearly" fixed in the above sence. Finally we note

$$N_{\nu p} = \Omega^{*}_{\nu p}\, \Omega_{\nu p}$$

$$[\, N_{\nu p}, \Omega_{\nu' p'},] = - B_{\nu\nu'}(q - q')\, \Omega_{\nu p}$$

$$[\, N_{\nu p}, \Omega_{\nu p}] = - \Omega_{\nu p} \qquad (B_{\nu\nu}(0) = 1).$$

So, $\Omega_{\nu p}$ is an ordinary destruction operator in each subspace ν, p. Only the different modes $\nu, p\,;\, \nu', p', \ldots .$ are not independent. But this is just what we need, because physical particles are never independent (except in the one particle state), as expressed already by the non-orthogonality of states $(1_{\nu p} \mid 1_{\nu' p'})$.

c) Interactions

The treatment of interactions is possible along two different ways. The first "orthodox" way, writes down one single H_{int} between the bare free particles, something like

$$\tilde{H}_{int} = g \int (\bar{\psi}(x) \Gamma \psi(x) (\bar{\psi}(x) \Gamma \psi(x)) dx ,$$

with $\psi(x)$ like (5), but with free spherical (or plane) Dirac solutions $w_{\underline{\nu}}(x)$, $\underline{\nu} = E j m \underline{n} i i_3 y \pi$, E continuous. One then has to relate the corresponding operators $N_{\underline{\nu}}$ with our N_0 and by this with the operators of the physical particles Ω_{ν}, M_{α} etc.

In our case - already having field operators describing approximately the physical particles - it is more natural to go another "pragmatic" way. We shall write down effective interactions expressing directly, that physical particles are scattered one another. In order to be concrete, consider the octet barion case of paper I, $\vec{n}_{\nu} = 8_{\nu}$, where the Ω_{ν} describe 8-barions, and the special case of nucleon-antinucleon scattering. Then we have

$$\tilde{H}_{int} = g \sum_{I} \sum_{i_3 \ldots} C^{I}_{12} C^{I}_{34} \mu \int (\Omega^{+}_{\underline{N}_1} {}^{*}(X) \Gamma \Omega^{-}_{N_2}(X)) (\Omega^{+}_{\underline{N}_3}(X) \Gamma \Omega^{-}_{N_4} {}^{*}(X)) dX + h.c. \quad (9)$$

The sums and C's means the proper coupling of isospins. Taking the $\Omega^{\pm}(X)$ from (1), (2) gives overall momentum conservation.

According to (2b) the incoming plane waves Ω_{N_p}, $(\bar{\Omega}_{N_p})$ contain $(\sum_{\lambda}^{0})$ admixtures of particles (resonances) λ with the SU_3 quantum numbers of the nucleon (antinucleon), but with other masses and spins. Examples are $N^{*}(1490)$ and $N^{*}(1520)$, but not the decuplet resonances. The same is true clearly for the outgoing waves too. Such effects are typical for the scattering of compound systems.

In $N\bar{N}$-scattering meson resonances can appear as intermediate states. Consider for this e.g. the operator $\Omega_{\underline{N}} \Omega_{N}$ contained in (9) after use of (1) and (2b). The same operator is contained in the meson operators M_{α} (I.24). So with $\sum_{\alpha} a^{\alpha}_{N\bar{N}} \mu^{\alpha}_{\nu\nu'} = \delta_{\nu N} \delta_{\nu' \bar{N}}$ we can write

$$\Omega_N \bar{\Omega}_{\bar{N}} = \sum_\alpha a_{N\bar{N}} M_\alpha^- \ .$$

From this we get for $\tilde{\tilde{H}}_{int}$ the "resonance" form

$$= \Omega_N^* \ \Omega_N^* \ M_\alpha^- + h.c. \ , \qquad (10)$$

leading indeed to resonance denominators at the meson masses ω_α as we shall see. It is not difficult to repeat this for πN -scattering, where $\tilde{\tilde{H}}_{int} = \Omega_N^* \ \Omega_N \ M_\pi$ is easily shown to contain e.g. $\Omega_N \ \Omega_\nu \ \bar{\Omega}_{\nu'}$. But just this is a part of a decuplet-barion, so that resonances like $\Delta_{3/2\ 3/2}$ now appear in πN -scattering. Obviously this method of producing resonances is essentially common to all theories building all particles out of a common material, as barions, or quarks, or Urmaterie.

d) Renormalization

Concerning coupling constants, there are really three: the depth of the potential $V(r)$, the G of H_{int} (paper 1) and the g of the effective scattering interaction $\tilde{\tilde{H}}_{int}$ (9). The first two need no renormalization at all, because there is no experiment to measure them directly. For they determine together with the range of the potential and m_{bare} all properties of the bound states, like masses, magnetic moments etc., but do not appear singly. Only g can be measured directly by scattering and so allowes (and requires) a renormalization. We perform it analogous to the Chew-Low-model, taking the ratio of the quasiparticle and the bare particle nmatrix element of $\tilde{\tilde{H}}_{int}$. Suppressing already c -number factors equal in numerator and denominator, we define

$$g_r = \frac{(1_{N_p} \bar{1}_{N_p'} | \ \Omega^*_{N_p} \bar{\Omega}^*_{N_p'} \ \bar{\Omega}_{N_p''} \Omega_{N_p'''} | \ 1_{N_p''} \bar{1}_{N_p'''})}{<1_{N_p} \bar{1}_{N_p'} | \ \Omega^*_{N_p} \bar{\Omega}^*_{N_p'} \ \bar{\Omega}_{N_p''} \Omega_{N_p'''} | \ 1_{N_p''} \bar{1}_{N_p'''}>} \cdot g \ .$$

This is a reasonable definition, because one gets easily for the ratio a number independent of the states $p; p', \ldots$:

$$g_r = u_N^{-4}\, g \equiv z_N^4\, g \geq g\,, \qquad (\text{NN - scattering}).$$

So we have a finite renormalization with the numerical value $z_N^4 = 1,68$ taken from table 1.

Table 1

Numerical values of u_ν^2 for paper I, case B

	u_ν^2		u_ν^2
N_{940}	0,77	N^*_{1520}	0,81
N^*_{1490}	0,23		0,19
Λ_{1115}	0,74	Λ^*_{1520}	0,81
Λ^*_{1665}	0,26		0,19
Σ_{1190}	0,72	Σ^*_{1660}	0,78
Σ^*_{1740}	0,28		0,22
Ξ_{1315}	0,70	Ξ^*_{1815}	0,76
Ξ^*_{1865}	0,30		0,24

This shows, that the problem of resonances can be disentangled from that of the divergencies along our way of a field theory of compound systems. It seems, that the divergencies are connected only with the non-resonance part of the scattering. Our coupling constant renormalisation z_N is simply related to the wave function renormalization z_ν of paper I, chapt.IIId:

$$Z_\nu = <1_\nu \mid 1_\nu) = Z_{\nu p} = <1_{\nu p} \mid 1_{\nu p}) = \frac{U_1}{u_\nu}$$

$$|1_{\nu p}) = \Omega^*_{\nu p} \mid 0)_{\nu p} = Z_\nu N^*_{\nu p} (1 + \sum_{\nu'} y_{\nu\nu'} N^*_{\nu' p} N^*_{-\nu' p} + \dots) |0>$$

$$Z_\nu = z_\nu U_1 \qquad U_1 = <0 \mid 0) = u_1 \dots\dots u_n \,,\; z_\nu = u_\nu^{-1} \,.$$

Table 1 gives $U_1 = 8{,}2 \cdot 10^{-19}$, therefore $Z_N = 9{,}3 \cdot 10^{-19}$ x). The ratio of both renormalization constants is $U_1 = <0|0)$, i.e. the decisive matrix dement measuring the "overlapping" of both Hilbert spaces. We have considered the mass and coupling constant renormalizations resulting from the intrinsic interaction H_{int}, i.e. from the transition from bare to quasiparticle states. The scattering interaction $\tilde{H}_{int}$ again causes such renormalizations. But our two-step proceeding (bound problem-scattering problem) requires, that already the first step gives about the right masses and the main structure of the particles. So it is a necessary condition for our theory, that the renormalization effects of $\tilde{H}_{int}$ are small. This has to be studied in more detailed calculations. The very small numerical value of Z_ν ($\approx 10^{-18}$.) by the way allowes a remark about bootstrap. In the latter, the limit $Z_\nu \to 0$ ("no elementary particle") is essential. From our quasiparticle standpoint indeed Z_ν can be very small, but not zero, because according to paper I, ch.III d, $Z_\nu = \frac{u'_1 \dots u_n}{u_\nu}$ is a finite product (number n of resonances finite) and all u_ν are nonvanishing (table1). It seems therefore interesting not to omit the case $Z_\nu \ll 1$, but Z_ν finite, in bootstrap ("nearly no elementary particle"). But this question has to be studied cautiously, because our Z_ν refers to bare particles, which already "feel" a large part of the total interaction: the potential $V(r)$ defining the orbits $\nu = E_{\bar{\nu}}, j_\nu m_\nu, i_\nu i_{3\nu} y_\nu, \pi_\nu$.

x) This very small value is due to the fact, that U_1 consists out of 48 factors $u_\nu < 1 : 2(2w) + 1(\Lambda) + 3(\Sigma) + 2(\Xi)) + 4(2 + 1 + 3 + 2) = 48$.

III. K- and S-matrix

We are now prepared to solve the main task of this paper, the embedding of the discrete states of paper I into the continuum of free particle scattering states of the preceding chapter. From nuclear reactions it is well known how to solve this task: The scattering waves filling out the full space and the stable state of a resonance within its small volume V (c.m.s.) have to be linked together at the surface of this small volume, so making this state quasistable (point 7 of our list in paper I). The mathematical instrument for this is the reactance matrix K (R-matric, derivation matrix), as well known. Recently Weidenmüller [x] gave a new treatment of the same problem, avoiding the intermediate step of a K-matrix. We give a short account of this method too, because it avoides the noncovariant device of a small radius or volume. [xx] Both derivations of resonance formulas are completely independent of the quasiparticle approximation.

a) Resonance formula I

Weidenmüller directly diagonalizes the Hamiltonian $\tilde{H}$ with the help of the states ψ_a^+, E_a and ϕ_α, w_α. For simplicity we confine to the case of one state ϕ_α only, $1=0$ and elastic scattering, Weidenmüllers chapter 3. One starts from

$$(\psi_a^+, \psi_b^+) = \delta(a-b) \qquad (\psi_a, \tilde{H}\psi_b^+) = E_a \delta(a-b)$$

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x) Weidenmüller, Nucl.Phys., 75, 189 (1966)

xx) The author is indebted to Prof.Rosenfeld for the hint to this method.

$$(\phi_\alpha, \phi_\alpha) = 1 \qquad (\phi_\alpha, \tilde{H}\,\phi_\alpha) = \epsilon_\alpha$$

$$(\psi^+_\alpha, \phi_\alpha) = g_{\alpha\alpha} \quad (\psi_\alpha, \tilde{H}\,\phi_\alpha) = y^+_{\alpha\alpha} \ .$$

One determines the asymptotic form of

$$\Psi_\alpha = \int C_\alpha(E_b)\,\psi^+_b\,dE_b + c_\alpha\,\phi_\alpha$$

$$H\Psi_\alpha = E_\alpha\Psi_\alpha \ ,$$

which is found to be $(\psi^+_\alpha(\infty) \approx \frac{1}{r}\sin(k_\alpha r + \delta_\alpha))$

$$\Psi_\alpha(\infty) \approx \frac{1}{r}\,e^{i\Delta_{\alpha\alpha}}\,(z_{\alpha\alpha}(E_\alpha) + i\pi)\sin(k_\alpha r + \delta_\alpha + \Delta_{\alpha\alpha}) \ .$$

From this one gets directly the S-matrix

$$S_{\alpha\alpha} = e^{2i(\delta_\alpha + \Delta_{\alpha\alpha})}$$

$$e^{2i\Delta_{\alpha\alpha}} = 1 - \frac{2\pi i\,|\,y_{\alpha\alpha\alpha}\,|^2}{E_\alpha - w_\alpha - G_{\alpha\alpha}(E_\alpha) + i\pi\,|\,y_{\alpha\alpha\alpha}\,|^2}$$

$$y_{\alpha b\alpha} = y_{b\alpha} - E_\alpha\,g_{b\alpha} \qquad G_{\alpha\alpha}(E_\alpha) = F_{\alpha\alpha}(E_\alpha) + \epsilon_\alpha - w_\alpha$$

$$F_{\alpha\alpha}(E_\alpha) = P\int \frac{|\,y_{\alpha b\alpha}\,|^2}{E_\alpha - E_b}\,dE_{\alpha b} \equiv \epsilon_\alpha - E_\alpha - z_{\alpha\alpha}(E_\alpha)\,|\,y_{\alpha\alpha\alpha}\,|^2 \ .$$

This differs from our later formula II mainly by the appearance of a level shift $G_{\alpha\alpha}(E_\alpha)$, and by the lack of the extra $\gamma_{\alpha\alpha}$ in the numerator and in $\Gamma_{\alpha\alpha}$. Such $\gamma_{\alpha\alpha}$ -containing the amplitudes at radius R- cannot appear here, because the present formulation avoides the introduction of a radius. This removes the decisive difficulty for a fully covariant formulation (!) and avoides the intermediate step of calculating a K- matrix.

Altogether we have

$$S_{\alpha\alpha}(E) = e^{2i\delta_\alpha}\left(1 - \frac{i\,\Gamma_{\alpha\alpha}}{E - \omega'_{\alpha\alpha} + \frac{i}{2}\,\Gamma_{\alpha\alpha}}\right)$$

$$w'_{\alpha\alpha} = w_\alpha + G_{\alpha\alpha}(E_\alpha)$$

$$\Gamma_{\alpha\alpha} = 2\pi\,|\,\gamma_{\alpha\alpha\alpha}\,|^2 \,.$$

b) Resonance formula II

Let us call ϕ_α the stable states within the small volume, i.e. our quasiparticle states $\Omega^*_\nu\,|0\rangle$, $M^*_k\,||0\rangle\rangle$, etc.:

$$H\Phi_\alpha = w_\alpha \Phi_\alpha \qquad \text{in } V,$$

where H is the approximated Hamiltonian of paper I. Besides we have the full space scattering states with Hamiltonian $\tilde{H}$, e.g. (q) :

$$H\Phi_\alpha = E_\alpha \Phi_\alpha \qquad \tilde{H} = \tilde{H}_0 + \tilde{H}_{int} \qquad \text{full space}$$

We use Φ_α in the Lippman- Schwinger form [x] for standing waves, in order to bring in the K- matrix:

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x) Schweber, Relativistic Quantum Field Theory, Chapter 11e.

$$\Phi_\alpha = |\psi_\alpha^1> = |\phi_\alpha> + P\,\frac{1}{E_0 - \tilde{H}_0}\sum_b K_{b\alpha}\,|\phi_b>$$

$$K_{b\alpha} = <\phi_b|\,\tilde{H}_{int}\,|\psi_\alpha^1>.$$

Besides we need incoming and outgoing waves

$$|\psi_\alpha^{\pm}> = |\phi_\alpha> + \frac{1}{E - \tilde{H}_0 \pm i\epsilon}\,\tilde{H}_{int}\,|\psi_\alpha^{\pm}> = \tag{11}$$

$$= \sum_c \left(\delta_{\alpha c} + \frac{1}{E_\alpha - E_c \pm i\epsilon}\,R^{\pm}_{c\alpha}\right)|\phi_c>$$

$$\Phi_\alpha = |\psi_\alpha^1> = \sum_b \left(\delta_{\alpha b} \mp i\pi\,\delta(E_\alpha - E_b)\,K_{b\alpha}\right)|\psi_b^{\pm}> \quad \text{full space.} \tag{12}$$

Here

$$\tilde{H}_0\,|\phi_\alpha> = E_\alpha\,|\phi_\alpha>. \qquad |\equiv \sum_b \Phi_\alpha^b$$

Define

$$y_{\alpha\alpha} = (\Phi_\alpha, H\Phi_\alpha) - (H\Phi_\alpha, \Phi_\alpha) = (E_\alpha - w_\alpha)(\Phi_\alpha, \Phi_\alpha).$$

Developing Φ_α within V in terms of the Φ_α, we get therefore

$$\Phi_\alpha = \sum_\alpha N_{\alpha\alpha}\,\Phi_\alpha \qquad N_{\alpha\alpha} = (\Phi_\alpha, \Phi_\alpha) =$$

$$= \sum_\alpha \frac{y_{\alpha\alpha}}{E_\alpha - w_\alpha}\,\Phi_\alpha \qquad \text{in V.} \tag{13}$$

The both expressions of Φ_α, (12), and (13), have to be equal at the surface of the small volume for each channel. More precisely this surface is really a hypersurface in the configuration space of $|\psi_b^{\pm}>$, because according to (11) the latter contains all channels e, each with its own variables. So we choose the channel radius for c=b so that $\frac{\partial}{\partial r_b}\,|\phi_b> = 0$, for c≠b so that $|\phi_c> = 0$ x),

x) Really these conditions can be fulfilled only for either the "large" or the "small" Dirac components. So in the following some "leakage" terms are neglected.

and call this set of channel radii $\{R\}_b$, defining our hypersurface. We require for each channel b

$$\Phi_\alpha(\text{in } V)_{R_b} = \Phi_\alpha^b(\text{outside } V)_{\{R\}_b}$$

$$\sum_\alpha \frac{y_{\alpha\alpha}\Phi_\alpha(R_b)}{E_\alpha - w_\alpha} = (\delta_{\alpha b} \mp i\pi\delta(E_\alpha - E_b)K_{b\alpha})|\psi_b^\pm>_{\{R\}_b}) \quad ,$$

where $|\psi_b^\pm>_{\{R\}_b}$ -according to (11) and the above conditions for the $|\phi_\alpha>$ -contains only $|\phi_b>$. Multiplying now with ${}_{R_b}<\phi_b|$ from the left, we get

$$\delta_{\alpha b} - \sum_\alpha \frac{y_{\alpha\alpha}\gamma^\pm_{\alpha b}}{E_\alpha - w_\alpha} = \mp i\pi\delta(E_\alpha - E_b)K^\pm_{b\alpha} = \mp i K^\pm_{b\alpha} \quad ,$$

$$\gamma^\pm_{\alpha b} = \frac{{}_{R_b}<\phi_b|\Phi_\alpha>_{R_b}}{{}_{R_b}<\phi_b|\psi_b>_{\{R\}_b}} \quad .$$

So the necessity to use either the + or the - system of orthogonal states $|\psi_b^\pm>$ leads primarily to two K-matrices. But K is connected with standing waves and therefore has to be symmetrical in outgoing and ingoing waves: The right K-matrix is

$$K_{b\alpha}(E) = \frac{1}{2}(K^+_{b\alpha} + K^-_{b\alpha}) = i\sum_\alpha \frac{y_{\alpha\alpha}\gamma_{\alpha b}}{E - w_\alpha} \qquad (14)$$

$$y_{\alpha b} = (\Phi_\alpha, \bar{\bar{H}}\Phi_\alpha) - (H\Phi_\alpha, \Phi_\alpha) = (\Phi_\alpha, (\bar{\bar{H}} - H)\Phi_\alpha)$$

$$y_{ab} = \frac{1}{2}\,(y^{+}_{ab} - y^{-}_{ab})\,.$$

First we observe the resonance denominator at w_α, appearing in K as a real pole, as it must be (nuclear reactions: real pole of first order). Let us assume, that the resonances are well separated.. Then it is easy to show according to the appendix, that the corresponding S-matrix has a Breit-Wigner form, so getting a formula for the width Γ_α of resonance α :

$$S_{b\alpha}(E) = \delta_{b\alpha} - \sum_{\alpha} \frac{2i\,\Gamma_{\alpha\alpha b}}{E - w_\alpha + i\,\Gamma_\alpha}$$

$$\Gamma_\alpha = \sum_{\alpha} \Gamma_{\alpha\alpha\alpha} \qquad \Gamma_{\alpha\alpha b} = y_{\alpha\alpha}\, y_{\alpha b}\,.$$

As well known, for definitely overlapping resonances K does not change its simple form (1), while S gets a more complicated non BW-form.[x)]

Secondly a resonance term in K appears only if the numerator does not vanish. $y_{\alpha\alpha}$,containing essentially the quotients of amplitudes ϕ_α / ψ_b. at the surface of V, is finite in general. For a non-vanishing $y_{\alpha\alpha}$ the difference $\tilde{H} - H$ -i.e. that "rest" part of the full space Hamiltonian $\tilde{H}$, not respected in our bound state calculation in paper I - has to make transitions between Φ_α and Φ_α . We have seen in (10), that indeed $\tilde{H}_{int}$ of e.g. $N\bar{N}$-scattering can be brought into the form creating and destroying a meson resonance, so giving definitely a nonvanishing numerator in (14). Analogous considerations can be have for other examples, so our "pragmatic" choice of effective interactions plus our K reproduces the usual (lowerst order) graph scheme, but for (approximately) physical particles, quite independently of the quasiparticle approximation of paper I. Concrete applications will be made in the next paper of this series.

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x) Two strongly overlapping resonances: Wigner, Phys.Rev., 70,606 (46).

We have written down only the resonance scattering. Besides there exists a nonresonance contribution, which can be calculated without the intermediate stage of K-matrix. Another neglected effect are cloud contributions. For we treated only the core, though we know that in our picture this dense many particle cloud of order 10^{-14} cm is surrounded by a dilute "corona", in which presumably the fermions (barions, quarks etc.) condensate to mesons. The scattering contributions from this cloud are intimately connected with that transient effects (Einschwingvorgänge) and leakage effects already mentioned.

c) Orientative calculation of matrix elements $y_{\alpha\alpha}$

As am example let us consider $y_{\alpha\alpha}$ of formula II.
It is useful to divide $y_{\alpha\alpha}$:

$$y^{ex}_{\alpha\alpha} = (\Phi_\alpha, \tilde{H}_{int}\,\Phi_\alpha) \qquad y^{in}_{\alpha\alpha} = (\Phi_\alpha, (\tilde{H}_0 - H)\,\Phi_\alpha).$$

Here $\tilde{H}_0$ is the free energy of the physical particles: example $N\bar{N}$-scattering:

$$\tilde{H}_0 = \tilde{H}_{0N} = \sum_p E_{Np}(\Omega^*_{Np}\,\Omega_{Np} + \bar{\Omega}^*_{Np}\,\bar{\Omega}_{Np}).$$

And H is the free energy of the (stable) states α at trest: example $N\bar{N}$-scattering: $H = \sum_\alpha \omega_\alpha M^*_\alpha M_\alpha$. So besides the "exterior" resonance scattering by $\tilde{H}_{int}$, we have an "intrinsic" resonance contribution by $\tilde{H}_0 - H$. For $\tilde{H}_0$ or H -though formally energies of free particles contain the intrinsic interactions $\bar{H}_{pdr}$ or $\bar{H}_{rest}$ already respected in paper I, which begin to act in scattering too, as soon as the particles begin to penetrate. Indeed the diagonal operator $\tilde{H}_0$, H nontheless connect α and α , namely the common part $\Omega_N\,\bar{\Omega}_N$ contained in α and α .

We calculate now $y_{\alpha\alpha}$ in quasiparticle approximation orientatively.

First the pure exterior contribution to K is

$$y^{ex}_{\alpha\alpha} \approx g_r \sum_{\lambda\lambda'\lambda''\lambda'''} x^{N}_{\lambda}(p)\, x^{\bar{N}}_{\lambda'}(-p)\, x^{N^*}_{\lambda''}(p')\, x^{\bar{N}^*}_{\lambda'''}(-p')\, a^{\alpha}_{\lambda''\lambda'''},$$

where p and p' are the center of mass momenta before and after the scattering. On the other hand with the approximation $(M_\alpha \| 0)) = 0)$

$$\| 0)) = U_2 |0) + \ldots . \approx U_2 | 0), \qquad \text{(orientatively)},$$

we get

$$y^{in}_{\alpha\alpha} = (2\omega_N - \omega_\alpha)\, \mu^{\alpha^*}_{N\bar{N}}\, \pi^*_{0\alpha}\, U_2 \qquad \pi_{c\alpha} = \delta_{c\alpha} + P \frac{1}{E_\alpha - E_c} K_{c\alpha}$$

$$c = 0 : \; p_c = p' = 0$$

So both parts of the vertex function $y_{\alpha\alpha}$ in our (quasiparticle) resonance approximation separate in the form $f_i(p)\, f_\alpha(0)\, f_f(p')$. This is the direct expression for the three step character of our approximation: $\Omega\bar{\Omega} - M_\alpha - \Omega\bar{\Omega}$. Therefore we can state that <u>not only the lines in Fig. 1</u>

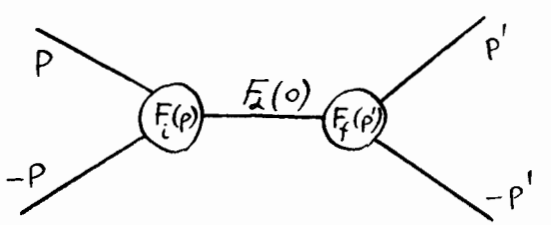

Fig. 1

<u>but the vertices too are the physical ones</u>. Altogether we were led to a scheme of compound system resonance scattering, which can be described by a graph scheme resembling that of dispersion relations. Based on this, one can develop a bootstrap scheme, e.g. for the graph of Fig.1 and its crossing transform.

IV. Conclusion

Using some simplifying assumptions, we were lead to a simple quantum field theory of compound systems , based on the quasiparticle approximation. For the resonance part of the scattering process this theory avoides divergencies and allowes a finite renormalization. The most imporatant methodical fact of our attempt is the two-step proceeding, performing first a bound state approximation (1) and only after that treating the scattering (II). Correspondingly we worked with two different Hamiltonians H_{int} (I) and $\overset{ss}{H}_{int}$ (II). which were found to lead to interior and exterier resonance contributions respectively.

We succeeded in deriving a resonance formula for the (K matrix and) S-matrix. By this we got a graph scheme resembling that of dispersion relations, with physical lines and vertices. Another interesting aspect in the connection of our formulation with bilocal (polylocal) fields.

The most important drawback up to now is the not full covariance of the formulation. There are different sources of this (Schrödinger picture , use of angular momentum states, of a static potential $V(r)$ etc.). In a following paper of this series we shall try to remove this. In the present paper we already prepared it by using Dirac spinors, Lorentz contraction etc.

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Appendix

In matrix form we have for one single resonance

$$K^{\alpha} = \frac{y \times y}{(E-\omega)} \equiv \frac{N}{\Delta E} \qquad y_{\alpha} = y_{\alpha\alpha} \quad y_{\alpha} = y_{\alpha\alpha} \quad .$$

Further

$$S = (1 - i\,K)(1 + iK)^{-1} = 1 - 2iK(1 + iK)^{-1}$$

$$= 1 - 2N(a + N)^{-1} \qquad a = -i\Delta E \, .$$

It is easily shown, that the Kronecker product N has the property

$$N \cdot N = \sigma N \qquad \sigma = \mathrm{Tr}\, N = \sum_{\alpha} y_{\alpha} y_{\alpha} \, .$$

Therefore

$$N \frac{1}{a + N} = N(a + \sigma) \frac{1}{(a + N)(a + \sigma)} =$$

$$= N(a + N) \frac{1}{(a + N)(a + \sigma)} = \frac{N}{a + \sigma} \, .$$

$$S^{\alpha} = 1 - 2 \frac{N^{\alpha}}{a_{\alpha} + \sigma_{\alpha}} \qquad \text{q. e. d.}$$

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