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# PHOTOPRODUCTION OF MESONS AND DIVERGENCE CONDITIONS

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It has been observed[1-5] that the interplay between strong interactions and electromagnetic or weak interactions is governed to some extent by divergence conditions on currents. In this context a very important point – recognized by M.Veltman[4] is that the majority of results of current algebra can be derived from divergence conditions.

In this paper we apply the divergence condition approach to the processes $\gamma p \to MB$ near threshold (where $B$ denotes a member of baryon octet of decuplet and $M$ stands for a pseudoscalar or vector meson). As a result, the cross sections near threshold can be expressed through certain matrix elements of the axial-vector and tensor currents. Some of these matrix elements are known from weak interactions, the others can be estimated applying the $U(6) \otimes U(6)$ symmetry at rest.

Essentially the same relations were already derived with current-algebraic technique[6-8] and it is very interesting to note that similar results follow from the entirely different assumptions of the composite particle quark-model[9].

1. First we deal with the photoproduction of pseudoscalar mesons. Neglecting wak interactions the divergence condition for the axial-vector currents $A_\mu(x)_i$ , $(i = 1, \dots, 8)$ is the following[10-12]:

$$\partial^\mu A_\mu(x)_i - e A^\mu(x)\left(f_{i3j} + \frac{1}{\sqrt{3}} f_{i8j}\right) A_\mu(x)_j = i m_i^2 f_i \phi_i(x) . \qquad (1)$$

Here $A_\mu(x)$ is the electromagnetic field, $\phi_i(x)$ is the pseudoscalar meson field with mass $m_i$ and by definition

$$\langle 0 | A_\mu(0)_i | p_i \rangle = \frac{f_i \, q_\mu}{\sqrt{2(2\pi)^3}} , \qquad (2)$$

where $| p_i \rangle$ is the one-pseudoscalar meson state with momentum $q$. The value of the constant $f_\pi$ ($f_K$) is known from the decay $\pi \to \mu\nu$ ($K \to \mu\nu$) to be $f_\pi = 0.138\, M_N$ ( $f_K = 1.28\, f_\pi$ ).

Let us define $T_{\gamma p \to p_i B}$ by

$${}_{out}\langle p_i B / \gamma p \rangle_{in} = i(2\pi)^4 \delta^4(q + p_B - p_p - k) \frac{T_{\gamma p \to p_i B}}{2(2\pi)^3} . \qquad (3)$$

( We introduce $k$ for the momentum and $\epsilon(k)$ for the polarization vector of the incoming photon). Up to the first order in $e$ one easily obtains from eq. (1):

$$T_{\gamma p \to p_i B} = \frac{m_i^2 - q^2}{m_i^2} \left( T^{(1)}_{\gamma p \to p_i B} + T^{(2)}_{\gamma p \to p_i B} \right) , \qquad (4)$$

where

$$T^{(1)}_{\gamma p \to p_i B} = \frac{\sqrt{2(2\pi)^3}}{f_i} q^\mu \langle B | A_\mu(0)_i | p \gamma \rangle_{in} = ; \qquad (5)$$

$V_\mu(x)_i$ $(i = 1, \dots, 8)$ being the vector currents, and

$$T^{(2)}_{\gamma p \to p_i B} = \frac{i e \epsilon^\mu(k)}{f_i} \langle B | ( f_{i3j} + \frac{1}{\sqrt{3}} f_{i8j} ) A_\mu(0)_j | p \rangle . \qquad (6)$$

Now we suppose that the physical amplitude does not differ very much from the off-mass-shell amplitude at $q^2 = 0$. Then near threshold one can neglect the term $T^{(1)}$ and hence the amplitude is expressed by a matrix element of the axial-vector current. For example in the case of the reaction $\gamma p \to \pi^+ n$ we have with $\kappa = p_n - p_p$ :



$$T_{\gamma p \to \pi^+ n} \approx T^{(2)}_{\gamma p \to \pi^+ n} = \frac{-i e \epsilon^\mu(k)}{(2\pi)^3 f_\pi} \bar{u}_n (\gamma_\mu F_1^A + \kappa_\mu F_2^A) \gamma_5 u_p . \qquad (7)$$

Here $F^A_{1,2}(\kappa^2)$ denote the form factors of the axialvector current $A_\mu(x)$ - between proton and neutron states. From eqs. (7) and (3) we have for the total cross section $\sigma(\pi^+ n)$ near threshold:

$$\sigma(\pi^+ n) = \frac{e^2}{4\pi} \frac{M_N^2}{f_\pi^2} \frac{|\vec{q}|}{k_0 (k_0 + p_{p0})^2} F(\pi^+ n) , \qquad (8)$$

where $M_N$ is the nucleon mass, $k_0$ and $p_{p0}$ are the c.m. energies of $\gamma$ and proton respectively, $\vec{q}$ is the c.m. momentum of pion and $F(\pi^+ n)^{1/2} = q_A / q_V$ is the renormalized axial vector coupling constant. Using the world-average data of J.T.Beale et al./13/: $\sigma(\pi^+ n) = 100 \pm 1\ \mu b$ at $k_{lab} = 185 \pm 2$ MeV one obtains $q_A / q_V = 1.20 \pm 0.02$ which agrees well with the known value $q_A / q_V = 1.18$ . ( Between the threshold at about 152 MeV and 185 MeV one would expect even a better agreement but there the data are few and the errors are larger. This motivates our choice of photon energy $k_{lab} = 185$ MeV). It is important to note that this agreement is achieved by putting for $f_\pi$ its measured value contrary to the general custom in current algebra, where the value

$$f_\pi^{(GT)} = \frac{\sqrt{2}\, M_N \, q_A / q_V}{g_r} = 0.125\, M_N$$

is used ( $g_r$ = renormalized $\pi N$ coupling constant). We consider the unequality of the two values $f_\pi$ and $f_\pi^{(GT)}$ as an evidence for $g_r(0) \neq g_r(m_\pi^2)$ and thus the use of $f_\pi$ as some sort of taking into account off-mass-shell effects. It is interesting that also the Adler-Weisberger sum rule[14] gives a good value for $g_A/g_V$ if one takes simply the measured value for $f_\pi$ and apart from this ignores off-mass-shell corrections. According to the new data[15] one thus gets $g_A/g_V = 1.15$ (instead of 1.16 of ref.[15]).

Eq. (6) can be used to determine the cross sections of other processes too if the relevant axial-vector matrix element is known. In the case of $\gamma p \to K^+ (\Lambda$ or $\Sigma^0)$ the constant $F(\pi^+ n)$ in eq. (8) must be replaced by

$$F(K^+ \Lambda) = 0.67 \frac{M_\Lambda}{M_N} \left(\frac{f_\pi}{f_K}\right)^2 ; \quad F(K^+\Sigma^0) = 0.061 \frac{M_{\Sigma^0}}{M_N} \left(\frac{f_\pi}{f_K}\right)^2 , \tag{9a}$$

respectively. This correspond to the value $g_A/g_V = 1.18$, $D/F = 1.84$ of C.E.Carlson[16]. In the cases where unknown matrix elements appear one can make use of the fact that in the compact $U(12)$ algebra $A_k(x)_i$ ($k = 1,2,3$) belong to the generators of the rest symmetry group $U(6) \times U(6)$. (It must be noted however that the $U(6) \times U(6)$ values give only crude approximations. It is well known e.q. that in the proton-neutron case $U(6) \times U(6)$ gives a value 1.67 instead of 1.18). In $U(6) \times U(6)$ a straightforward calculation gives

$$F(\pi^+ N^{*0}) = 0.90 \frac{M_{N^*}}{M_N} ; \tag{9b}$$

$$F(\pi^- N^{*++}) = 2.7 \frac{M_{N^*}}{M_N} ; \tag{9c}$$

$$F(K^+ Y_1^{*0}) = 0.45 \frac{M_{Y_1^*}}{M_N} \left(\frac{f_\pi}{f_K}\right)^2 ; \tag{9d}$$

$$F(\pi^0 B) = F(\eta B) = F(K^0 B) = 0 . \tag{9e}$$



The interesting prediction (9e) of eq. (1) is the vanishing of

$$\sigma \frac{k_0 (k_0 + p_{p0})^2}{|\vec{q}\,|}$$

near threshold for the neutral members of the octet. The process $\gamma p \to \pi^0 p$ shows such a behaviour indeed. Nevertheless the cross section rises rapidly above 200 MeV and have a large maximum about 300 MeV. As $T^{(2)}$ vanishes, in this region the term $T^{(1)}$ gives a large contribution, which is due to the $P_{33}$ resonance $N^*(1236)$. Recent experiments show that the same happens also for $\eta$ - production, but there the maximum is in the immediate neighbourhood (about 760 MeV) of the threshold (at 710 MeV). The last measurement[17] has given an evidence for the $S_{11}$ resonance $N^*(1570)$ being the relevant intermediate state.

Table 1 shows the predictions of eq. (1) compared to experimental data. For charged pion production the agreement is encouraging indeed, the discrepancy in the $\pi^0$ and $\eta$ production can be explained by the presence of resonances, but for $K^+$ mesons (at least in the $\gamma p \to K^+ \Lambda$ case) a serious disagreement is found.

Eq. (1) has been derived from the assumptions of PCAC and the minimality of electromagnetic interactions, therefore (maintaining minimality) a possible explanation would be a complete failure of PCAC for $K$ mesons (expressed for example by the rapid variation of formfactors between $q^2 = m_K^2$ and $q^2 = 0$). But if PCAC gives a reasonable approximation, then this seems to show that there is a considerable SU(3) breaking in the axial-vector matrix elements resulting in the suppression on of strangeness changing weak interactions. In any case this point needs further investigations.

2. Now we briefly describe how the same approach can be applyed to the photoproduction of vector mesons. It has been proposed[18-20] recently that in analogy with PCAC also the tensor current $T_{\mu\nu}(x)_i$ ($i = 0,1...8$) may be "partially conserved" that is

$$\partial^\mu T_{\mu\nu}(x)_i = L_\nu(x)_i , \tag{10}$$

where the matrix elements of the operator $L_\nu(x)$ are dominated by the vector meson pole (or simply $L_\nu(x)_i = iF_i M_i^2 \phi_\nu(x)_i$ ; $\phi_\nu(x)_i$ being the fields of vector mesons with mass $M_i$ ). This hypothesis is called PCTC. The strong analogy between PCAC and PCTC suggests that one can try the description of photoproduction of vector mesons near threshold along the same lines as for pseudoscalar mesons.

First of all in the quark model it is possible to derive the following divergence condition for $T_{\mu\nu}(x)_i$, taking into account electromagnetic interactions:

$$\partial^\mu T_{\mu\nu}(x)_i - eA^\mu(x)\left[\left(f_{i3j} + \frac{1}{\sqrt{3}} f_{i8j}\right) T_{\mu\nu}(x)_j - g_{\mu\nu}\left(d_{i3j} + \frac{1}{\sqrt{3}} d_{i8j}\right) S(x)_j\right] \tag{11}$$

$$= i M_i^2 F_i \phi_\nu(x)_i \quad .$$

Here $S(x)_i$ stands for the scalar densities in the quark model. This equation shows that the "principle of minimality of interactions" does not work here as simply as for vector and axial-vector currents. Here eq.(11) cannot be derived from eq. (10) substituting simply $\partial_\mu$ by $\partial_\mu \pm ieA_\mu$ . We have to make such a substitution for the "fundamental fields" (in our case the quark fields).

As $T_{ok}(x)_i$ and $S(x)_i$ belong also to the subalgebra $U(6) \otimes U(6)$ we can use also here $U(6) \otimes U(6)$ for the approximate determination of matrix elements. A calculation analogous to those of the preceding gives that in eq. (8) we must substitute respectively

$$F(\rho^+ n) = 5.7\left(\frac{f_\pi}{F_\rho}\right)^2 ; \quad F(\rho^+ N^{*0}) = 1.8\frac{M_{N^*}}{M_N}\left(\frac{f_\pi}{F_\rho}\right)^2 ; \tag{12a}$$

$$F(\rho^- N^{*++}) = 5.4\frac{M_{N^*}}{M_N}\left(\frac{f_\pi}{F_\rho}\right)^2 ; \quad F(\rho^\circ p) = 5.5\left(\frac{f_\pi}{F_\rho}\right)^2 ; \tag{12b}$$



$$F(\rho^0 N^{*+}) = F(\phi p) = F(\phi N^{*+}) = F(\omega N^{*+}) = F(K^{*0} Y_1^{*+}) = 0 ;$$

$$F(\omega p) = 2\left(\frac{f_\pi}{F_\phi}\right)^2 ; \tag{12c}$$

$$F(K^{*+}\Lambda) = 3.2\frac{M_\Lambda}{M_N}\left(\frac{f_\pi}{F_{K^*}}\right)^2 ; \quad F(K^{*+} Y^{*0}) = 0.89\frac{M_{Y_1^*}}{M_N}\left(\frac{f_\pi}{F_{K^*}}\right)^2 ; \tag{12d}$$

$$F(K^{*+}\Sigma^0) = 0.18\frac{M_{\Sigma^0}}{M_N}\left(\frac{f_\pi}{F_{K^*}}\right)^2 ; \quad F(K^{*0}\Sigma^+) = 0.45\frac{M_{\Sigma^+}}{M_N}\left(\frac{f_\pi}{F_{K^*}}\right)^2$$

Experimentally the process $\gamma p \to \rho^\circ p$ is known the most accurately/21,22/. The comparison with data shows that eq. (12b) gives considerably larger cross sections than the experiments if we take $F_V = f_\rho$ . This indicates $F_V > f_\rho$ or (and) a renormalization with respect to $U(6) \times U(6)$ values. Reasonable agreement can be achieved (see Table 1) taking $F_\nu = \sqrt{2}\, f_\rho$ and a renormalization factor $\frac{1}{2}$ in $F(\rho^\circ p)$ (like in the case of $g_A / g_V$ !).

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## Table 1

| Process | Refer. | $k_{lob}$(Mew) | $\sigma(k_{lob})(\mu b)$ | $\alpha_{exp}$ | $\alpha$ theor |
|---|---|---|---|---|---|
| $\gamma p \to \pi^+ n$ | 13 | 185± 2 | 100 ±1 | 215± 8 | 208 |
| $\gamma p \to \pi^0 p$ | 13 | 180± 5 | 9 ± 1 | 20 ±4 | 0 |
| $\gamma p \to \pi^- N^{*++}$ | 21 | 625± 25 | 72± 5 | 480± 60 | 530 |
| $\gamma p \to \eta p$ | 17 | 730± 5 | 0.23±0.2 | 2.0± 1.8 | 0 |
| $\gamma p \to K^+ \Lambda$ | 24 | 1000±10 | 1.9±0.2 | 17± 3 | 73 |
| $\gamma p \to K^+ \Sigma^0$ | 24 | 1160±20 | 0.7±0.2 | 6±2 | 7 |
| $\gamma p \to \rho^0 p$ | 21 | 1400±50 | 20±3 | 130±30 | 200 |
| | 22 | 1400±100 | 30±8 | 190±80 | |

$$\alpha \equiv \sigma \frac{k_0 (k_0 + p_{po})^2}{|\vec{q}|}$$