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V.Z. Baluny

GENERALIZATION OF THE CABIBBO–RADICATI RELATION TO THE CASE OF NONMINIMAL ELECTROMAGNETIC INTERACTIONS

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GENERALIZATION OF THE CABIBBO–RADICATI RELATION TO THE CASE OF NONMINIMAL ELECTROMAGNETIC INTERACTIONS

Recently the current algebra method has been widely applied in deriving integral sum rules[1]. It is well known that this method suffers from two serious shortcomings, giving rise to some criticizm concerning the before-mentioned sum rules. The first difficulty is connected with the so-called Schwinger terms. In some cases it is avoided by considering commutators of currents or some combinations of these commutators in which these terms do not contribute. The second has a dynamical nature. The algebraic structure depends essentially on the model and on the assumed structure of currents, in particular. We point out that the equal-time commutation relations can not be proved even for simple models, if derivation of spinors, for example, enter into currents.

In view of these and other difficulties the method of integral ("superconvergent") sum rules without applying the current algebra is widely discussed[2].

The method is based on the local properties of currents and on a strong assumption about the sufficiently rapid decrease of the amplitude at high energies.

Veltman[3] and Terentyev[4], assuming a definite structure of the vector and axial-vector currents divergences, have obtained many results of the current algebra. We point out, that though, the method does not exhibit the flexibility of the current algebra, it does not suffer from its shortcomings.

Recently it has been shown[5], that Veltman condition for current divergences can be obtained in the framework of the quark model with the minimal electromagnetic interactions (e.m.). This brought a possibility of penetrating into dynamics of interactions without utilization of the canonical equal-time commutation relations.

In the present note we shall derive the modified Cabibbo-Radicati relation by means of the generalized Veltman condition, which includes the contribution from the possible nonminimality of e.m. interactions.

The Cabibbo-Radicati relation has been derived in many works. Several authors[6] have used current algebra for this purpose, assuming minimality of e.m. interactions and vanishing of the corresponding invariant amplitude at high energies. Another group of authors[7] has derived this relation in the form of a superconvergent sum rule assuming rapid decrease of the invariant amplitude at high energies. We, however, shall demand the vanishing of this amplitude at high energies only,

Veltman condition for the divergence of the axial-vector current with e.m. interactions only is

$$\frac{\partial J^{k}_{\mu}}{\partial x_{\mu}} = \epsilon_{k\ell 1} \, J^{\ell}_{\mu}(x) \, e \, A^{1}_{\mu}(x) \, , \qquad (1)$$

where A^{1}_{μ} is the isotopic vector of the e.m. field. We stress that the relation (1) holds strictly in any theory provided the e.m. interactions are switched on in a minimal way[4,7]. Therefore, any deviation from (1) results from nonminimality of e.m. interactions.

Assume that e.m. interactions are switched on in a nonminimal way. Then instead of the condition (1) we get[x]

$$\frac{\partial J^{k}_{\mu}(x)}{\partial x_{\mu}} = \epsilon_{k\ell 1} \, e \left\{ J^{\ell}_{\mu}(x) A^{1}_{\mu}(x) + g \, \frac{\partial J^{\ell}_{\mu\nu}(x) A^{1}_{\mu}(x)}{\partial x_{\nu}} \right\} , \qquad (2)$$

where $J^{\ell}_{\mu\nu}(x)$ is a antisymmetrical tensor current and g is a constant.

To derive the Cabibbo-Radicati relation we shall formally introduce charged components of the photon. Consider the matrix element of scattering of isovector photons on nucleons

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[x] Eq. (2) may be proved, e.g. in the quark model, if the Pauli term is included into the interaction Lagrangian.

$$S = {}_{out}\langle N(p_2)\, e^i(k) / N(p_1)\, e^k(q) \rangle_{in} \quad ,$$

where $e^i(k)$ is the isovector photon polarization vector with the momentum k .

Applying the reduction formula we get

$$S = ie \frac{e^k_\beta}{\sqrt{2q^0}} \int e^{-iqx}\, dx \langle N(p_2) e^i(k) / J^k_\beta(x) / N(p_1) \rangle = ie \frac{e^k_\beta}{\sqrt{2q^0}} M^i_\beta \quad (3)$$

Now, taking into account (2), we get in the lowest order in e.m. interactions

$$q_\beta M^k_{\alpha\beta} = -\int e^{-iqx} \langle N(p_2) e^i(k) / \partial_\mu J^k_\mu(x) / N(p_1) \rangle = \quad (4)$$

$$= i\epsilon_{k\ell i} \frac{e^i_\alpha}{\sqrt{2q^0}}\, e \int e^{i(k-q)x} \langle N(p_2) / J^\ell_\alpha(x) + giq_\gamma J^\ell_{\alpha\gamma}(x) / N(p_1) \rangle .$$

Thus, the matrix element (3) takes the form:

$$S = ie^2 \frac{e^i_\alpha\, e^k_\beta}{\sqrt{4k_0q_0}} (2\pi)^4\, \delta(p_1 + q - p_2 - k)\, \bar{U}(p_2) M^{ik}_{\alpha\beta} U(p_1), \quad (5)$$

where $M^{ik}_{\alpha\beta}$ satisfies the condition:

$$M^{ik}_{\alpha\beta} q_\beta = i\epsilon_{ik\ell} \{ G^\ell_\alpha(p_2, p_1) + igq^\gamma G_{\alpha\gamma}(p_2, p_1) \}$$

$$\bar{U}(p_2) G^\ell_\alpha(p_2, p_1) U(p_1) = \langle N(p_2) / J^\ell_\alpha(0) / N(p_1) \rangle \quad (6)$$

$$\bar{U}(p_2) G^\ell_{\alpha\gamma}(p_2, p_1) U(p_1) = \langle N(p_2) / J^\ell_{\alpha\gamma}(0) / N(p_1) \rangle .$$

Analogously, it can be shown that

$$k_\alpha M^{ik}_{\alpha\beta} = i\epsilon_{ik\ell} \{ G^{\ell}_{\beta}(p_2, p_1) - igk^{\gamma} G^{\ell}_{\beta\gamma}(p_2, p_1) \} . \tag{7}$$

In the following we shall consider the isovector part of $M^{ik}_{\alpha\beta}$ only. Following Gribove, Ioffe and Schechter /6/ we construct a tensor:

$$\tilde{\tilde{M}}^{v}_{\alpha\beta} = Sp \left\{ \frac{\hat{p}_2 + m}{2m} M^{v}_{\alpha\beta} \frac{\hat{p}_1 + m}{2m} \right\} .$$

It may be written in the form:

$$\tilde{\tilde{M}}^{v}_{\alpha\beta} = A_1 P_\alpha k_\beta + A_2 P_\alpha P_\beta + \ldots\ldots ,$$

where $P = p_1 + p_2$ and the ten-functions A_i depend on two invariants $Q^2 = -(k-q)^2$ and $\nu = \frac{Pq}{2m}$. From (6) and (7) it is easy to get relations between the invariant amplitudes A_i. In the following we shall need only one of them

$$\left\{ A_1 + g \frac{G(Q^2)}{m^2} \right\} \frac{Q^2}{2} + 2m\nu A_2 = - \frac{2G_E(Q^2)}{m} , \tag{8}$$

where $G_E(Q^2)$ is the Sacks formfactor of the proton and $G(Q^2)$ is defined by the equality

$$Sp \{ (\hat{p}_2 + m) G_{\alpha\beta} (\hat{p}_1 + m) \} = 2i \{ p^\alpha Q^\beta - p^\beta Q^\alpha \} G(Q^2) .$$

$$(G(0) = 1)$$

If $g = 0$, these relations coincide with those obtained in Gribov et al. work[6].

Untill now no assumption was involved in our considerations. Assume now that a dispersion relation without subtraction can be written for $A_1(\nu, Q^2)$. Then from (8) we obtain

$$-\{G_E(Q^2) + g\frac{G(Q^2)}{4m} Q^2\} = \frac{mQ^2}{4} A_1(0, Q^2) = \frac{mQ^2}{4}\int\frac{d\nu}{\nu} \operatorname{Im} A_1(\nu, Q^2). \quad (9)$$

Using the optical theorem and differentiating both sides with respect to Q^2 we get finally

$$\frac{1}{2m} g = -\frac{(r_0)_p^2 - (r_0)_n^2}{3} - \frac{(\mu_p - \mu_n)^2}{2m} + \frac{1}{2\pi^2 a}\int\frac{d\nu}{\nu}(2\sigma^V_{1/2} - \sigma^V_{3/2}) \quad . \quad (10)$$

Thus, in particular, if $g \neq 0$ then the assumption of rapid decrease of the invariant amplitude contains the assumption of nonminimality of e.m. interactions.

The constant g is not likely to differ much from zero, if at all it does. Therefore a scrupulous test of (10) would finally solve the old dilemma concerning the minimality of e.m. interactions[x].

We point out, finally, that expression (10) does not coincide with the corresponding formula in[10] derived on the basis of equal-time commutation relation method the shortcomings of which are indicated at the beginning of this note. This deviation shows that the before-mentioned method and the Veltman method used here are not always equivalent.

We add, that in contrast to[10], the calculations in this paper are carried out in a relativistically covariant form. This is another advantage of the Veltman method.

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[x] An attempt to test this relation has been made in[9].

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