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EXAMPLE OF AN INFINITE-COMPONENT LOCAL FIELD WHICH ANNIHILATES THE VACUUM

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I.

It is well-known (see e.g.[1] chapter 4) that if in conventional local field theory the field $\phi(x)$ annihilates the vacuum-vector, then it vanishes identically. Otherwise stated, a field with definite sign frequency, say,

$$\phi(x) = \int_{H_m^+} a(p) e^{-ipx} \frac{d^3 p}{p^0}, \qquad (1)$$

where H_m^+ stands for the upper hyperboloid:

$$H_m^+ = \{ \underline{p},\ p^0 = \sqrt{m^2 + \underline{p}^2} \} \qquad (2)$$

can never be local. We shall show that the known infinite component Majorana field[2] (studied also by Gel'fand and Yaglom[3]) supplies a counter-example to this statement, showing that the assumption that all fields transform under finite dimensional (non-unitary) representations of the "index Lorentz group" SL(2,C) is quite essential for conventional field theory. This example will also throw some new light on the problem discussed recently[4-8] about spin and statistics for infinite multiplets. Much of the result of the present note is implicit in[3] but seems to have been generally overlooked.

II.

We recall first some general facts about self-coupled representations of SL(2,C) (ref.[3]).

A representation of SL(2,C) is called self-coupled if it is contained in the direct product of this representation with the four-dimensional vector representation of the Lorentz group. The only irreducible self-coupled representations of SL(2,C) are the uniary representations[x]

$$[0,\tfrac{1}{2}] \text{ and } [\tfrac{1}{2},0]. \qquad (3)$$

In the space X of each of these representations an unitary representation of a larger group, namely of the 10 parameters group of real symplectic transformations in four dimensions Sp(4,R) can be defined. This means that we can introduce in the Lie algebra of (3), besides the Lorentz generators $S^{\mu\nu}$ a fourvector of infinitesimal (hermitian) operators L^μ satisfying

$$\frac{1}{4}[L^\mu, L^\nu] = S^{\mu\nu}, \; [S^{\mu\nu}, L^\lambda] = i(g^{\nu\lambda}L^\mu - g^{\lambda\mu}L^\nu). \qquad (4)$$

For each irreducible representation of SL(2,C) we shall make use of the canonical basis $|\ell\ell_3\rangle$ corresponding to the reduction with respect to SU(2) ($\ell = \ell_0, \ell_0+1, \dots$). The generator L^0 is diagonal in this basis:

$$L^0|\ell\ell_3\rangle = (2\ell+1)|\ell\ell_3\rangle. \qquad (5)$$

This allows to solve the eigenvalue problem

$$L^\mu p_\mu u(\underline{p}) \equiv (\omega L^0 - \underline{L}\,\underline{p})u(\underline{p}) = \kappa\, u(\underline{p}), \; \kappa > 0. \qquad (6)$$

The operator $L^\mu p_\mu$ in the left-hand side of (6) commutes with the spin-

x/ We use the notation $[\ell_0, \ell_1]$ of ref.[3] for the irreducible representations of the Lorentz group. ℓ_0 is half-integer or integer and gives the minimal value of the spin contained in $[\ell_0, \ell_1]$. For the unitary representations of SL(2,C) ℓ_1 is either pure imaginary (principal series) or for $\ell_0 = 0, -1, -1 < \ell_1 < 1$ (supplementary series).

vector $w_\mu = \frac{1}{2}\epsilon_{\rho\lambda\nu\mu} p^\rho S^{\lambda\nu}$ so that it can be diagonalized simultaneously with the total spin s and the spin projection s_3. The energy $p^0 = \omega$ has infinitely many discrete eigenvalues ω_s decreasing with the spin s:

$$\omega_s = \sqrt{m_s^2 + \underline{p}^2} > 0, \qquad m_s = \frac{\kappa}{2s+1} \qquad (7)$$

(To see this it is sufficient to transform (6) to the rest frame and to use (5). Only positive energy appears because of the positive definiteness of L^0 (5)). The corresponding eigenvectors will be denoted by $u_{ss_3}(\underline{p})$

$$u_{ss_3}(\underline{p}) = u_{ss_3}^{\ell\ell_3}(\underline{p})|\ell\ell_3\rangle = V(\Lambda_{p_s})_{ss_3}^{\ell\ell_3}|\ell\ell_3\rangle, \qquad (8)$$

where $V(\Lambda_{p_s})$ is the unitary operator representing the pure Lorentz transformation ("boost") Λ_{p_s} which takes a particle of mass m_s from rest to momentum p_s. For $\underline{p} \neq 0$ the vectors (8) are not orthonormalized, but they satisfy the following summation rules:

$$\bar{u}_{ss_3}(\underline{p}) L^0 u_{s's_3'}(\underline{p}) = \frac{\kappa\,\omega_s}{m_s^2}\delta_{ss'}\delta_{s_3s_3'} \qquad (9)$$

$$\sum_{s,s_3} \frac{m_s^2}{\omega_s} u_{ss_3}^{\ell\ell_3}(\underline{p})(L^0 u_{ss_3}(\underline{p}))^{\ell'\ell_3'} = \kappa\,\delta_{\ell\ell'}\delta_{\ell_3\ell_3'} \qquad (10)$$

which follow from (6).

III.

Let $\psi(x)$ be an infinite component field, transforming under some of the self-coupled representations (3) and satisfying the free field equation

$$i L^{\mu}\partial_{\mu}\psi(x) = \kappa\,\psi(x) \qquad \left(\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}}\right), \tag{11}$$

corresponding to the Lagrangian

$$\mathfrak{L}(x) = \frac{i}{2}\{\psi^*(x) L^{\mu}\partial_{\mu}\psi(x) - \partial_{\mu}\psi^*(x) L^{\mu}\psi(x)\} - \kappa\,\psi^*(x)\psi(x). \tag{12}$$

The properly normalized solution of Eq. (11) has the form

$$\psi(x) = \sqrt{\frac{2}{(2\pi)^3\kappa}}\ \sum_{s s_3} m_s \int_{H^+_{m_s}} a_{s s_3}(\underline{p})\, e^{-ip_s x}\, u_{s s_3}(\underline{p})\frac{d^3p}{\omega_s}, \tag{13}$$

where $a_{ss_3}(\underline{p})$ are annihilation operators, satisfying the invariant (anti) commutation rules:

$$[a_{ss_3}(\underline{p}), a_{s's'_3}(\underline{p}')]_{\pm} = 0$$

$$[a_{ss_3}(\underline{p}), a^*_{s's'_3}(\underline{q})]_{\pm} = \omega_s\,\delta_{ss'}\,\delta_{s_3 s'_3}\,\delta(\underline{p}-\underline{q}). \tag{14}$$

The expression for the energy operator E derived from (12) and (13) is

$$E = \sum_{ss_3}\int_{H^+_{m_s}} a^*_{ss_3}(\underline{p})\, a_{ss_3}(\underline{p})\, d^3p \tag{15}$$

(here we have made use of (9)). It is positive definite for both Bose and Fermi statistics (independently of the spin) i.e. of whether we take the representation $[0,\frac{1}{2}]$ or $[\frac{1}{2},0]$ Furthermore, the field $\psi(x)$ is local. To see this we mention that the conjugate momentum

$$\pi(t,\underline{x}) \equiv \frac{\partial\mathfrak{L}}{\partial(\partial\psi/\partial t.)} = \frac{i}{2}\psi^*(t,\underline{x})L^0 \tag{16}$$

satisfied the well known (anti)-commutation relations for equal time, so that

$$[\psi(t,\underline{x}), \frac{1}{2}\psi^*(t,\underline{y})L^0]_{\pm} = \delta(\underline{x}-\underline{y}), [\psi(\underline{x}),\psi(\underline{y})]_{\pm} = 0 \tag{17}$$

((17) is a direct consequence of (14) and of (10)). Eqs. (17) together with Lorentz invariance imply locality i.e. vanishing of the (anti-) commutators for all space-like distances.

On the other hand, as far as $\psi(x)$ is a superposition of annihilation operators only, we have

$$\psi(x)|0\rangle = 0. \tag{18}$$

In contradistinction with conventional field theory, however, (18) does not imply the vanishing of the two-point function. Actually it follows from (17) that for equal times

$$\langle 0|\psi(t,\underline{x})\psi^*(t,\underline{y})|0\rangle = 2(L^0)^{-1}\delta(\underline{x}-\underline{y}) \tag{19}$$

(the operator L^0 has an inverse because of (5)). The classical reasoning[1] which proves the vanishing of the two-point function is not applicable in our case, because the matrix elements of the infinite-dimensional representations of SL(2,C) do not allow analytic continuation for all complex values of the group parameters, so that the well-known Bargmen-Hall-Wightman theorem is not valid for such representations.

IV.

A field of type (13) naturally leads to a theory without crossing symmetry. It may serve for the construction of a relativistic Lee-type model. Actually, infinite-component fields may be used not only for such pathological examples. Using reducible representations of SL(2,C) we can construct Fermi fields which contain particles and antiparticles with opposite parity. For this purpose one can consider the Lagrangian

$$\mathfrak{L}(x) = \sum_{\epsilon=\pm} \{ \frac{1}{2} [:\psi^*_\epsilon(x) L^\mu \partial_\mu \psi_\epsilon(x): - :\partial_\mu \psi^*_\epsilon(x) L^\mu \psi_\epsilon(x):] - \kappa\epsilon :\psi^*_\epsilon(x)\psi_\epsilon(x): \}. \qquad (20)$$

It has to be mentioned that all examples of anticommuting infinite component fields which contradict theorem II of[5] correspond to theories with mass-spectrum destroying the invariance with respect to the so-called "auxiliary group".

The unphysical decreasing mass spectrum may be corrected by introducing a suitable interaction term or (for the case of a reducible representation of SL(2,C)) by including a (scalar) mass term, which is not a multiple of the identity.

A more detailed account of the remarks of this last section is in preparation.

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