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ON THE PROBLEM OF CONSTRUCTION OF THE UNITARY RENORMALIZABLE FIELD THEORY WITHOUT ULTRAVIOLET DIVERGENCES

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1. Introduction

A possibility of construction of a unitary relativistic model of quantum field theory without ultraviolet divergences was investigated by the author [1,2]. The Bialynicki-Birula's lagrangian

$$L_{int} = -g : \bar{\psi}(x) \tau_1 \gamma_\nu \psi(x) \partial_\nu \phi(x) : - \Delta m : \bar{\psi}(x) \tau_3 \psi(x) : \quad (1)$$

was investigated, where τ_1 and τ_3 are the isotopic spin matrices, γ_ν are the Dirac matrices, $\psi(x)$ is the spinor field operator, $\phi(x)$ is the scalar field operator. In the first paper the model is considered in the two-dimensional space-time and restrictions on the coupling constant g are found under which all the physical quantities are definite. In the second paper the model is investigated in the four-dimensional space-time. A finite two-point Green function is constructed for any coupling constant values, in terms of which are expressed all physical quantities in the second perturbation order in Δm . In higher perturbation order such functions correspond to each pair of vertices. This function satisfies the well-known unitarity and causality conditions.

However, in ref. [2] only the second perturbation order in Δm was considered. In this order the unitarity condition included only the imaginary parts of the two-point Green functions. In order to check that the real parts of the investigated functions are found correctly too, it is necessary to consider higher perturbation orders and to verify the validity of the unitarity. Besides, the investigation of higher orders is interesting for the determination of the integrals of the product of the functions considered and for checking the absence of ultraviolet divergences in the theory.

The present paper is devoted to the above problems. It deals with the lagrangian

$$L_{int}(x) = \Delta m : \bar{\psi}(x)\gamma_5 e^{-i2g\gamma_5 \phi(x)} \psi(x) : , \tag{2}$$

which is similar to (1). (If the sign of the normal product is not ascribed to the operators $\phi(x)$, then (2) is got from (1) by means of unitary transformation $\psi'(x) = \psi(x)\exp\{ig\gamma_5\ \phi(x)\}$). For the two-point Green functions, corresponding to the Fourier transforms of the quantitles

$$(S(x))^n \exp\{\pm i(2g)^2\ \Delta^0(x)\}, \tag{3}$$

where
$S(x-x') = i < T(\psi(x)\bar{\psi}(x')) >_0$, $\Delta^0(x-x') = i < T(\phi(x)\phi(x')) >_0$
spectral representations are constructed and used to prove the validity of the unitarity. Another integral representation of these functions was used to determine the integrals of their products and to show absence of ultraviolet divergences in any perturbation order in Δm. The masses of all the free particles are assumed to be zero. In this case in the third order in Δm all the integrals are explicitly calculated and the results are expressed through the well convergent series which are finite functions.

2. Spectral Representations of the Functions $\Psi(p)$ and $\Phi(p)$

To solve the problems formulated above, in addition to the function $\Pi(p)$ studied in ref./2/, two more functions are needed, which we denote by $\Psi(p)$ and $\Phi(p)$. In this section we shall obtain for them necessary integral and spectral representations. The function $\Pi(p)$ has similar representations, but they are not written down here.

Let us consider, first of all, the spinor Green function $\Psi(p)$

$$\Psi(p) = \int d^4x\, S(x) \exp i\{px - (2g)^2\ \Delta^0(x)\}. \tag{4}$$

Using the method developed in ref./2/ it is easy to obtain the following integral representation for this function

$$\Psi(p) = \hat{p}\, i \frac{\pi\kappa}{2} \int_L dz \frac{\operatorname{ctg}\pi z\, e^{-i\pi z}[\kappa(p^2+i\epsilon)]^{z-2}}{\sin\pi z\, \Gamma(z-1)\Gamma(z)\Gamma(z+1)} \tag{5}$$

which corresponds to the well convergent power series in κ. The contour L is shown in Fig.1., $\hat{p} = p_\nu\gamma^\nu$, $\kappa = (\frac{g}{2\pi})^2$, $\Gamma(z)$ is the gamma function, and ϵ is an infinitesimal positive quantity. In deriving (5) an intermediate regularization of (4) was used, which was removed in obtaining the final results. The regularized function is as follows:

$$\Psi_\delta(p) = -i\hat{p}\frac{\kappa}{2}\int_L dz \frac{e^{-i\pi z}(p^2+i\epsilon)^{z-2}}{\sin\pi z\, \Gamma(z)\Gamma(z+1)} f_\delta(z), \tag{6}$$

where $f_\delta(z)$ in the range $0 \le \operatorname{Re} z < 2$ has the representation

$$f_\delta(z) = \frac{1}{2}\int d\lambda\, \lambda^z \left[\frac{\exp(\frac{\kappa}{\lambda+i\delta})}{(\lambda+i\delta)^3} + \frac{\exp(\frac{\kappa}{\lambda-i\delta})}{(\lambda-i\delta)^3}\right] \tag{7}$$

and can be analytically continued throughout the whole right half-plane z, for the exception of the real positive axis, where it possesses the pole singularities. Then the function $\Psi(p)$ is a limit of $\Psi_\delta(p)$ at $\delta \to 0$. In this case, in (7) it is necessary to perform beforehand rotations of the integration contours such that the sign minus appears in the exponential of the integrand.

In the integral (6) the integration contour may be straightened so that it will be parallel to the imaginary axis (Fig.2.), what is forbidden in the integral (5). In investigating higher perturbation orders we use the Green function representation of the type of (6) with the straightened integration contour C and in order to go over to the limit $\delta \to 0$ we should integrate over all the moments and return again to the contour L.

To eq. (5) there may correspond another regularized function $\bar{\Psi}_\delta(p)$, which is obtained from $\Psi_\delta(p)$ if we shift the integration contour to the right by unity, single out from (6) a term corresponding to the first order pole of the integrand at $z = 1$, let δ tend to zero and in the remaining integral we straighten the integration contour

$$\bar{\Psi}_\delta(p) = -\hat{p}\left\{\frac{1}{p^2+i\epsilon} + i\frac{\kappa}{2}\int_C dz \frac{e^{-i\pi z}[p^2+i\epsilon]^{(z-1)}}{\sin\pi z\, \Gamma(z+1)\Gamma(z+2)} f_\delta(z+1)\right\}. \tag{8}$$

Noticing that in the region $0 < \operatorname{Re} z < 1$ we have the integral equation

$$-\pi[p^2+i\epsilon]^{(z-1)}\frac{e^{-i\pi z}}{\sin\pi z} = \int_0^\infty dm^2 \frac{m^{2(z-1)}}{m^2-p^2-i\epsilon}, \tag{9}$$

we can rewrite (8) in the form

$$\bar{\Psi}_\delta(p) = -\hat{p}\left\{\frac{1}{p^2+i\epsilon} + \frac{\kappa}{2\pi i}\int_C dz \frac{f_\delta(z+1)}{\Gamma(z+1)\Gamma(z+2)}\int_0^\infty dm^2 \frac{m^{2(z-1)}}{m^2-p^2-i\epsilon}\right\}. \tag{10}$$

Eq. (10) may be considered as a spectral representation of the function $\bar{\Psi}_\delta(p)$.

In passing to the limit $\delta \to 0$ both functions give the same function $\Psi(p)$

$$\Psi(p) = \lim_{\delta \to 0} \Psi_\delta(p) = \lim_{\delta \to 0} \bar{\bar{\Psi}}_\delta(p) \ . \tag{11}$$

Now consider the scalar function $\Phi(p)$

$$\Phi(p) = i \int d^4x \exp i \{ px + (2g)^2 \Delta^0(x) \} \ . \tag{12}$$

Similar procedures lead to the following spectral representation of the regularized function corresponding to $\Phi(p)$

$$\bar{\Phi}_\delta(p) = i(2\pi)^4 \delta^{(4)}(p) + (4\pi)^2 \kappa \{ \frac{1}{p^2 + i\epsilon} + \frac{i\kappa}{2\pi} \int_C dz \frac{\phi_\delta(z+1)}{\Gamma(z+1)\Gamma(z+2)} \int^\infty dm^2 \frac{m^{2(z-1)}}{m^2 - p^2 - i\epsilon} \} \tag{13}$$

where $\phi_\delta(z)$ in the range $0 \le \mathrm{Re}\, z < 2$ is represented by the integral

$$\phi_\delta(z) = \frac{1}{2} \int_0^\infty d\lambda \lambda^z \left[\frac{\exp\left(-\frac{\kappa}{\lambda + i\delta}\right)}{(\lambda + i\delta)^3} + \frac{\exp\left(-\frac{\kappa}{\lambda - i\delta}\right)}{(\lambda - i\delta)^3} \right] . \tag{14}$$

We give also another type of the regularized function $\Phi_\delta(z)$ similar to (6) for $\Psi(p)$

$$\Phi_\delta(p) = i(2\pi)^4 \delta^{(4)}(p) - i 8\pi^2 \kappa^2 \int_C dz \frac{e^{-i\pi z} (p^2 + i\epsilon)^{z-2}}{\sin \pi z\, \Gamma(z) \Gamma(z+1)} \phi_\delta(z) . \tag{15}$$

The most essential difference of the function $\Phi(p)$ from $\Psi(p)$ and $\Pi(p)$ consists in that the scalar particle propagator in the exponential has opposite sign. Due to this fact, it is unnecessary in the integral (14) to rotate the contour in passing to the limit $\delta \to 0$ as we have to do in finding similar limits of the functions $\Psi_\delta(p)$ and $\Pi_\delta(p)$. The regularization itself is here necessary only for straightening the integration contour in the z-plane from L to C.

Using the obtained representations for the two-point Green function it is easy to prove the unitarity of the S-matrix and the absence of ultraviolet divergences in the model studied.

3. Proof of the Unitarity

We consider the matrix element of interaction of spinor particle with vacuum in the third perturbation order in Δm (Fig.3.). The unitarity condition $SS^+ = 1$ gives the equation for the matrix elements $(S = \sum_0^\infty (\Delta m)^n S_n)$

$$\mathrm{Re} \langle b_p | S_3 | b_p^+ \rangle = -\mathrm{Re} \langle b_p | S_2 S_1^+ | b_p^+ \rangle , \tag{16}$$

where b_p^+ and b_p are the production and annihilation operators of a spinor particle, respectively. The left-hand side of eq.(16) is expressed in terms of the well known functions $\Psi(p)$ and $\Phi(p)$

$$\mathrm{Re} \langle b_p | S_3 | b_p^+ \rangle = r_3 \frac{\delta^{(4)}(p - p')}{(2\pi)^3} \bar{v}_\nu^{\alpha+}(\vec{p}) v_\mu^{\beta-}(\vec{p}) \mathrm{Re} \int d^4k \Psi^2(k) \Phi(p-k) , \tag{17}$$

where $\bar{v}_\nu^{\alpha+}(\vec{p})$ and $\bar{v}_\mu^{\beta-}(\vec{p})$ are the orthonormalized spinors. We write the real part of (17) as an integral of the product of the real and imaginary parts of the functions $\Psi(p)$ and $\Phi(p)$ and the corresponding functions $\theta(p^0) \left(\begin{matrix} \theta(p^0) = 1; p^0 \ge 0 \\ \theta(p^0) = 0, p^0 < 0 \end{matrix} \right)$. For this it is convenient, using the spectral representations of the two-point Green functions derived in the previous section, to divide them into parts corresponding to $G^{ret}(p)$ and $G^+(p)$, namely

$$\Psi(p) = \hat{p} \int_0^\infty dm^2 \rho(m^2) [\frac{1}{m^2 - p^2 + i 2\epsilon p^0} + i 2\pi \delta(m^2 - p^2) \theta(p^0)] \tag{18}$$

and we do the same for $\Phi(p)$. Using the property

$$\int d^4k (\Psi^{ret}(k))^2 \Phi^{ret}(k-p) = 0 \tag{19}$$

it is easy to write (17) in the form $(p^0 > 0)$

$$\mathrm{Re} \int d^4k \Psi^2(k) \Phi(p-k) = -2(2\pi)^4 \mathrm{Re}\Psi(p) \mathrm{Im}\Psi(p) - 4 \int d^4k \mathrm{Re}\Psi(k) \mathrm{Im}\Psi(k) \mathrm{Im}\bar{\bar{\Phi}}(p-k) \theta(k^0) \theta(p^0 - k^0) , \tag{20}$$

where

$$\bar{\bar{\Phi}}(p) = \Phi(p) - i(2\pi)^4 \delta^{(4)}(p) .$$

Introducing the intermediate states between S_2 and S_1^+ we get for the right-hand side of (16):

$$\mathrm{Re} \sum_0^\infty \langle b_p | S_2 | n \rangle \langle n | S_1^+ | b_p^+ \rangle = r_3 2\pi^2 \delta^{(4)}(p - p') \bar{v}_\nu^{\alpha+}(\vec{p}) v_\mu^{\beta-}(\vec{p}) \int d^4k \mathrm{Re}\Psi(k) \sum_0^\infty \frac{(\frac{\kappa}{\pi})^n}{n!} \Omega_{n+1}(k) \sum_0^\infty \frac{(\frac{\kappa}{\pi})^m}{m!} \Omega_m(p-k) , \tag{21}$$

where $\hat{\Omega}_{n+1}(k)$ and $\Omega_m(p-k)$ are the phase volumes of particles

$$\Omega_m(k) = \delta^{(4)}(k)\delta_{mo} + 2\delta(k^2)\theta(k^0)\delta_{m1} + 2\pi \frac{(\pi p^2)^{m-2}}{\Gamma(m)\Gamma(m-1)}\theta(p^2)\theta(p^0)\theta(m-2), \tag{22}$$

$$\hat{\Omega}_{n+1}(k) = \hat{k}\,\{2\delta(k^2)\theta(k^0)\delta_{no} + 2\pi \frac{(\pi k^2)^{n-1}}{\Gamma(n)\Gamma(n+2)}\theta(k^2)\theta(k^0)\theta(n-1)\}, \tag{23}$$

where $\delta_{mn} = \left\{ \begin{array}{ll} 0 & m \neq n \\ 1 & m = n \end{array} \right.$.

Now we write down the expressions for the imaginary parts of the functions $\Psi(p)$ and $\Phi(p)$

$$\mathrm{Im}\,\Psi(p) = \pi \hat{p}\,[\delta(p^2) + \kappa \sum_0^\infty \frac{(\kappa p^2)^n}{n!(n+1)!(n+2)!}]\theta(p^2), \tag{24}$$

$$\mathrm{Im}\,\Phi(p) = (2\pi)^4\,\delta^{(4)}(p) + 16\pi^3\kappa[-\delta(p^2) + \kappa \sum_0^\infty \frac{(-\kappa p^2)^n}{n!(n+1)!(n+2)!}]\theta(p^2). \tag{25}$$

Inserting (22) and (23) into (21) and comparing the obtained expression with (24) and (25) it is easy to see that for (21) the following relation:

$$\mathrm{Re} \langle b_p | S_2 S_1^+ | b_p^+ \rangle = r_3 \frac{\delta^{(4)}(p-p')}{(2\pi)^3}\, \bar{v}_\nu^{\alpha+}(\vec{p})\, v_\mu^{\beta-}(\vec{p})\,[2(2\pi)^4\, \mathrm{Re}\,\Psi(p)\,\mathrm{Im}\,\Psi(p) +$$

$$+ 4\int d^4k\, \mathrm{Re}\,\Psi(k)\,\mathrm{Im}\,\Psi(k)\,\mathrm{Im}\,\bar{\Phi}(p-k)\,\theta(k^0)\,\theta(p^0-k^0)\,] \tag{26}$$

is valid. Inserting (20) in (17) and comparing the obtained expression with (26) it is easy to see that eq. (16) is valid. Thus, we have proved that in the third perturbation order in Δm the theory is unitary. The author expects that the obtained spectral representations for the two-point Green functions would provide the validity of the unitarity in higher order in Δm as well.

Eq. (16) may be checked without recourse to the spectral representation of the Green functions, by using the direct calculation. The value of the integral (17) is given in the appendix, and the integral in (21) is easly calculated for each member of the sum.

4. Proof of the Absence of Ultraviolet Divergences

Consider the integral, we have delt, with which enters eq. (17) and calculate it using the integral representations of the functions $\Psi(p)$ and $\Phi(p)$ (see (6) and (15))

$$\int d^4k\, \Psi^2(k)\, \bar{\Phi}(p-k) = i2\pi^2\kappa^4 \int_{C_1}\int_{C_2}\int_{C_3} dz_1\, dz_2\, dz_3 \frac{\exp[-i\pi(z_1+z_2+z_3)]\, f_\delta(z_1)\, f_\delta(z_2)\, \phi_\delta(z_3)}{\sin\pi z_1 \sin\pi z_2 \sin\pi z_3\, \Gamma(z_1)\Gamma(z_2)\Gamma(z_3)}$$

$$\times \frac{1}{\Gamma(z_1+1)\Gamma(z_2+1)\Gamma(z_3+1)} \int d^4k\,(k^2+i\epsilon)^{z_1+z_2-3}\,[(p-k)^2+i\epsilon]^{z_3-2}. \tag{27}$$

The requirement that the integral over k must be free of ultraviolet divergences imposes the following restrictions on the variables z_i

$$\mathrm{Re}(z_1 + z_2 + z_3) < 3. \tag{28}$$

Since the contours C_i lie in the range $0 \leq \mathrm{Re}\, z_i < 1$, this requirement is fulfilled. The integral over k is:

$$\int d^4k(k^2+i\epsilon)^{z_1+z_2-3}[(p-k)^2+i\epsilon]^{z_3-2} = -i\pi(p^2+i\epsilon)^{z_1+z_2+z_3-3}\frac{\sin\pi z_3 \sin\pi(z_1+z_2)}{\sin\pi(z_1+z_2+z_3)} \times$$

$$\times \frac{\Gamma(z_3-1)\Gamma(z_3)\Gamma(z_1+z_2-2)\Gamma(z_1+z_2-1)}{\Gamma(z_1+z_2+z_3-2)\Gamma(z_1+z_2+z_3-1)} \tag{29}$$

Inserting (29) into (28) and rotating the contours C_i so that they pass above and below the real positive axes in the appropri ate z_i planes and using the residues at the poles we explicitly calculate all the integrals and give the results in the appendix.

Let us prove that ultraviolet divergences are absent in the n-th perturbation order in Δm too. To this end we consider a diagrams with n vertices, two external spinor lines and an arbitrary number of external scalar lines (Fig.4.). All the vertices are connected by pair by the lines each of which corresponds to one of the Green functions considered by us and having the integral representation like (6) or (15). We consider the case when all the vertices are connected by a continuous spinor line corresponding to the $n-1$ functions $\Psi(k_i)$ .Then, in addition to these functions, $\frac{(n-1)(n-2)}{2}$ scalar functions like $\Phi(k_j)$ will correspond to the diagram. The product of all these functions will have the sign of $2(n-1)(n-2)$ fold integral over k_ℓ . Then the requirement of the absence of ultraviolet divergences is written as

$$2(n-1)(n-2) + n-1-2n(n-1) + 2\,\mathrm{Re}\sum_{1}^{\frac{n(n-1)}{2}} z_i < 0 \tag{30}$$

hence, it follows

$$\mathrm{Re}\sum_{1}^{\frac{n(n-1)}{2}} z_i < \frac{3}{2}(n-1)\,. \tag{31}$$

Assuming all z_i to be equal to each other ($z_i = z$) we get

$$\mathrm{Re}\, z < \frac{3}{n}\,. \tag{32}$$

Since the contour C in the integral representations of our Green functions may exactly coincide with the imaginary axis then the condition (32) is well satisfied.

We prove in a similar manner the absence of ultraviolet divergences in higher orders in Δm in somewhat other terms. To this end, following the work by N.N. Bogolubov and D.V. Shirkov, we introduce the notion of maximum vertex index and calculate it in the framework of our model

$$\omega_i^{max} = \frac{1}{2}\sum_{\ell_{int}} (r_\ell + 2\bar{z}_\ell) - 4 = -3\,. \tag{33}$$

Here the summation is made over internal lines, r_ℓ is unity for spinor lines and is zero for scalar lines and $\bar{z}_\ell = \mathrm{Re}\, z_\ell$ are assumed to be zero. From the inequality $\omega_i^{max} < 0$ it follows that if ultraviolet divergences are absent in lower perturbation orders then they can not appear in higher orders too.

Thus it is proved that in the considered model ultraviolet divergences are absent in any perturbation order in Δm .

5. Conclusion

Thus, we have constructed the renormalizable unitary theory free of ultraviolet divergences with interaction essentially non-linear in the field $\phi(x)$. We have not considered here the diagrams which correspond to

the renormalization of the spinor particle mass, through they appear already in the third order. It is supposed that all the diagrams may be excluded from the consideration by introducing the corresponding counter-terms into the Lagrangian. The problem of infrared divergences which may, perhaps, occur in higher perturbation orders is not investigated too. However, the main problems - the check of the absence of ultraviolet divergences in the model and the fulfilment of the conditions imposed by the S-matrix unitarity - may be considered solved in positive.

It is important, in our opinion, to try to treat in more detail the n-th order of perturbation theory and generalize the obtained results to the case of non-zero masses of particles. It is also interesting to study other interactions leading to the Green functions like those considered in the present paper (see., e.g. /4-10/).

In conclusion the author expresses his deep gratitude to Prof. D.I. Blokhintsev for the interest in the work and fruitfull discussion and G.V. Efimov who proposed a very useful integral representation for the Green functions.

APPENDIX

In order to demonstrate explicitly the fact that the theory is finite in the third order in Δm , we represent the integral (27) as power series. It is not difficult to see that these series rapidly converge.

$$\mathrm{Im}\int' d^4k\, \Psi^2(k)\vec{\Phi}(p-k) = -2(2\pi)^4\kappa^2p^2\sum_0^\infty \frac{(-\kappa p^2)^n}{(n+1)!(n+2)!}\sum_0^n\frac{(-1)^m}{(m+2)!(n-m+1)!}[\ln\kappa p^2 -$$

$$-\Psi(n+2)-\Psi(n+3)-\Psi(n-m+2)] - (2\pi)^4\kappa^8p^4\sum_0^\infty\frac{(-\kappa p^2)^n}{(n+2)!(n+3)!}\sum_0^n\frac{(-1)^m(m+1)!(m+2)!}{(n-m+1)!}\times$$

$$\times\sum_0^m\frac{1}{k!(k+1)!(k+2)!(m-k)!(m-k+1)!(m-k+2)!}\{\Psi'(n-m+2)-\Psi'(n+3)-\Psi'(n+4)-\pi^2+$$

$$+\ln\kappa p^2[\ln\kappa p^2+2(\Psi(m+2)+\Psi(m+3)-\Psi(n+3)-\Psi(n+4))-\Psi(k+1)-\Psi(k+2)-\Psi(k+3)-$$

$$-\Psi(m-k+1)-\Psi(m-k+2)-\Psi(m-k+3)]+[\Psi(n+3)+\Psi(n+4)]^2-\Psi^2(n-m+2)-$$

$$-[\Psi(n+3)+\Psi(n+4)+\Psi(n-m+2)][2(\Psi(m+2)+\Psi(m+3))-\Psi(k+1)-$$

$$-\Psi(k+2)-\Psi(k+3)-\Psi(m-k+1)-\Psi(m-k+2)-\Psi(m-k+3)]\}, \qquad (A.1)$$

$$\mathrm{Re}\int' d^4k\,\Psi^2(k)\vec{\Phi}(p-k) = -(2\pi)^5\kappa^2p^2\sum_0^\infty\frac{(-\kappa p^2)^n}{(n+1)!(n+2)!}\sum_0^n\frac{(-1)^k}{(k+2)!(n-k+1)!}-$$

$$-(2\pi)^5\kappa^8p^4\sum_0^\infty\frac{(-\kappa p^2)^n}{(n+2)!(n+3)!}\sum_0^n\frac{(-1)^m(m+1)!(m+2)!}{(n-m+1)!}\sum_0^m\frac{1}{k!(k+1)!(k+2)!(m-k)!(m-k+1)!(m-k+2)!}\times$$

$$\times[\ln\kappa p^2-\Psi(k+1)-\Psi(k+2)-\Psi(k+3)+\Psi(m+2)+\Psi(m+3)-\Psi(n+3)-\Psi(n+4)], \qquad (A.2)$$

where $\Psi(n)$ is the Euler function. The prime $\int'$ denotes that in calculating this integral we omitted the terms corresponding to the diagram

of Fig.5. since we agreed not to consider the diagrams corresponding to the mass renormalization of a spinor particle. For completeness we give here these terms

$$\int dk\,\Psi^2(k)\vec{\Phi}(p-k)-\int' dk\,\Psi^2(k)\vec{\Phi}(p-k) = i\frac{(2\pi)^4}{p^2}\sum\frac{(-\kappa p^2)^n}{n!n!(n-1)!}[\ln(\kappa p^2e^{-i\pi})-\Psi(n)-2\Psi(n+1)]. \qquad (A.3)$$

In order to exclude similar terms from (26) it is sufficient in $\mathrm{Im}\Psi(p)$ to reject a term containing $\delta(p^2)$.

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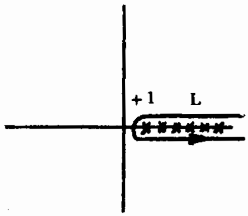

Fig. 1.

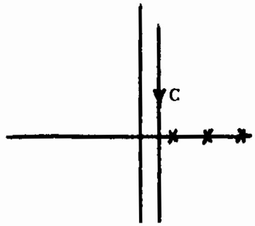

Fig. 2.

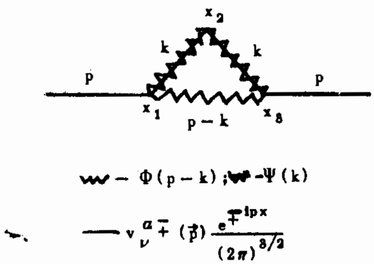

Fig. 3.

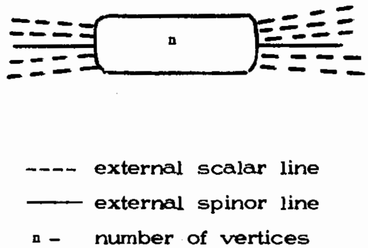

Fig. 4.

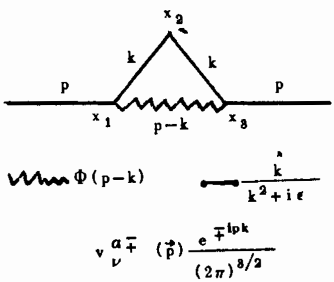

Fig. 5.