ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-3224

A.V. Efremov

ISOTOPIC INVARIANCE VIOLATION AND PION FORM FACTOR

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1967.

E2-3224

4897/2 *пр.*

A.V. Efremov

ISOTOPIC INVARIANCE VIOLATION AND PION FORM FACTOR

I. Introduction.

At first sight the title of the paper is somewhat paradoxal. One would think what does the form factor, which is, as well known, related to the isotopically invariant strong interaction, care for the violation of the invariance. Nevertheless these things are found to be rather closely connected. Firstly, the concept of form-factor itself implies the presence of the electromagnetic interaction violating the isotopic invariance. Secondly, recall how the problem of the electromagnetic mass is formulated in classical field theory. The pion is there represented as a ball which can be either neutral, or charged. Then the mass difference of the balls should depend on their radius: the smaller the charged ball is, the heavier it is than the neutral one by the quantity $\Delta m = \frac{2}{3}\frac{\alpha}{r c^2}$. Thus, knowing the mass difference of pions we can determine their "radius" r.

Such a relationship between the mass difference and the "size" of the pion is not a privilege of classical electrodynamics. In quantum field theory there is also a formula which relates the mass difference with the form-factor of the pion/1,2/, although here the derivation is based on some approximation. However both in classical and quantum field theories we delt, as yet, with a global isotopic invariance violation, i.e. with a simple mass splitting inside an isomultiplet due to the total charge. It is difficult to expect that one number available, Δm, can give sufficient information about the charge distributions if, of course, we do not specify beforhand its form.

In local theories, we are usually dealing with, the isotopic invariance violation should be also local, i.e. should have the form $\partial_\mu I_\mu = \rho$, what may give further evidenoe about the form of the charge distribution, i.e. about the form factor. But what we have to put on the place of I_μ and ρ in the case of isotopic invariance violation? This question, in fact, was answered in many papers /3,4,5,6/ as follows: when the electromagnetic interaction is switched out the theory is isotopically invariant. This means that together with the vector of the electro magnetio current (more accurately, with isovector part of it j_μ^0) there exist local operators $j_\mu^\pm$ for whioh

$$\partial_\mu j_\mu (x) = 0$$

and space integrals the time components of which form the isotopic group generators. If now we switch on a minimal electromagnetic interaction then it turns out that

$$i\partial_\mu J_\mu^Q(x) = Qe A_\mu(x) J_\mu^Q(x) \, , \tag{I}$$

where J_μ^Q denotes the current when the electromagnetic intéraction is switched on and A_μ is the electromagnetic potential. In Section II we give a somewhat different definition of the charged vector current than that presented in the abovementioned papers and find its divergenoe within the accuracy up to e^2 .By applying then the obtained expression to pions we get within a certain assumption a non-linear integral equation for the pion form factor. Section III is devoted to an approximate analytic solution of this equation by expanding it in the eigenfunctions of the group of motion of the Lobachevsky space and by the asympto tic solution of the obtained functional equation. At the end of this section an expression for the r.m.s. radius is given the numerical value of which is found to be 0.3 fermi.

II. Current Density, the Divergence and the Equations for the Pion Form Factor

As was already said in Introduction it is not apparently difficult to determine the density of the charged current when the electromagnetic interaction is switched out. When the electromagnetic interaction is switched on the current is "dressed" with the electromagnetic vertices. Therefore it is natural to determine such a current in terms of "electromagnetic" S-matrix

$$J_\mu^Q(x) = T(j_\mu^Q(x), S_{em})S^+ = \underset{>}{exp}\left\{ie\int_{x_0}^{\infty} L_{em}^{int}(t)dt\right\} j_\mu^Q(x)\, \underset{<}{exp}\left\{-ie\int_{x_0}^{\infty} L_{em}^{int}(t)dt\right\},$$

where $>$ and $<$ denote the corresponding ordering in time. Writing the right-hand side of this expression by perturbation theory it is not difficult to get that, up to the terms e^2

$$J_\mu^Q(x) = j_\mu^Q(x) + ie\int d^4y\,[\mathcal{L}_{em}^{int}(y), j_\mu^Q(x)]\,\theta(y_0-x_0) - \tag{2}$$

$$-e^2\int d^4y_1\, d^4y\,[\mathcal{L}_{em}^{int}(y_1),[\mathcal{L}_{em}^{int}(y), j_\mu^Q(x)]]\,\theta(y_{10}-y_0)\,\theta(y_0-x_0)\,.$$

Now assume that the electromagnetic interaction is of the usual form of the type "current × potential", i.e.

$$\mathcal{L}_{em}^{int}(x) = a_\mu(x)\, j_\mu(x)\,, \tag{3}$$

where a_μ is the electromagnetic field operator (it is not renormalized by the strong interaction), and j_μ is the electromagnetic current density, consisting of the isoscalar part and the third component of the isovector.

Further we assume that the commutator $[j_\mu, j_0^Q]$ vanishes outside the light cone and has a singularity of the δ -function type at the cone vertex. (This assumption may be justified by the models such as quark model [7] or model with nonabelian group of gauge transformations [8]). This yields

$$[j_\mu(y), j_0^Q(x)]\,\delta(x_0-y_0) = Q\, j_\mu^Q(x)\,\delta(x-y)\,. \tag{4}$$

The Schwinger terms which usually appear in the commutation relations for space- and time components are non-essential for us, since they appear only in commutators with identical isotopic indices. Now it is sufficient to take the divergence of both sides of the equality (2) and using eqs.(3) and (4) as well as the conservation of j_μ^Q to get an equality which will be valid for any matrix elements without photons

$$\partial_\mu J_\mu^Q(x) = = -\frac{1}{2} Q e^2 \int d^4y \left(D^{ret}(y-x) \{ j_\nu(y), j_\nu^Q(x) \} + D^1(y-x)\theta(y_0-x_0)[j_\nu(y), j_\nu^Q(x)] \right), \tag{5}$$

where D^{ret} and D^1 are the retarded function of photons and the vacuum expectation of the photon field anticommutator (see, e.g.[9]). The same formula can be derived from eq. (I) by expanding J_μ^Q and A_μ in the right-hand side in a power of e.

Consider the problem of the relativistic invariance of eq.(5). The invariance of the first term on the right hand side is obvious and that of the second term is based on vanishing of the current commutator outside the light cone. However, further we shall expand the matrix elements of the product of currents entering the commutator and the anticommutator in a complete set of the intermediate states and then cut off this series. For the anticommutator this procedure harbours nothing dangerous while for the commutator this cutoff will lead to the violation of the local commutativity and, as a consequence, to the loss of the relativistic invariance. To avoid this we follow the Dyson advice [IO] and represent the commutator in the form

$$\theta((x-y)^2)[j_\nu(y), j_\nu^Q(x)]$$

being aware that due to the uncertainty of the product of distributions such a procedure of separating $\theta((x-y)^2)$ can lead to a divergence, to the necessity of renormalization and, as a consequence, to the appearance of uncertain constants. But, as will be seen, this ambiguity is non-essential in our case.

Thus, applying the Dyson procedure we obtain a relativistically invariant co-multiplier.

$$\Delta(y-x) = D^1(y-x)\theta(y_0-x_0)\theta((y-x)^2) = D^1(y-x)\, V_+(y-x) \quad (6)$$

in front of the current commutator. Now we transform eq.(5) to the momentum representation taking into account condition (6). After the ordinary manipulation we get

$$\langle \vec{p}\,' f | \mathcal{D}^Q(0) | \vec{p}\, i \rangle =$$
$$= \frac{e^2 Q}{2}\int d\vec{q}^{\,3} \sum_n \Big\{ D^{ret}(q-p')\big[\langle \vec{p}\,' f | j_\nu(0) | \vec{q}\, n\rangle \langle \vec{q}\, n | j^Q_\nu(0) | \vec{p}\, i\rangle + j_\nu \leftrightarrow j^Q_\nu\big] + \Delta(q-p')\big[\text{—— " —— " ——} - \text{—— " ——}\big]\Big\}, \quad (7)$$

where $\mathcal{D}^Q(0) = i\partial_\mu J^Q_\mu(0)$, and $D^{ret}(\kappa) = \frac{-1}{\kappa^2 - i\varepsilon\kappa_0}$ (see[9]).

We go over to the calculation of $\Delta(\kappa)$. By integrating we obtain that the Fourier transform of the characteristic function of the upper light cone V_+ is

$$V_+(p) = \int d^4x\, V_+(x)\, e^{ipx} = \frac{8\pi}{p^2 - i\varepsilon p_0} = -8\pi \int_0^\infty d\alpha\, \alpha\, e^{i(p^2 - i\varepsilon p_0)\alpha}$$

and after the contraction of the obtained expression with the Fourier transform D^1 we find

$$\Delta(\kappa) = \frac{1}{(2\pi)^4}\int d^4p\, D^1(p)\, V_+(\kappa-p) = -\frac{i}{\pi(\kappa^2 - i\varepsilon\kappa_0)}\int_0^\infty \frac{d\alpha}{\alpha}\, e^{i\alpha(\kappa^2 - i\varepsilon\kappa_0)} \quad (8)$$

However the written expression is meaningless since the integral diverges. In order to make the function Δ meaningfull we note that the formal expression (8) satisfies the differential equation (here and in what follows we omit the indication on the going round the poles because it is non-essential for us).

$$\frac{d\Delta}{d\kappa^2} = \frac{1}{\kappa^2}\Delta(\kappa) - \frac{i}{\pi(\kappa^2)^2}$$

whose solution is

$$\Delta(\kappa^2) = \frac{i}{\pi(\kappa^2)^2}\left(\ln|\kappa^2| + C\right),$$

where C is an arbitrary real constant. It is not difficult to see that such a procedure is equivalent to the ordinary subtraction. Thus, an indefinite constant has appeared in the term with the current anticommutator as a consequence of the Dyson procedure. However, in the case we are interested in there appears a condition which allows to determine this constant.

Now let us go over immediately to the derivation of the equation for the pion form factor. To this end, as the initial and final states in eq. (7) we take the one-pion states with charges s and s' and in the sum over the intermediate states we also restrict ourselves to a one-pion state only. Then the matrix elements of the current in the right-hand side are written, using the well-known formula, through the form factor

$$\langle \vec{P}'s' | J_\mu^Q(0) | \vec{P}s \rangle = \frac{(P'+P)_\mu}{(2\pi)^3\sqrt{4P_0'P_0}}\, T^Q_{s's}\, F(P'P)\,, \qquad (9)$$

where $T^Q_{s's}$ are isotopic matrices of the triplet. However for the left-hand side matrix element $\langle \vec{P}'s' | \mathcal{D}^Q(0) | \vec{P}s \rangle$, we may not use the isotopic invariance. In accordance with our approximation we keep in this matrix element only the term of the order of e^2 and the term proportional to the mass difference. Now employing the transformation properties of $\langle \vec{P}'s' | \mathcal{D}^Q(0) | P, s \rangle$ with respect to the T and C operations and under the complex conjugation, taking into account the conservation law of electric current $(\mathcal{D}^0 = 0)$ and the fact that the π^+-meson antiparticle is π^--meson it is not difficult to obtain

$$\langle \vec{P}'s' | \mathcal{D}^Q | \vec{P}, s \rangle = \frac{Q}{(2\pi)^3\sqrt{4P_0'P_0}}\left(2m\Delta m\{T^0, T^Q\}_{s's} F(P'P) + i\alpha[T^0, T^Q]_{s's}\, G(P'P)\right), \qquad (10)$$

where $\Delta m = m_{\pi^\pm} - m_{\pi^0}$ and $\alpha = e^2/4\pi$ is the fine structure constant. It is also easy to establish the conneotion of the function F and G with the invariant funotions of the matrix element of J_μ^Q , which,up to terms contributing to the divergence of the order of e^2 , is of the form

$$\langle \vec{p}' s' | J_\mu^Q | \vec{p} s \rangle = \frac{1}{(2\pi)^3 \sqrt{4 p_0' p_0}} \left(T_{s's}^Q (p'+p)_\mu F(p'p) + i\alpha T_{s's}^Q |Q| (p'-p)_\mu \tilde{G}(p'p) \right) .$$

Hence we conclude that the function F in eq.(IO) is the meson form factor and $G(m^2)=0$. Inserting eqs.(9) and (IO) into eq.(7) we see that in the right- and left-hand sides there appear two terms in each which transform in different ways under rotation in the isotopio space: one of them as the anticommutator $\{T^0, T^Q\}$ and the other as the commutator $[T^0, T^Q]$. Besides, the first of them is purely real and the second one is purely imaginary. This makes it possible to equate these terms and obtain two equations:

$$\boxed{F(p'p) = \frac{\alpha}{32\pi^2 m \Delta m} \int \frac{d^3 q}{q_0} \frac{(p'+q)\cdot(p+q)}{p'q - m^2} F(p'q) F(qp)} \qquad \text{(II)}$$

$$G(p'p) = \frac{1}{16\pi^3} \int \frac{d^3 q}{q_0} \frac{(p'+q)(p+q)}{p'q - m^2} \left[\ln\left(\frac{p'q}{m^2} - 1\right) + C \right] F(p'q) F(pq) \qquad \text{(I2)}$$

with the additional conditions $G(m^2)=0$ and $F(m^2)=1$ the first of which allows to determine the constant C . Thus, we have derived a non-linear integral equation for the pion form factor which will be investigated in the next section. As to the second relation connecting the radiative correaction for the β -decay of a pion with the form factor we do not know as yet whether any useful information can be obtained from it.

III. Solution of the Equation for the Form Factor, the Root-Mean Square Radius.

Now we are engaged in the solution of eq.(II). For this it is convenient to go over to the space velocity [11] which is the Lobachevsky space, the irrespective velocity of a particle being represented as a point in this space and the scalar product of two momenta by the masses of the corresponding particles as the hyperbolic oosine of the distance between the points oorresponding to the velocities of these particles. Thus, $\frac{p'p}{m^2} = ch\,a$, $\frac{p'q}{m^2} = ch\,b$, $\frac{pq}{m^2} = ch\,c$ and the sides a , b , c form a triangle in the Lobachevsky space (Fig. I)

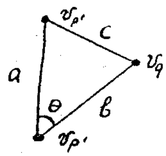

Fig. I.

i.e. $ch\,c = ch\,a\,ch\,b - sh\,a\,sh\,b\cos\theta$. Rewriting eq. (II) in terms of these variables, after the integration over the azimuthal angle, we have

$$F(a) = \frac{\alpha m}{16\pi \Delta m}\int d\mu(b)\,\frac{ch\,a + ch\,b + ch\,c + 1}{ch\,b - 1}\,F(b)F(c) = \qquad (I2)$$

$$= \mathcal{E}'\left\{\Psi_1(a) + \Psi_2(a) + \Psi_3(a)\right\},$$

where $\mathcal{E}' = \frac{\alpha m}{16\pi\Delta m}$, $d\mu(b) = sh^2 b\,\sin\theta\, db\, d\theta$ is the invariant measure, and

$$\Psi_1 = \int d\mu(b)\,\frac{ch\,b + 1}{ch\,b - 1}\,F(b)F(c) = \int d\mu(b)\left(\Phi_1(b) + \Phi(b)\right)F(c)$$

$$\Psi_2 = \int d\mu(b)\,\frac{ch\,c}{ch\,b - 1}\,F(b)F(c) = \int d\mu(b)\,\Phi(b)\,F_1(c)$$

$$\Psi_3' = \Psi_3(a)/ch\,a = \int d\mu(b)\,\frac{F(b)}{ch\,b - 1}\,F(c) = \int d\mu(b)\,\Phi(b)\,F(c) .$$

(The definition of the functions Φ , Φ_1 and F_1 is obvious from these equalities). Notice that each of the functions is a contraction on the Lobachevsky space motion group; therefore it is natural to make the expansion in the "radial" eigenfunctions of this group [12,13]

$$P_\rho(a) = \frac{P^{-1/2}_{-1/2+i\rho}(ch\,a)}{\sqrt{sh\,a}} = \sqrt{\frac{2}{\pi}}\;\frac{\sin\rho a}{\rho\, sh\,a} . \qquad (I4)$$

Thus, let

$$F(a)=\int_0^\infty f(\rho)\,P_\rho(a)\,\rho^2 d\rho \tag{I5}$$

and inversely

$$f(\rho)=\int d\ell\, sh^2\ell\, F(\ell)\,P_\rho(\ell)\ . \tag{I6}$$

These formulas and a particular case of the "multiplication theorem"[13] for the functions P_ρ :

$$\int_0^\pi P_\rho(c)\sin\theta\, d\theta=\sqrt{2\pi}\,P_\rho(a)\,P_\rho(\ell),$$

where a , ℓ and c are the side lengths of the triangle given in Fig.I, yield

$$\begin{aligned}\psi_1(\rho)&=\sqrt{2\pi}\,(\varphi_1(\rho)+\varphi(\rho))\,f(\rho)\\ \psi_2(\rho)&=\sqrt{2\pi}\ \ \varphi(\rho)\,f_1(\rho)\\ \psi_3'(\rho)&=\sqrt{2\pi}\,\varphi(\rho)\,f(\rho)\ .\end{aligned} \tag{I7}$$

If, in addition, the known identity is used[14]

$$ch\,a\,P_\rho(a)=\frac{1}{2}\left(\frac{\rho+i}{\rho}P_{\rho+i}(a)+\frac{\rho-i}{\rho}P_{\rho-i}(a)\right)$$

it may be found that

$$\begin{pmatrix}\psi_3\\ \varphi_1\\ f_1\end{pmatrix}(\rho)=\frac{1}{2}\left[\frac{\rho+i}{\rho}\begin{pmatrix}\psi_3'\\ \varphi\\ f\end{pmatrix}(\rho+i)+\frac{\rho-i}{\rho}\begin{pmatrix}\psi_3'\\ \varphi\\ f\end{pmatrix}(\rho-i)\right] \tag{I8}$$

and

$$f(\rho)=\frac{1}{2}\left(\frac{\rho+i}{\rho}\varphi(\rho+i)+\frac{\rho-i}{\rho}\varphi(\rho-i)\right)-\varphi(\rho)\ . \tag{I9}$$

Inserting eqs.(I7) and (I8) into the obvious equality

$$f(\rho)=\varepsilon'\left(\psi_1(\rho)+\psi_2(\rho)+\psi_3(\rho)\right)$$

we are led, instead of the integral equation, to the following funotional equation:

$$2f(\rho) = \sqrt{2\pi}\,\mathcal{E}'\left\{\frac{\rho+i}{\rho}\left(\varphi(\rho+i)+\varphi(\rho)\right)\left(f(\rho+i)+f(\rho)\right)+ \right. \tag{20}$$
$$\left. + \frac{\rho-i}{\rho}\left(\varphi(\rho-i)+\varphi(\rho)\right)\left(f(\rho-i)+f(\rho)\right)\right\}$$

to which eq. (I9) relating φ with f is to be necessarily added.

As is seen, the obtained non-linear functional system for the exact solution is not at all simpler than the initial integral equation. However, it has the advantage that there appears a possibility to investigate various limiting cases, in particular, the case of large ρ, which corresponds to small momentum transfers, according to (I4) and (I5).

Now we turn to the asymptotic behaviour of $f(\rho)$. A simple integration by part of eq.(I6) will lead to that all the coefficients in the expansion in $1/\rho$ vanish. Therefore to find the asymptotic behaviour of $f(\rho)$ it is necessary to know the analytical properties of $F(a)$ in the complex a-plane /15/. As is known, the pion form factor is analytical in the plane $t=-2m(cha-1)$ for the exception of the cut $[4m^2, \infty)$. In the a-plane this will lead to a singularity on the lines $Im\,a = \pi \pm 2\pi n$ so that the maximum value by which the contour of integration may be displaced in the expression

$$f(\rho) = \sqrt{\frac{2}{\pi}}\frac{1}{\rho}\int_0^\infty F(a)\,sha\,\sin\rho a\,da = \sqrt{\frac{2}{\pi}}\frac{1}{2i\rho}\int_{-\infty}^{\infty} F(a)\,sha\,e^{i\rho a}\,da$$

is $i\pi$. This will lead to the following asymptotic behaviour for $\rho\to\infty$: $f(\rho)\sim O(e^{-\pi\rho}/\rho)$. In the case of

$$\varphi(\rho) = \sqrt{\frac{2}{\pi}}\frac{1}{\rho}\int_0^\infty \frac{F(a)\,sha}{cha-1}\sin\rho a\,da = \frac{1}{\sqrt{2\pi}\,i\rho}\int_{-\infty}^{\infty}\frac{F(a)\,sha}{cha-1}e^{i\rho a}\,da$$

when displacing the contour of integration it is necessary to go round the pole at the point $a=0$, what lead to the behaviour

$$\varphi(\rho)\sim\frac{\sqrt{2\pi}}{\rho}+O(e^{-\pi\rho}/\rho)\,. \tag{2I}$$

Our approximation is that by inserting eq. (21) into eq.(20) we reject exponentially small terms and obtain a linear equation for $f(\rho)$:

$$2\rho^2 f(\rho) = \varepsilon\left[(2\rho+i)f(\rho+i) + (2\rho-i)f(\rho-i) + 4\rho f(\rho)\right], \qquad (22)$$

where $\varepsilon = 2\pi\varepsilon'$. Such a rejection is practically equivalent to the following replacement in eq.(12)

$$\Phi(\ell) = \frac{F(\ell)}{ch\ell - 1} \longrightarrow \frac{2}{\ell sh \ell} \; . \qquad (23)$$

The solution for eq. (22) is found in the form

$$f(\rho) = \int_{-\infty}^{\infty} h(x)\, e^{-i\rho x} dx \, , \qquad (24)$$

where $h(x)$ together with its first derivative vanishes when $x \to \infty$. As to the limit $x \to -\infty$, owing to that $f(\rho)$ is real, $h(-x) = h^*(x)$ and consequently must also decrease. These boundary conditions follow immediately from the connection between h and F (see below) and from the rate of the decrease which is required by the integral equation for F in the above approximation. Indeed, eq. (15) can be rewrtten in the form

$$F(a) = \sqrt{\frac{2}{\pi}}\, \frac{1}{sh a}\, \frac{\partial}{\partial a} \int_{-\infty}^{\infty} d\rho\, e^{i\rho a}\, \frac{f(\rho) + f(-\rho)}{2}$$

from where taking into account (24) we obtain immediately

$$F(a) = \frac{1}{sh a}\, \frac{d}{da}\left(\frac{h(a) + h(-a)}{2}\right) \qquad (25)$$

(The numerical coefficient in the right-hand side is non-essential for us since the function h is determined up to an arbitrary constant).

Thus, we may conclude that

$$f(\rho) = \frac{1}{i\rho} \int_{-\infty}^{\infty} h'(x)\, e^{-i\rho x} dx = -\frac{1}{\rho^2} \int_{-\infty}^{\infty} h''(x)\, e^{-i\rho x} dx$$

and inserting it in eq. (22) we are led to the following differential equation for $\eta(x)$

$$\eta''(x) - i\varepsilon\left[2\eta'(x)(chx+1) + \eta(x)shx\right] = 0 \quad . \tag{26}$$

Although the asymptotic of equation (26) can not be given any physical meaning because in our approximation we may believe the solutions only in the region of small x, the investigation of asymptotic behaviour is necessary in order to fix the suitable solution for eq. (26) by choosing one of the arbitrary constants. (The second arbitrary constant will be used to normalize the form factor F).

To choose the suitable asymptotic solution and understand what it goes into for the region of small x we make the replacement

$$\eta(x) = e^{i\varepsilon(shx+1)}\xi(x) \tag{27}$$

since as two linearly independent solutions of the equation for ξ:

$$\xi''(x) + \varepsilon^2(chx+1)^2\xi(x) = 0 \tag{28}$$

we can choose the solution of definite parity with respect to the replacement $x \to -x$:

$$\xi(x) = C_0\xi_{even}(x) + C_1\xi_{odd}(x) \ ,$$

where $\xi_{even}(0) = 1$ and $\xi'_{odd}(0) = 1$. After the substitution $S = shx$ it is not difficult to conclude that at $x \to \infty$ two linearly independent solutions are of the form $\xi(x) \sim \frac{1}{\sqrt{chx}}\begin{cases} sin(\varepsilon shx) \\ cos(\varepsilon shx) \end{cases}$. Taking into account eq. (27) we find that to provide the decrease of $\eta'(x)$ it is necessary to put

$$C_1 = -i\,C_0 \quad . \tag{29}$$

Thus, eqs.(25), (27) and (28), the condition (29) and the normalization condition determine unambiguously the pion form factor.

However, owing to the fact that our approximation is valid only for small X and the information on the pion form factor is very poor and contradiotory we restrict ourselves only to the calculation of the rms. radius. The simplest way to do this is the following. Inserting in eq. (26)

$$h(x) = a_0 + a_1 x + a_2 \frac{x^2}{2!} + \cdots$$

and equating the coefficients for the identical powers of x, we express a_n in terms, say, of a_0 and a_2 which may be chosen arbitrarily. Using the replacement(27) it isn't difficult to find the condition (29) to be now

$$a_0 = \frac{a_2}{4\mathcal{E}(1-2\mathcal{E})} .$$

Taking into account this condition

$$a_4 = \left(1 - 16\mathcal{E}^2 - \frac{\mathcal{E}}{1-2\mathcal{E}}\right) a_2$$

and using the expression (25) we find the first two terms of the expansion of the form factor

$$F(x) = 1 - \frac{x^2}{6}\left(1 - \frac{a_4}{a_2}\right)$$

which gives

$$\boxed{\langle r \rangle = \sqrt{\frac{\mathcal{E}}{1-2\mathcal{E}} + 16\mathcal{E}^2}} , \qquad \text{where } \boxed{\mathcal{E} = \frac{\alpha m}{8\Delta m}}$$

Substituting the numerical values $\alpha = 1/137$, $m = 135_{mev}$ and $\Delta m = 4.5_{mev}$ we get

$$\boxed{\langle r \rangle = 0.23 \frac{1}{m} = 0.3\, f}$$

IV. Conclusion.

The r.m.s. radius of a pion obtained in the previous section is based on three approximations. Let us consider these approximations more closely.

The first one is the restriction up toterms of the order of e^2 in deriving the expression of divergence. The validity of it apparently beyond any doubt since such an expansion proved itself to be excelent in quantum electrodynamics.

The second approximation, i.e. the rejection of all the states, but the one-particle one, in the expansion of the current produot in a complete set, is more confused . Frankly speaking, we are not able to give to this approximation at least some wordy justification of the type "nearby singularities" or "saturation condition". The only justification appears to be the fact that if in eq.(II) we put $\rho'\rho = m^2$ we are led to a formula which connects the mass differences with the form factor. Inserting in this formula the pion form factor in the ρ -meson approximation we obtain the mass difference which is in rather good agreement with experiment. This allows to expect a reasonable value for the form factor not too far from the point $\rho'\rho = m^2$ as well. This fact is also seen from the comparison of the obtained radius with the ρ -meson radius of the pion which is found to be about 0.6f.Unfortunately, we have no more to compare with since the experimental data available are rather inaccurate and as a rule are based on assumptions of theoretical nature such as "Chew-Low pole diagram". The question naturally arises: why we did not use the nucleon form factor which is known rather well today to check the validity of a similar equation? However, the use of the isotopic invariance violation in the form (I) does not allow us to write a closed set of equations for the nucleon form factors, at least due to the fact that the isoscalar form factor will enter the righthand side of the equation together with the isovector one. Thus, some more equations are needed for the system to be closed.

But we have no one of them. (Of course, if the supposition such as the similarity of the behaviour of all nucleon form factors with the momentum transfer is not made.) So, the only possibility to check the validity of one-meson approximation would be the account of the next intermediate states and, first of all, three-pion state, at least in the form of ω -meson.

Finally, the third approximation, namely the rejection of the exponentially small terms when $\rho \to \infty$ (what corresponds to small momentum transfer), can be easily overcame by the numerical caloulation of the initial integral equation with the help of an electronic computer. However, it seems prematurally to do this untill the question about the validity of the second approximation is solved.

In conclusion I take the opportunity to thank D.I.Blokhintsev, I.T.Todorov, A.T.Filippov and especially M.Micu for many interesting and rather fruitful discussions.

References:

I. Riazuddin. Phys.Rev., II4, II84 (1959).

2. L.D.Soloviev, A.V.Shchelkachev. Nucl.Phys., 76, (1966),684

3. М.В.Терентьев. Несохранение векторного тока и минимальность электромагнитного взаимодействия. Препринт ИТЭФ, №463, 1966.

4. M.Veltman. Phys.Rev.Lett., I7, 553 (1966).

5. C.Bouchiat, G.Flamand, J.M.Kaplan. "A derivation of Veltman's Equation in Classical Field Theory". Preprint, 1966.

6. S.L.Adler. Phys.Rev., I39, B I638 (1965).

7. M.Gell-Mann. Physics, I, 63 (1964).

8. See for instance L.S.Brown "Gauge Invariance, Lorentz Covariance and Current Correlation Functions." Preprint, 1966.

9. Н.Н.Боголюбов и Д.В.Широков. "Введение в теорию квантовых полей" Гостехиздат, Москва, 1957.

IO. F.Dyson. Phys.Rev., IIO, (1958).

II. See for instance Н.А.Черников, Международная зимняя школа по теоретической физ., Дубна, 1964, т.II, стр.151, также: НДВШ физ.-мат. наук, № 2, 158 (1958).

I2. N.Ya.Vilenkin, Ya.A.Smorodinsky. Preprint E-1503,Dubna,1964.

I3. Н.Я.Виленкин. Специальные функции и теория представлений групп, Физматгиз, Москва, изд. "Наука", 1965.

I4. И.М.Рыжик и И.С.Градштейн. "Таблицы интегралов... ", Гостехиздат, Москва, 1951, стр. 384

I5. М.А.Евграфов. Асимптотические оценки и целые функции. ФМ, Москва, 1962, стр. 28.

Received by Publishing
Department on March 15, 1967.