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L.D. Soloviev

SYMMETRIES AND CURRENT ALGEBRAS FOR ELECTROMAGNETIC INTERACTIONS

Talk at the International Conference on Low and Intermediate Energy Electromagnetic Interactions,

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1. Introduction

It is the SU(6) symmetry suggested by Gursey and Radicati, and Sakita[1] two and a half years ago that we owe a number of succesful relations and essential predictions, particularly for electromagnetic interactions[2]. These results are being understood in two ways. The first way - quark model - which is a simple and natural interpretation of the symmetry - is based on the crucial assumption that particles are composite. The second way - current algebra - suggested by Gell-Mann[3] is an attempt to interpret symmetry by means of usual concepts of the local relativistic theory. The starting point of this way is to postulate equal-time commutators of some local operators. Following this way, use can be made of such an achievement of the local theory as dispersion relations (it was done by Fubini, Furlan and Rossetti[4]). At the same time this method employs the notion of local operators at a fixed moment of time which is indefinite in the local theory.

In the summer of 1965 N.N. Bogolubov suggested to use ordinary dispersion relations to obtain some results of this approach without postulating any commutators. For this purpose use was made of the so-called superconvergent sum rules[6] which has been well-known since 1959[5]. The Italian physicists[7] derived similar sum rules starting from current algebra. These sum rules have attracted recently considerable attention.

In general, current algebras applied to electromagnetic interactions were touched upon, only in 1966, in a hundred papers, what in no way makes my task easier.

SU(6) Symmetry

We consider in brief the main predictions of SU(6) symmetry for electromagnetic interactions. The greatest success of this symmetry has been achieved in describing the static limit of the three-particle vertex

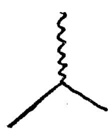


Fig.1.

$$\mu_p \xrightarrow{SU(6)} 20 \text{ constants}$$

$$\Big\downarrow \quad \text{quarks}$$

$$\Gamma(\omega \to \pi\gamma) \xrightarrow[SU(6)]{} 10 \text{ constants}$$

where the wavy line stands for the photon, and each solid line is a particle or a resonance. The SU(6) symmetry allows one to express in terms of the proton magnetic moment the magnetic moments and the magnetic-dipole decay constants of the remaining seven baryons and ten nearest baryon resonances, i.e. 20 isotopically independent quantities[2]. Similarly, 10 magnetic constants of nine vector mesons[2,8-10] are expressed in terms of the $\omega \to \pi\gamma$ decay width. I would like to note that the quark model goes still farhter and succeeds in relating the $\omega \to \pi\gamma$ decay width to the proton magnetic moment[11-13,9].

However, great mass difference of particles involved in the decay, especially in meson decay, makes one to introduce an additional assumption showing which constants the predictions of symmetry are referred to. As those, we usually take the constants entering the simplest relativistic-invariant effective Lagrangians.

The number of the magnetic constants measured are much less than that predicted by the symmetry. The experimental data are discussed in reports by Chuvilo and Khachaturian.

The comparison which experiment is at present as follows (see also[24a])

	$-\frac{\mu_p}{\mu_n}$	$\frac{\mu_{\Sigma^+}}{e/2m_p}$	$-\frac{\mu_\Lambda}{e/2m_p}$	$\frac{\mu(N^* \to N\gamma)}{2\sqrt{2}\mu_p/3}$
theory	1.5	2.79	0.96	1
experiment	1.46	2.1 ± 0.8	0.79 ± 0.20	1.28 ± 0.02
				1.25 ± 0.02
				1.20 ± 0.08

The second and third predictions in this table follow from SU(3). The experimental estimates of $\mu(N^* \to N\gamma)$ have been obtained from the data on photoproduction in the first resonance region: the first two estimates - from π^0 -photoproduction[14,6] and the third - from π^+-photoproduction[15]. There are also preliminary data on μ_{Σ^0} and $\mu_{\Sigma\Lambda}$ from an analysis of strange particle photoproduction[16,17].

Theory	Experiment
$\Gamma(\rho \to \pi\gamma)$,	MeV
0.13 ± 0.03	0.6 upper bound of direct measurement, CERN[18]
	0.2, $\gamma p \to \pi^+ n$, $\gamma n \to \pi^- p$, 0^0, 180^0 The Lebedev Phys.Institute, Moscow[19]. This estimate is the least sensitive to possible uncertainties in theoretical formulae for photoproduction.
	0.1 $\gamma p \to \rho^0 p$, Hamburg[20]. The estimate is obatined with the aid of diffraction model.
	0.4 $\gamma p \to \rho^- N^{*++}$, Hamburg[21].
	0.7 ρ -Production by pion through photon exchange with nucleus, Paris[22].

$\Gamma(\phi \to \eta\gamma)/\Gamma(\phi_{tot}),\%$	
8 ± 2	9 ± 11 Sacle [22a]. 0 ± 8 Berkeley[22b].

We see that the agreement with the available meagre experimental data is good. It is very important to proceed with measurements of the magnetic moments and radiative decay widths, as well as decays of vector mesons into lepton pairs (see Khachaturyan's talk) and pseudoscalar mesons -into two photons.

The collinear group $SU(6)_W$ is a relativistic generalization of the group SU(6)[23]. For the electromagnetic vertex this collinear group simply generalizes the static SU(6) relations to the corresponding form-factors.

In particular, we have

$$G_M^{(p)}(q^2) \mid G_M^{(n)}(q^2) = -3/2\,,\quad G_E^{(n)}(q^2) = 0,$$

$$G_{M1}^{(N^* \to N\gamma)}(q^2) = \frac{2\sqrt{2}}{3}\, G_M^{(p)}(q^2)\,,\quad (N^* \to N\gamma\,,\, E2\,, L2) = 0,$$

$$G_E(4m^2) = G_M(4m^2) = 0,$$

see, e.g.[24]. These relations are fulfilled in experiment[25].

The SU(6) symmetry was also applied to electromagnetic mass differences (see Chuvilo's talk) and four-particle vertices[25]. As far as the four-particle vertices are concerned it is easy to think of dynamical models in which symmetry breaking may be large(e.g. due to the mass differences of virtual particles in propagators) even if in the three-particle vertices the symmetry is not broken. Therefore, one should be aware here either of the mechanism of the process, or be able to take into account the symmetry breaking. The results of the current algebra mainly refers to the three-particle vertices.

Current Algebras

The SU(6) symmetry is an approxiamate dynamic symmetry: it can be applied only to static and collinear processes (in contrast to SU(3)). It is not an easy matter to guess relativistic equations which would admit solutions having such a symmetry. Besides, even in the cases to which the unitary symmetry is applicable, it is broken, what follows from great mass differences in the multiplets (this refers to the SU(3) symmetry as well). The method of the current algebra[3] is an attempt to construct a relativistic theory of broken symmetry which would allow one to obtain relations of the SU(6) symmetry without postulating this group as a symmetry of physical states. The question concerning the origin of this symmetry becomes of minor importance.

The main notion of this method is a local current $j(x)$. First of all, the vector and axial currents $j_\mu(x)$ and $\bar{j}_\mu(x)$ are considered which describe interactions of hadrons with photons and leptons. These currents are local since such are photons and leptons, regardless of the fact whether hadrons are local or composite. The locality of currents permits one to use dispersion relations in this approach[4]. At the same time use is made of the concept of the local operator at a fixed moment of time which is uncertain in the local theory. This uncertainty is demonstrated, on the one band, in possible presence of indefinite Schwinger terms in commutators. On the other hand, use is made of the amplitudes expressed in terms of the T-products of the currents which are, generally speaking, different from the physical amplitudes. It would be nice to overcome these difficulties as rigorously as was done, for instance in proving dispersion relations.

The axiomatic formulation of the local theory suggested by Bogolubov and co-workers[26] which is just a theory of the local current may serve as a basis for this purpose. The above difficulties have been treated in papers by Schwinger[27], Johnson[28], Okubo[29] and by other authors[30-36].

The main idea of the current algebra approach is as follows. Consider first a theory with exact SU(3) symmetry. In this theory the vector current is a conserving octet

$$\partial_\mu j_\mu^{(a)}(x) = 0. \tag{1}$$

This means that the corresponding charges

$$Q^{(a)} = \int j_0^{(a)}(x)\, d\vec{x} \tag{2}$$

are independent of time and, therefore, commute with the Hamiltonian. On the other hand, in the theory with exact SU(3) symmetry there must exist eight generators of the group commuting eith the Hamiltonian. One of them - the electric charge operator - is present among $\Xi Q^{(a)}$. Therefore, all the $Q^{(a)}$ are the generators of the group and satisfy the commutation relations of the SU(3) algebra

$$[Q^{(a)}, Q^{(\beta)}] = i f_{a\beta\gamma} Q^{(\gamma)}. \tag{3}$$

If now the symmetry is broken, the operators $Q^{(a)}$ cannot commute with the Hamiltonian and, therefore, are time-dependent. The main assumption of the current algebra approach is that the commutators of these operators with each other at the same moment remain unchanged

$$[Q^{(a)}(t), Q^{(\beta)}(t)] = i f_{a\beta\gamma} Q^{(\gamma)}(t). \tag{4}$$

This assumption is generalized to the charges of the axial current $\bar{Q}^{(a)}$ (determined by the formula (2) with $\bar{j}$ instead of j)

$$[Q^{(a)}(t), \bar{Q}^{(\beta)}(t)]\, h = i f_{a\beta\gamma} \bar{Q}^{(\gamma)}(t), \tag{5}$$

$$[\bar{Q}^{(a)}(t), \bar{Q}^{(\beta)}(t)] = i f_{a\beta\gamma} Q^{(\gamma)}(t). \tag{6}$$

These are the relations of the algebra $SU(3) \times SU(3)$. The coincidence of the right-hand sides in eqs. (4) and (6) implies that the vector and axial currents are similar and in some sense "equal in magnitude".

A further generalization of these assumptions, which is more dangerous from the point of view of the local theory, is to postulate commutators between charges and currents, e.g.

$$[Q^{(\alpha)}(t), j^{(\beta)}(x)]_{x_0=t} = i f_{\alpha\beta\gamma} j^{(\gamma)}(x), \qquad (7)$$

and, finally, between the currents themselves, e.g. for the spatial components of the vector current ($i,j=1,2,3$)

$$[j_i^{(\alpha)}(x), j_j^{(\beta)}(y)]_{x_0=y_0} = i f_{\alpha\beta\gamma}\delta_{ij}\ \delta(\vec{x}-\vec{y}) J_0^{(\gamma)}(x) + S^{(\alpha\beta)}. \qquad (8)$$

In this formula $S^{(\alpha\beta)}$ are Schwinger terms containing derivatives of the δ function and symmetrical with respect to $\alpha \to \beta$ and $J_0^{(\gamma)}$ is a certain unknown operator different from $j_0^{(\gamma)}$ by operator terms. As was shown by Buccella, Veneziano, Gatto and Okubo[30] the equality $J_0^{(\gamma)}(x) = j_0^{(\gamma)}(x)$ contradicts locality if the Jacobi identity is postulated for the currents at a fixed moment of time.

There exist a number of methods for obtaining information from the postulated commutators[4,37-40]. We first consider the commutator (8). Using this, it is easy to find the commutator for one of the components of the magnetic moment operator

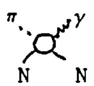
$$m_i^{(\alpha)} = \frac{1}{2}\epsilon_{ijk}\int x_j j_k^{(\alpha)}(x)\, d\vec{x}, \qquad (9)$$

$$[m_i^{(\alpha)}, m_i^{(\alpha)}] = f_{\alpha\beta\gamma} M^{(\gamma)} \qquad (10)$$

where $M^{(\gamma)}$ is some operator. Following Lee-Dachen-Gell-Mann[37] we consider the matrix elements of eq. (10) between the states of the baryon octet and decuplet and the nonets of the pseudoscalar and vector mesons with zero momenta.

We expand the operator product in a complete system of states and restrict ouselves in this system by the same states (i.e. by the octet and /decuplet or the nonets). Then we get a system of equations for the magnetic constants of the baryons and mesons, whose consistency condition yields all the results of the static SU(6) symmetry for the electromagnetic vertex[37,41-43]. No new results are available (the operator M is unknown), what settles a contradiction between the relations of Lee-Dachen-Gell-Mann[37] and Cabibbo-Radicati[44] in favour of the latter one. The contradiction arises if in eq. (8) we put $J_0^{(\gamma)} = j_0^{(\gamma)}$ [x]. Thus, we obtained the result of SU(6) symmetry without assuming its existence. However, it is clear that this result depends essentially on which states are taken into account in the sum over the intermediate states in the commutator. The assumption on these states, i.e. on the saturation of commutators is the second principal postulate of this method for obtaining the results of SU(6) symmetry.

One can give a lot of examples showing how essential this postulate is. So, if the commutators between charges and currents (7) and similar commutators for the axial quantities are saturated by the baryon octet and decuplet, then one obtains a zero solution for the anomalous magnetic moments of the baryons and the decay constants of the resonances[45, 46] what is, of course, unsatisfactory. On the other hand, it is only the assumption about saturation, without using any algebras, that may yield relations of the SU(6) symmetry type[6]. To show this, we consider the amplitude of the virtual photoproduction of pions on nucleon



$$= T_\mu = \langle q p' | j_\mu(0) | p \rangle = u(p')\gamma_5 (k_\mu \gamma k - \gamma_\mu k^2) u(p) f(\nu, t, k^2) + \ldots, \qquad (11)$$

where $j_\mu(x)$ is the electromagnetic current, $k = p' + q - p$ is the momentum of the virtual photon, and the points indicate the terms we are not interested in. Only the term written down has two energy factors k_μ and γk in front of the invariant amplitude f. Therefore, if at high energies ν the whole amplitude $f T_\mu$ increases slower than ν, then ν^{-1} decreases faster than

$$\nu \to \infty \quad |T_\mu| < \text{const}\ \nu \to |f| < \frac{\text{const}'}{\nu} \qquad (12)$$

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x/ This equality is used in papers[67] where the commutators for higher moments of the current and their application to photoproduction of higher resonances are considered.

Using now the analytic properties of f and applying the Cauchy theorem to this function, we see that the integral over the large circle Fig.2. tends to zero, and therefore, theintegral over the cut vanishes

$$\int_{-\infty}^{\infty} \mathrm{Im}\, f(\nu, t, k^2)\, d\nu = 0. \qquad ^{x)} \qquad (13)$$

Fig.2.

Now we restrict ourselves in this so-called superconvergent sum rule only to the nucleon and nearest resonance contributions. We neglect the finite resonance width (note that such an approximation is always made in the SU(6) symmetry. In calculating the resonance contribution it may give an error as large as 30%). With this reserve, we obtain the following results (at $K^2 = 0$)

$$\mu'_S = 0 \qquad (14)$$

$$g_{NN\pi}\mu'_V = G_{N^*N\pi}\mu(N^* \to N\gamma)\, m \left[\left(1+\frac{m}{M}\right)^2 - \frac{m_\pi^2}{M^2} - \frac{3t}{mM} \right] \qquad (15)$$

Here $\mu'_{V,S}$ are the isovector and isoscalar nucleon magnetic moments, m and M are the nucleon and resonance masses, and the underlined constants are known from the πN scattering. The first of these relations is in agreement with experiment. The second one depends on t, what points to its approximate nature. However, for those t which correspond to the energy region of the nearest resonance and only for which the resonance approximation in dispersion integrals holds true, the dependence upon t changes the right-hand side of eq. (15) less than by 10% and serves as a certain indication to the accuracy of the approximation made. For the threshold value $t = -m_\pi^2(1 + m_\pi/M)^{-1}$ at which

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x) It is worth noting that this sum rule was used to show the coexistence of two sets of dispersion relations for photoproduction/72,73/, corresponding to two different decompositions in invariants, exactly in the same way, as it was recently done to show the coexistence of linear and quadratic mass formulae for baryons following from the current algebra/76/.

there is no unobservable region in the dispersion relations we have

$$\mu(N^* \to N\gamma) \,/\, \frac{2\sqrt{2}}{3}\,\mu_p = 1.28, \tag{16}$$

what is in good agreement with SU(6) and experiment.

If the relations (14) and (16) are supplemented by the SU(3) relations for the octet and decuplet, we get almost all the relations of the SU(6) type for the baryon 56-plet (except the relations for the magnetic moments of the resonances).

We compare this result with what the current algebra gives us. This approach allows us to obtain, with the help of the PCAC hypothesis, relations of the form

$$g(\nu = t = m_\pi^2 = 0) = C \tag{17}$$

where g is a certain amplitude and C is a constant known from the commutator. The direct check of such relations is difficult since it requires an extrapolation to the nonphysical region. However, if it is assumed that for g the dispersion relation without subtractions holds true, eq. (17) allows one to write down the sum rule.

$$C = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\operatorname{Im} g(\nu, t = m_\pi^2 = 0)}{\nu}\, d\nu . \tag{18}$$

We see that if $C = 0$ (zero commutator) this relation looks like a superconvergent sum rule. It was the zero commutator that was used by Fubini Furlan and Rosetti[4] for photoproduction. It is possible to show that in this case the only difference between the amplitude f entering the superconvergent sum rule (13), and the amplitude g/ν at $t = m_\pi^2 = 0$ is that the former one involves longitudinal amplitudes of the virtual photoproduction, and the latter - does not. Therefore, in principle, a difference between them is possible.

It follows, however, from experiment or from dispersion theory that the longitudinal amplitudes are small, the relations (13) and (18) practically coincide and we get the result of the current algebra[4,47] directly from eq. (15), putting $t = m_\pi^2 = 0$.

An approximate account of the next resonance[47] shows that its contribution is of the same order (5%) as a possible error in the first resonance contribution due to inaccurate knowledge of the quadrupole amplitude.

The sum rules for photoproduction of strange particles were treated by Mathur and Pandit[47a] using the current algebra and by Pisarenko[47b] using superconvergence.

We consider next the example of the Compton scattering. Several years ago Lapidus and Chou Kuang Chao[48] and Gerasimov[49] obtained, with the help of the low-energy theorem, the following sum rule

$$\mu_p'^2 \mp \mu_n^2 = \frac{m^2}{2\pi^2 a} \int \frac{d\nu}{\nu} [\sigma_-^{(p)} - \sigma_+^{(p)} \mp \sigma_-^{(n)} \pm \sigma_+^{(n)}], \qquad (19)$$

where σ is the total cross section for photon-nucleon scattering. The upper index corresponds to the proton or the neutron and the lower index indicates that the photon helicity is parallel or antiparallel to the nucleon spin. As it was recently show by Drell and Hearn[50], the sum of these relations does not contradict to the available experimental data.

Aznaurian[51] has considered the saturation of these relations by the nearest resonance. Let me remind you that the sum rule for photoproduction (13) have led to the relation (15) which contains the pion-nucleon constants besides the magnetic ones. The sum rules (19) make it possible to connect only the magnetic constants

$$\mu_p'^2 - \mu_n^2 = 0 \qquad (20)$$

$$\mu_p'^2 + \mu_n^2 = \frac{(1+r^2)(1+r)}{8r} \mu^2 (N^* \to N\gamma), \qquad (21)$$

where $r = m/M$. The latter rule gives

$$\mu(N^* \to N\gamma) / \frac{2\sqrt{2}}{3} \mu_p = 1.11 \qquad (22)$$

in accordance with photoproduction.

The Compton effect on each of the baryons has been considered in the same manner. It turns out that the restriction to the nearest resonances leads to results which agree with SU(6) symmetry everywhere except the channel into which the resonance Λ (1405) gives a contribution. This resonance does not belong to the decuplet. If we took into account only the decuplet, we would obtain zero magnetic moment of the Λ -hyperon. However, even the account of Λ (1405) leads to a contradiction. What is clear is that the saturation problem seems to be rather complicated. It is possible to indicate a criterion for choosing superconvergent amplitudes (de Alfaro, Fubini, Furlan and Rosetti[7/] suggested to make use of the Regge pole hypothesis), but it is much more difficult to give a dynamical criterion showing which of them can be saturated by the nearest resonances.

The main role in studying the problem of saturation belongs to experimentators. If at present we can estimate the contributions of the known higher resonances, then the question concerning the role of the non-resonance background is still completely open. The study of this background which is an effect of higher symmetry breaking requires the phase shift analysis or - at higher energies - the determination of the Regge type parameters. From this point of view, to understand higher symmetries, one should study first their breakdowns.

However, the current algebra method allows one to formulate the problem of higher symmetries the other way round. Indeed, I have not yet said anything about a relationship of the algebras with the SU(6) quarks.

The commutators of vector and axial currents which were discussed above, are a generalization of the $SU(3)_W$ symmetry. Using the quark model it is also possible to introduce tensor currents and to find their commutators. Fubini, Segre and Walecka[52/] have shown that the relations thus obtained are close to the algebra of $SU(6)_W$. Therefore, saturating these commuators by one-particle states one can obtain the SU(6) results without assuming explicitly the symmetry to exist[52-55/]. However, the tensor currents like quarks are unobservable, at least for the present. Therefore, in order to interpret these results one has to make additional assumptions. It is peculiar that if the one-pole model is postulated for all vertices into which the vector mesons can give a contribution (this is rather a rough approximation) then one obtains not only the SU(6) results for the magnetic moments but also succeds in relating the magnetic constants of mesons with that of baryons like it is done in the quark model. This result obtained by Petsakos, Segre and Walecka[55a] will be reported at the Conference. I only note that the saturation criterion in such an approach reduces to a requirement that the equations obtained after the saturation of the commutators were self-consistent[x)].

Thus, the problem of obtaining higher symmetries from the lower ones by the current algebra method is rather complicated. Discussion of higher symmetries should be postponed, at least up to the next today's review report devoted to quarks. We come to the question about the symmetry breaking, namely, the SU(3) symmetry breaking.

As I have already said the main hypothesis of the current algebra method is that the equal-time commutation relations (6,7,5)

$$[\bar{Q}, \bar{j}] \approx j \qquad (6a)$$

$$[\bar{Q}, j] \approx \bar{j} \qquad (5a)$$

$$[Q, j] \approx j \qquad (7a)$$

hold true even if the symmetry is broken. Therefore, they allow one to calculate the corrections to the SU(3) formulae. Fubini, Furlan and Rossetti have shown[4] that the sum over higher intermediate states in these commutators can be expressed through a dispersion integral of the imaginary part of the scattering amplitude for some process, either real or hypothetical. If this integral does not require subtractions (this is an additional assumption which can be verified in the Regge pole model if one disregards the Schwinger terms) then from (5a-7a) we obtain the SU(3) relations with corrections which are given by the dispersion integrals.

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x) The one-pole model for the form-factors in a superconvergence approach was considered by Oehme[71].

The commutator (6a) yields the well-known Adler-Weisberger[56,57] relation which connects the correction to the weak axial constant with an integral of the pion-nucleon cross section. Fortunately this integral can be calculated from experiment and gives a good result. A similar relation for the electromagnetic interaction follows from (5a). It connects a weak nucleon form-factor $G(t)$ with the integral of the electroproduction amplitude[58,59] δ

$$G(t) = F_1^V(t) + \delta(t). \tag{23}$$

If the integral is neglected we have a result of the SU(3) - symmetry. Unfortunately, the integral δ can be calculated now only under an assumption about saturation. In the given case we believe that this assumption is justified. Taking into account in δ only the contribution from the (33) resonance Furlan, Jendo, and Remiddi[58] have obtained

$$G(t) = F_1^V(t) - \frac{t}{4m^2} F_2^V(t), \tag{24}$$

where F_1^V and F_2^V are the Dirac and Pauli isovector form-factors of the nucleon. We see that at small t the correction is small.

The commutator (7a) gives corrections for the magnetic moments. Unfortunately, the corrections in this case are integrals of the amplitudes which can be interpreted as photoproduction amplitudes of scalar particles whose existence has not yet been established. Therefore, these integrals can be estimated only under model assumptions, even if we saturate them by lowest states. Using some model[60] it was shown that the corrections to the SU(3) formulae for the magnetic moments are small (of the order of 20 %). Finally, the commutator $[\rho, \partial_\mu j_\mu]$ leads to the mass formulae with corrections due to SU(3)[4] and or SU(2) breaking. The case of SU(2) symmetry breaking was considered in details by Faustov[74] who succeded in finding the mass formulae, involving electromagnetic mass differences, which take into account the interference of electromagnetic and medium-strong interactions. In this case the current algebra approach seems to give some additional dynamical information as compared with the pure group-theoretical method. A similar question is also considered in

Efremov's report to this Conference. The electromagnetic mass differences on the basis of superconvergence were recently considered by Harari[75].

As far as the commutators between vector currents of the type (7a) are concerned when symmetry is not broken (isotopic symmetry for the isospin currents or SU(3) in the general case) then the sum rules following from the commutators (e.g. the Cabibbo-Radicati relation[44]) can be obtained directly from the gauge invariance and certain assumptions about the high-energy behaviour of the amplitudes (60a-f, 35a).

The commutator for the electromagnetic current of the type $[j_\mu, j_\lambda] \approx \epsilon_{0\mu\nu\lambda} \bar{j}_\lambda$ can be obtained in the quark model. It allows one to connect the weak constant g_A with an integral of the total cross section for the absorbtion of polarized photons on polarized nucleons, into which the high energy region gives the main contribution[68]; this commutator also leads to an approximate relation between g_A and π^0 lifetime[69].

I'll mention some other applications of the current algebras. Using current algebra Weinberg[61] formulated a so-called soft pion method which makes it possible to connect the processes with different number of not very energetic pions. This method proved to be succesful for weak interactions. Its application to electromagnetic processes allowed one to succeed in connecting the decay widths of $\eta \to 2\pi\gamma$ and $\eta \to 2\gamma$ [62,63], as well as $X^0 \to 2\pi\gamma$ and $X^0 \to 2\gamma$ [63a]. Formulae have been also obtained which connect the cross sections for the processes $e^+e^- \to \pi^+\pi^-$ and $\pi\pi \to \pi\pi$ [64]. It is worthwhile noting, however, that the validity of such an approximation is not yet quite clear. Its application to the process $\omega \to 2\pi\gamma$ gives zero in contrast to experiment[65].

Bjorken[32] applied the current algebra to estimate the photo-absorption cross sections at high energies and to calculate the hyperfine structure of the hydrogen spectrum. He found that the contribution from very virtual photons in the diagram of the virtual Compton scattering is less than $4 \cdot 10^{-6}$ in agreement with what Prof. Yennie said yesterday about the new experimental value of the hyperfine structure due to a new determinetion of the fine structure constant. The application of commutators to calculations of radiative corrections to the β decay constant in cut-off models was considered in refs.[32,70]. Finally, attempts were made to establish a relationship between the πN scattering lengths and nucleon magnetic moment[66]. The latter results are still only speculative. We see, however, that the potential possibility of applying the current algebra method to electromagnetic interactions may be considerable.

Before I stop I would like to repeat once more that symmetry is beautiful, but we can appreciate it only through its breakdowns. One could appeal to experimentators and ask them to search for symmetry breakdowns if they themselves did not find these breakdowns rather often.

The theory of the symmetry breaking should point to where these breakdowns are to be sought for in the first turn.

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