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THE DIVERGENCES OF VECTOR AND AXIAL-VECTOR CURRENTS IN STRONG INTERACTION

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1. Introduction

It has been proposed in the works of Gell-Mann[1] that a part of strong interactions may have a symmetry larger than SU(3) namely SU(3)x SU(3) generated by the vector and axial-vector currents (charges) of the hadrons. This means that considering only this part of the strong interactions we have

$$-i\partial^{\mu} V_{\mu}(x)^{a}_{b} = 0, \qquad (1)$$

$$-i\partial^{\mu} A_{\mu}(x)^{a}_{b} = 0;$$

where $V_{\mu}(x)^{a}_{b}$ and $A_{\mu}(x)^{a}_{b}$ are the vector and axial-vector currents, respectively (a,b =1,2,3). On the other hand, it was quite clear from the beginning that eqs. (1) can have only very approximate validity if we consider all the strong interactions. In fact, the PCAC hypothesis[2] states that

$$-i\partial^{\mu} A_{\mu}(x)^{a}_{b} = \text{const} \cdot \Phi(x)^{a}_{b} \qquad (2)$$

($\Phi(x)^{a}_{b}$ being the fields of pseudoscalar mesons), thus we are left with SU(3) symmetry only generated by the vector currents. Further, SU(3) is also violated in a way that the really conseved quantitites in strong interactions are only the isospin and hypercharge: SU(2) xU(Y) symmetry.

Recently it has been pointed out by M.Veltman[3] that an essential part of informations about the dynamics of weak and electromagnetic interactions is contained in the divergence conditions, i.e. in the explicit expressions for $\partial^{\mu} V_{\mu}$ and $\partial^{\mu} A_{\mu}$. It is obvious that this statement holds for the strong interactions as well (if not better). Thus there is an interest in considering models of symmetry breaking strong interactions which give certain definite divergence conditions for the vector and axial-vector currents. The purpose of this note is to construct such models and to investigate several consequences of them.

2. Models of SU(3) x SU(3) Symmetry Breaking Strong Interactions

Let us consider the divergence conditions

$$[P^{\mu}, V_{\mu}(x)^a_b] = -i\partial^{\mu} V_{\mu}(x)^a_b = W(x)^a_b ,$$
$$[P^{\mu}, A_{\mu}(x)^a_b] = -i\partial^{\mu} A_{\mu}(x)^a_b = M(x)^a_b . \qquad (3)$$

First of all we derive a general relation expressing the algebraic properties of the "divergences" $W(x)^a_b$ and $M(x)^a_b$. Assuming the equal time commutation relations /1/ between charges (V^a_b, A^a_b) and current densities ($V_{\mu}(x)^a_b$, $A_{\mu}(x)^a_b$) we have from the Jacobi identity and eq.(3):

$$[V^a_b, W(x)^c_d] - [V^c_d, W(x)^a_b] = \delta^c_b W(x)^a_d - \delta^a_d W(x)^c_b ,$$
$$[A^a_b, M(x)^c_d] - [A^c_d, M(x)^a_b] = \delta^c_b W(x)^a_d - \delta^a_d W(x)^c_b , \qquad (4)$$
$$[V^a_b, M(x)^c_d] - [A^c_d, W(x)^a_b] = \delta^{c}_b M(x)^a_d - \delta^a_d M(x)^c_b .$$

In our models, neglecting electromagnetic and weak interactions we assume a Lagrangian $\mathfrak{L} = \mathfrak{L}_0 + \mathfrak{L}_{int}$. where $\mathfrak{L}_0$ is SU(3) x SU(3) symmetric. We suppose further that $\mathfrak{L}_{int}$ depends on the currents $V_{\mu}{}^b_a = \bar{q} A^a_b \gamma_{\mu} q$; $A_{\mu}{}^a_b = -i\bar{q} A^a_b \gamma_{\mu}\gamma_5 q$ and on "phenomenological" fields like Φ^a_b . We consider the quark fields (q) as the fundamental variables, consequently in $P^{\mu} = P^{\mu}_0 + P^{\mu}_{int}$ we have $P^k_{int} = 0$ (k=1,2,3) and therefore:

$$W(x)^a_b = [P^0_{int}, V_0(x)^a_b] ,$$
$$M(x)^a_b = [P^0_{int}, A_0(x)^a_b] . \qquad (5)$$

In this case it is easy to show the fulfillment of eq. (4).

Supposing the existence of the scalar meson nonet (σ - singlet, σ^a_b octet) our model-Lagrangian is the following

$$\mathfrak{L}_{int} = \mathfrak{L}_s + \mathfrak{L}_m ,$$
$$\mathfrak{L}_s = -\frac{a_0}{2}\sigma(x) - \frac{b}{2}(V^{\mu}(x)^r_s V_{\mu}(x)^s_r - A_{\mu}(x)^r_s A_{\mu}(x)^s_r) ,$$
$$\mathfrak{L}_m = -g_0\sigma(x)^3_3 - \sum_{r,s}(g^V_{(rs)} V^{\mu}(x)^r_s V_{\mu}(x)^s_r + g^A_{(rs)} A^{\mu}(x)^r_s A_{\mu}(x)^s_r) , \qquad (6)$$
$$g_{(ik)} = g_1 \quad (i,k=1,2); \quad g_{(3i)} = g_{(i3)} = g_2 ; \quad g_{(33)} = g_3 .$$

Here $a_0, b, g_0, g^V_1, \ldots, g^A_3$ are constants playing the role of coupling constants. It is obvious that $\mathfrak{L}_s$ is SU(3)-symmetric whereas $\mathfrak{L}_m$ breaks SU(3). Using the equal-time commutation rules of currents /1/ and the commutation relations expressing that pseudoscalar and scalar mesons belong to $(3^*, 3) + (3, 3^*)$ in SU(3) x SU(3) /5/ :

$$[V^a_b, \tilde{\tilde{\sigma}}^c_d] = \delta^c_b \tilde{\tilde{\sigma}}^a_d - \delta^a_d \tilde{\tilde{\sigma}}^c_b ,$$
$$[A^a_b, \tilde{\tilde{\sigma}}^c_d] = -\delta^c_b \tilde{\tilde{\Phi}}^a_d - \delta^a_d \tilde{\tilde{\Phi}}^c_b + \frac{2}{3}\delta^a_b \tilde{\tilde{\Phi}}^c_d , \qquad (7)$$
$$\tilde{\tilde{\Phi}}^a_b = \Phi^a_b + \frac{1}{3}\delta^a_b \Phi , \quad \tilde{\tilde{\sigma}}^a_b = \sigma^a_b + \frac{1}{3}\delta^a_b \sigma .$$

we obtain from eqs. (5) and (6) the divergences

$$W(x)^a_b = g_0(\delta^a_3 \sigma^3_b - \delta^3_b \sigma^a_3) + g^V_{(rs)}(\delta^a_r V^{rs}_{sb} - \delta^s_b V^{ra}_{sr}) +$$
$$+ g^A_{(rs)}(\delta^a_r A^{rs}_{sb} - \delta^s_b A^{ra}_{sr}) ,$$

$$M(x)^a_b = a_0 \Phi(x)^a_b + b\, T^{3a}_{b3} + g_0 \left[\delta^3_b \Phi^a_3 + \delta^a_3 \Phi^3_b - \frac{2}{3}\delta^a_b \Phi^3_3 + \right.$$

$$\left. + \left(\frac{2}{3}\delta^3_b \delta^a_3 - \frac{2}{9}\delta^a_b \right)\Phi \right] + g^S_{(rs)} \left(\delta^a_r S^{rs}_{sb} - \delta^s_b S^{ra}_{sr} \right) + \qquad (8)$$

$$+ g^T_{(rs)} \left(\delta^a_r T^{rs}_{sb} - \delta^s_b T^{ra}_{sr} \right) .$$

Here we used the notations $V^{ab}_{cd} = [V^a_c, V^b_d]_+$, $A^{ab}_{cd} = [A^a_c, A^b_d]_+$;

$$S^{ab}_{cd} = [A^a_c, V^b_d]_+ + [A^b_d, V^a_c]_+ , \quad g^S = \frac{1}{2}(g^V + g^A) ;$$

$$T^{ab}_{cd} = [A^a_c, V^b_d]_+ - [A^b_d, V^a_c]_+ , \quad g^T = \frac{1}{2}(-g^V + g^A) .$$

Let us consider special cases:

A. <u>The generalized σ-model.</u> $(b = g^V_{(rs)} = g^A_{(rs)} = 0)$.
This is clearly a generalization of the σ-model of Gell-Mann and Lévy [2] to the case of SU(3)xSU(3). It is also interesting to note that the term σ^3_3 in $\mathcal{L}_{int}$ is something like the η' tadpole of Glashow and Coleman [6] responsible for the SU(3) breaking.

B. <u>The current x current model.</u> $(a_0 = g_0 = 0)$.
In this case the term of the "old PCAC" disappears, thus the question of PCAC must be reinvestigated (see in Sec. 3).

C. <u>The hibrid model.</u> This was considered already (with $g_0 = 0$) in an earlier paper [7] where we dealt with SU(3) breaking effects is strong interactions: the decuplet decays and the mass splitting of baryon octet.

3. The PDDAC Hypothesis

It was pointed out already by J. Bernstein, S.Fubini, M.Gell-Mann, and W.Thirring [8] and independently by Ch.H.Chao [9] that the "field theoretic version" (2) of PCAC can be replaced by a "dispersion-theoretic version" (PDDAC):

$$\langle B | M(0)^b_a | A \rangle \cong \frac{\mu^2_{(ab)} f_{(ab)}}{\mu^2_{(ab)} - k^2} T_{A \to BM^a_b} . \qquad (9)$$

where $T_{A \to BM^a_b}$ is connected with the amplitude $T'_{A \to BM^a_b}$ of the process $A \to B + M^a_b$ in the following way:

$$T'_{A \to BM^a_b} = {}_{out}\langle BM^a_b | A \rangle_{in} = (2\pi)^4 \delta^4 (p_B + k - p_A) \frac{i\, T_{A \to BM^a_b}}{\sqrt{2(2\pi)^3 k_0}} ; \qquad (10)$$

$\mu_{(ab)}$ and k are the mass and momentum of the pseudoscalar meson M^a_b, respectively, and finally $f_{(ab)}$ is defined through

$$\langle 0 | A_\mu (0)^b_a | M^a_b \rangle = f_{(ab)} k_\mu . \qquad (11)$$

This version of PCAC seems to be more general than eq. (2) since it expresses only that $\partial^\mu A_\mu$ obeys an unsubtracted dispersion relation dominated by the meson pole.

It is well known [10] that in the current-algebraic applications eq.(2) is not needed and the results can be obtain also using only eq.(9). The question of field-theoretic or dispersion-theoretic version of PCAC arises also in connection with divergence conditions. Here we note that the equations (9) and (3) are of different character: eq. (9) can be looked upon as an approximate consequence of properties of strong interactions, whereas the equations in (3) express that the SU(3) xSU(3) symmetry is broken in a given, definite way. On the other hand, it seems to us that the divergence conditions must be used together with and not instead of current algebra, therefore in the following we suppose the simultaneous validity of PDDAC in eq. (9) and the divergence conditions in eqs. (3), (8).

4. Photoproduction of Mesons Near Threshold

In this Section as an example we consider the photoproduction of pseudoscalar mesons in the hibrid model C) unifying models A) and B). (For other applications see ref. [7], decays of decuplet $3/2^+$ and mass splitting in the octet $1/2^+$).

Taking into account also the electromagnetic interactions we have

$$-i\,\partial^{\mu} A_{\mu}(x)^{a}_{b} = M(x)^{a}_{b} + M_{em}(x)^{a}_{b} \quad ; \qquad (12)$$

where $M(x)^{a}_{b}$ is given in eq. (8), and following ref.[3]

$$M_{em}(x)^{a}_{b} = e\,\mathfrak{A}^{\mu}(x)\,[\,j_{\mu}(x)_{em}, A^{a}_{b}\,], \qquad (13)$$

$$j_{\mu}(x)_{em} = \frac{1}{2}\left(V_{\mu}(x)_{1}^{1} - V_{\mu}(x)_{2}^{2} - V_{\mu}(x)_{3}^{3} \right)$$

($\mathfrak{A}^{\mu}(x)$ is the photon field). From eq. (9) follows for the process $N\gamma \to BM^{a}_{b}$ (N = nucleon, $B = \frac{1}{2}^{+}$ or $\frac{3}{2}^{+}$ baryons) at the unphysical point $k = 0$:

$$T_{N\gamma \to BM^{a}_{b}} = f^{-1}_{(ab)} \langle B \,|\, -i\partial^{\mu} A_{\mu}(0)^{b}_{a} - M_{em}(0)^{b}_{a} \,|\, N_{\gamma} \rangle = \qquad (14)$$

$$= -\,f^{-1}_{(ab)} \langle B \,|\, M_{em}(0)^{b}_{a} \,|\, N_{\gamma} \rangle .$$

Eqs. (13), (14) up to the first order in e give the result

$$T_{N\gamma \to BM^{a}_{b}} = -\,e\,f^{-1}_{(ab)} \frac{\epsilon^{\mu}(q)}{\sqrt{2(2\pi)^{3}}} \langle B \,|\, [\, j_{\mu}(0)_{em}, A^{b}_{a}\,] \,|\, N \rangle , \qquad (15)$$

where $\epsilon^{\mu}(q)$ and q are the polarization vector and momentum of the photon, respectively. As we have put $k = 0$, eq. (15) can be expected to hold only near threshold. And even there it is approximate for two reasons: first, it is valid only to the first order in e , secondly, it is approximate since we used PDDAC.

We evaluate the matrix element $T(M^{a}_{b}B) = \langle B|..|N\rangle$ in eq. (15) taking into account the SU(3) breaking in model C). It is easy to show that for $k \cong 0$ the SU(3) transformation properties of axial-vector currents $A_{\mu}(x)^{a}_{b}$ are the same as those of $M(x)^{a}_{b}$, thus from eqs. (15) and (8) we obtain

$$T(\pi^{0}B) = T(K^{0}B) = T(\eta B) = 0 , \qquad (16a)$$

$$T(\pi^{+}N^{0}) = T(\pi^{-}N^{+}) , \qquad (16b)$$

$$\sqrt{2}\,T(K^{+}\Sigma^{0}) = T(K^{+}\Sigma^{-}) , \qquad (16c)$$

$$\sqrt{3}\,T(\pi^{+}N^{*0}) = T(\pi^{+}N^{*-}) = -T(\pi^{-}N^{*++}) = -\sqrt{3}\,T(\pi^{-}N^{*+}) , \qquad (16d)$$

$$\sqrt{2}\,T(K^{+}Y_{1}^{*0}) = T(K^{+}Y_{1}^{*-}) . \qquad (16e)$$

In a recent paper of P. de Baenst et al.[11] eq. (15) was derived from current algebra for S-wave photoproduction. There the SU(3) breaking was neglected, consequently there are further relations, namely,

$$\sqrt{2}\,T(\pi^{+}N^{0}) - \sqrt{3}\,T(K^{+}\Lambda) + T(K^{+}\Sigma^{0}) = 0 , \qquad (17a)$$

$$T(\pi^{+}N^{*0}) - \sqrt{2}\,T(K^{+}Y_{1}^{*0}) = 0 . \qquad (17b)$$

Taking into account SU(3) breaking eqs. (17a,b) are no longer valid. E.g. in the σ -model instead of (17a,b) we have

$$\sqrt{2}\;T(\pi^{+}N^{0}) - \sqrt{3}\,T(K^{+}\Lambda) + T(K^{+}\Sigma^{0}) = \sqrt{2}\;g_{0}(\Phi - \delta) ,$$

$$T(\pi^{+}N^{*0}) - \sqrt{2}\,T(K^{+}Y_{1}^{*0}) = \frac{1}{\sqrt{6}}\,g_{0}\,\omega \quad ;$$

where Φ , δ and ω are reduced matrix elements of the pseudoscalar meson fields between octet-octet and decuplet-octet states, respectively.

The experimental data on meson photoproduction near threshold are at present too poor for checking the most of relations in eq. (16). Nevertheless it is known that : 1) $\sigma(\gamma N^{+} \to \pi^{0} N^{+})$ decreases much more rapidly near threshold[12,13] than $\sigma(\gamma N^{+} \to \pi^{+} N^{0})$; 2) $\sigma(\gamma N^{+} \to \pi^{+} N^{0}) / \sigma(\gamma N^{0} \to \pi^{-} N^{+}) \cong 1,2$ near threshold[14] in reasonable agreement with eq. (16b). We remark also that the poor existing data on $\gamma N^{+} \to K^{+} Y_{1}^{*0}$ seem to show that eq. (17b) is not well satisfied, thus the SU(3) breaking effects can play a role in this case.

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