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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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COMPARISION OF THE DISPERSION SUM RULES WITH THE EQUAL-TIME COMMUTATOR SUM RULES

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The problem of the number of the subtractions in the dispersion relations for various invariant amplitudes, i.e. the problem of the dynamics of interaction can be associated with the analysis of the dispersion sum rules.

It is in fact so in the virtue of that the dispersion sum rules are the exact consequence of certain assumptions about the number of the subtractions in dispersion relations.

A certain assumption about the high-energy behaviour of the one of the invariant amplitudes L for πN -virtual photoproduction have been made in the ref.[5].

The unsubtracted dispersion relation in the variable ν for the quantities $L(\nu)$ and $\nu \cdot L(\nu)$ have been obtained from these assumptions and a certain sum rules follow from them. In the resonance model the account of nucleon and the N_{33} isobar gives sum rules which are in good agreement with the experimental data.

It is interesting to recognize if the unsubtracted dispersion relations are valid for the other invariant photoproduction amplitudes.

In sec. I of this note we investigate the dispersion sum rules which are consequences of the definite assumptions about the high-energy behaviour of various invariant amplitudes.

In sec. 2 we consider the sum rules which are obtained from the high-energy behaviour of the amplitudes together with the assumptions about the validity of the equal-time commutator relations.

Finally we compare results, which are obtained by these methods.

I. The Dispersion Sum Rules for πN -Virtual Photoproduction.

Consider the amplitude-like quantuty: .

$$T_n = \int \theta(x) e^{-ikx} \langle p's' | [j_n(x), \mathrm{div} A_\rho(0)] | ps \rangle d^4 x , \tag{1.1}$$

where k is the photon momenta, $\mathrm{div}\, A_\rho(x) \equiv \partial^\mu A_{\mu\rho}(x)$ is the divergence of the axial current with the same quantum number as a pion labled with index ρ , $|p's'\rangle$, $|ps\rangle$ are the one-nucleon states, $j_n(x)$ is the electromagnetic current. The quantity T_n has the all transformation properties of the virtual πN -photoproduction amplitude. In the virtue of it the quantity T_n can be expanded on 6 gauge invariant quantities:

$$R_1^n = \bar{u}^{s'}(p')\gamma^5 [p^n (p' \cdot k) - (p \cdot k) p'^n] u^s(p)$$

$$R_{2,3}^n = \bar{u}^{s'}(p')\gamma^5 [\gamma^n ((p' \pm p) \cdot k) - \gamma \cdot k (p' \pm p)^n] u^s(p)$$

$$R_4^n = - \bar{u}^{s'}(p')\gamma^5 [\gamma^n \gamma \cdot k - \gamma \cdot k \gamma^n] u^s(p) \tag{1.2}$$

$$R_5^n = \bar{u}^{s'}(p')\gamma^5 [(p - p')^n k^2 - ((p - p') \cdot k) k^n] u^s(p)$$

$$R_6^n = \bar{u}^{s'}(p')\gamma^5 \gamma \cdot k [\gamma^n \gamma \cdot k - \gamma \cdot k \gamma^n] u^s(p) .$$

With the account of isotopic structure T_n can be written as follows:

$$T^n = \sum_{i=1}^{6} R_i^n \{ \tau_\rho L_i^{(1)} + \delta_{\rho 3} L_i^{(2)} + \frac{1}{2} [\tau_\rho , \tau_3] - L_i^{(3)} \} . \tag{1.3}$$

From the crossing symmetry of the amplitude we get the following properties:

$$L^{(a)*}_{1,2,4}(\nu) = \pm L^{(a)}_{1,2,4}(-\nu), \quad \left\{ \begin{array}{l} +,\ a = 1,2 \\ -,\ a = 3 \end{array} \right.$$

$$L^{(a)*}_{3,5,6}(\nu) = \mp L^{(a)}_{3,5,6}(-\nu), \left\{ \begin{array}{l} -,\ a = 1,2 \\ +,\ a = 3, \end{array} \right. \tag{1.4}$$

where the invariant variables are defined as follows:

$$\nu = \frac{(p + p')k}{2m}, \qquad s = (p + k)^2$$

$$u = (p' - k)^2, \qquad t = (p - p')^2 .$$

Now we assume a high-energy behaviour of all amplitudes which enables us to write down the unsubtracted dispersion relation for all quantities $L_i(\nu)$ and $\nu \cdot L_i(\nu)$ in the variable ν x/.

From these assumptions we get:

$$\int_{-\infty}^{\infty} \mathrm{Im}\, L\ (\nu', k^2, t)\, d\nu' = 0, \quad (i = 1, 2, 3, 4, 5, 6) . \tag{1.5}$$

The relations (1.5) in the frame of the made assumptions are exact.

We consider approximate consequences which the relations (1.5) give.

Now we analyse the relations (1.5) in the point of view of the resonance model taking into account the one-nucleon and the N_{33} -isobar contributions in these relations only.

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x/ Now it is important to emphasize that these total assumptions for the all invariant amplitudes (1.4), of course, have a methodical, formal character. In fact, assume that the high-energy asymptotic of the quantity T is determined with the Regge poles exchance at the t -channel. (Regge-poles with the same quantum numbers as ρ, ω -mesons).

In account of $a_\rho(0) \approx 0,5 \pm 0.3$; $a_\omega(0) \approx 0.5 \pm 0.1$ we see that some invariant amplitudes L_j may generally have a high-energy behaviour which do not allow us to write down the unsubtracted dispersion relations for the quantities $\nu \cdot L_j(\nu)$ and for these amplitudes $L_i(\nu)$ itselves too. The discussion about it to be continued.

We make use of the Goldberger-Treiman relation and define the coupling constants as follows:

$$\langle N(p') \mid \operatorname{div} A_\rho\ (0) \mid N(p) \rangle = \frac{i a\, g_{NN\pi}(q^2)}{m_\pi^2 - q^2}\, \bar{u}(p')\, r_\rho \gamma_5\, \bar{u}(p);$$

$$\langle N(p') \mid \operatorname{div} A_\rho\ (0) \mid N_{33_\mu}\ (p) \rangle = \qquad (1.6)$$

$$= \frac{i a \cdot g_{N^*N\pi}(q^2)}{m_\pi^2 - q^2}\, \bar{u}(p')\,(p-p')_\mu\, r_\rho\, u_\mu(p);$$

$$\langle N(p') \mid (j \cdot \epsilon) \mid N_{33_\mu}\ (p) \rangle = \frac{i\, 3\mu\,(N^* \to N\gamma)}{2\sqrt{2}} \times$$

$$\times\, \bar{u}(p')\Big[\Big(-\hat{k}\gamma_5 + \frac{\gamma_5\,(p \cdot k)}{M}\Big)\epsilon_\mu + \hat{\epsilon}\, k_\mu - \frac{\gamma_5 \hat{\epsilon}}{M}\Big]\, u_\mu(p).$$

where a = [illegible], M is the isobar mass, ε is the photon polarization vector, q = (p+k−p') is the pion momentum.

From the above assumptions and eqs. (1.6), (1.5) we obtain the following relations.

For amplitude $L_1^{(3)}, (k^2 = 0)$ we get:

$$\frac{2e\ g_{NN\pi}}{t - m_\pi^2} + \left(-\frac{2}{3}\right) \frac{3\mu\,(N^* \to N\gamma)\, g_{N^*N\pi}(-M - m)}{2\sqrt{2} \cdot M} = 0, \qquad (1.7)$$

where m, m_π, M -are the masses of nucleon, pion and the N_{33} -isobar, respectively, $\mu(N^* \to N\gamma)$ is the magnetic moment of $N^* \to N + \gamma$ transition.

This relation for forward photoproduction does not hold for experimental values of the quantities which are contained in eq. (1.7).

The discussion concerning this fact will be given below. For invariant amplitude $L_2^{(3)}$ we get:

$$\mu'^{V}(N)\, g_{NN\pi} + \left(-\frac{2}{3}\right) \frac{3\mu(N^* \to N\gamma)\; g_{N^*N\pi}}{2\sqrt{2}} \times$$

$$\times \left[\frac{(M+m)^2(5M-m) + m_\pi^2\,(m-2M)}{12 M^2} + \frac{t}{4M} \right] = 0. \qquad (1.8)$$

If the experimental value $\mu'^{V}(N) = 1,85 \frac{e}{2m_p}$ is used, for the forward photoproduction the value of the quantity $\mu(N^* \to N\gamma)$ can be obtained from eq. (1.8).

We make use of the value of ratio $\frac{g_{NN\pi}}{g_{N^*N\pi}}$ obtained from the dispersion sum rules for the πN -forward scattering[x/].

Making use of eqs. (1.8), (1.12) yields:

$$\frac{3}{2\sqrt{2}} \mu\,(N^* \to N\gamma) = 0,95\, \mu(P)\,,$$

where $\mu(P)$ is the total isovector magnetic moment of the proton. Experiment gives for this quantity the following value:

$$\frac{3}{2\sqrt{2}} \mu_{exp}\,(N^* \to N\gamma) = (1,25 \pm 0,02)\, \mu(P)\,.$$

For the amplitude $L_3^{(1)}$ we get:

$$\mu'^{S}(N)\, g_{NN\pi} = 0 \qquad (1.9)$$

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x/ In refs. /5,8/ the relation has been obtained for πN -forward scattering as follows:

$$\left(\frac{g_{NN\pi}}{g_{N^*N\pi}}\right)^2 = \frac{4}{3} \left\{ \frac{4m^2}{3} - \frac{(M^2+m^2-m_\pi^2)\,[(M-m)^2 - m_\pi^2]}{6M^2} \right\}. \qquad (1.12)$$

From eq. (1.12) we have

$$\frac{g_{NN\pi}}{g_{N^*N\pi}} = 1,235(\text{GeV}).$$

This value differs from the value $\frac{g_{NN\pi}}{g_{N^*N\pi}} = 1,252(\text{GeV})$ obtained from exp. decay width $\Gamma(N^* \to N\pi) = 120$ MeV and $g_{NN\pi\,exp} \Rightarrow 13,55$.

for the amplitude $L_3^{(2)}$ we have:

$$\mu'^{V}(N)\, g_{NN\pi}+\frac{4}{3}\cdot\frac{3\mu(N^*\to N\gamma)\quad g_{N^*N\pi}}{2\sqrt{2}}\times \tag{1.10}$$

$$\times\left[\frac{-(m+M)^3+m_\pi^2(m-2M)}{12M^2}+\frac{t}{4M}\right]=0.$$

From eq. (1.10) for forward photoproduction we obtain:

$$\frac{3}{2\sqrt{2}}\mu\ (N^*\to N\gamma)=1,15\,\mu\,(P).$$

From eq. (1.6) for the amplitude $L_4^{(3)}$ we have:

$$m\cdot\mu^{V}(N)\, g_{NN\pi}+\left(-\frac{2}{3}\right)\cdot\frac{3\mu(N^*\to N\gamma)\quad g_{N^*N\pi}}{2\sqrt{2}}\times \tag{1.11}$$

$$\times\left\{\frac{3m^2}{2}-\frac{t}{4}+\frac{[(m-M)^2-2mM-m_\pi^2]\cdot[2mM-(m-M)^2]}{24M^2}\right\}=0,$$

where $\mu^{V}(N)$ is the total isovector nucleon magnetic moment:

$$\mu^{V}(N)=\frac{\mu(P)-\mu(n)}{2},$$

Making use of the eq. (1.11) we receive:

$$\frac{3}{2\sqrt{2}}\mu\,(N^*\to N\gamma)=1,29\,\mu\,(p)\ .$$

For the amplitude $L_5^{(1)}$ we get:

$$\frac{e\,g_{NN\pi}}{m_\pi^2-t}=0 \tag{1.13}$$

and for the amplitude $L_5^{(2)}$ we have:

$$\frac{e\, g_{NN\pi}}{m_\pi^2 - t} + \frac{4}{3} \cdot \frac{3\, g_{N^*N\pi}\, \mu(N^* \to N\gamma)}{2\sqrt{2}}$$

$$\times \left\{ \frac{m}{M} + \frac{(m^2 - M^2)[(m+M)^2 - m_\pi^2]}{(m_\pi^2 - t)\, 6M^2} \right\} = 0. \qquad (1.14)$$

Eqs. (1.13) and (1.14) do not hold for the forward photoproduction.

Finally, for the amplitude $L_6^{(1)}$ we get:

$$\frac{\mu'^{s}(N)\, g_{NN\pi}}{2} = 0. \qquad (1.15)$$

and for the amplitude $L_6^{(2)}$ we receive:

$$\frac{\mu'^{V}(N)\, g_{NN\pi}}{2} + \frac{4}{3}\, \frac{3\mu(N^* \to N\gamma)\, g_{N^*N\pi}}{2\sqrt{2}} \times$$

$$\times \left\{ \frac{m[m_\pi^2 - (m+M)^2]}{12M^2} + \frac{t}{4M} \right\} = 0. \qquad (1.16)$$

For the forward photoproduction from eq. (1.16) we obtain:

$$\frac{3}{2\sqrt{2}}\, \mu(N^* \to N\gamma) = 1{,}28\, \mu(P) \quad .$$

The sum rules for the other ampliutde are trivial.

The relations (1.15) and (1.16) have been obtained in ref./5/. As we see, eq. (1.8) is hold with the accuracy about 22%, eq. (1.10) is hold with the accuracy about 8% ,and the relations (1.11) and (1.15) are hold with the experimental accuracy.

The eqs. (1.9) and (1.15) coincide and read:

$$\mu(P) + \mu(n) = \frac{e}{2m_p}$$

while experiment gives:

$$[\mu(p)+\mu(n)]_{exp} = 0{,}88 \frac{e}{2m_p}.$$

As it have been shown in ref. /8/ taking into account of the contributions of nucleon, the N_{11} (1518)-resonance and the N_{33}- isobar in the sum rules (1.6) gives:

$$\mu(p)+\mu(n) = (0{,}85 \pm 0{,}03)\frac{e}{2m_p} .$$

As to the relations (1.7), (1.13), and (1.14), they do not hold (for $t=0$). Perhaps, this circumstance shows that the unsubtracted dispersion relations do not hold for the quantities $\nu \cdot L_1(\nu), \nu \cdot L_5(\nu)$ and for the amplitudes $L_1(\nu)$, $L_5(\nu)$ itselves too.

It is possible that for these amplitudes the contribution of the high-energy region coming from Regge-pole exchange in t-channel and the mid-energy contribution are important.

Note, that the amplitudes L_1 and L_5 are these for which the one-nucleon term has a pole in the variable t at the point $t = m_\pi^2 - k^2$.

As it is easy seen, the amplitudes L_1 and L_5 correspond just to the invariants, which vanish for the forward photoproduction x/.

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x/ In fact, take the Breit coordinat system. In this system we get

$$\vec{p} + \vec{p}\,' = 0 \; ; \quad E = k_0 \; ;$$

$$\vec{k} = \vec{\lambda} - (1+\delta)\,\vec{p} \; ; \quad \vec{q} = \vec{\lambda} + (1-\delta)\,\vec{p}$$

$$\delta = \frac{m_\pi^2 - k^2}{4\vec{p}^{\,2}} \, , \quad |\vec{\lambda}(E)| = \sqrt{E^2 - \vec{k}^2 - (1+\delta)^2 \vec{p}^{\,2}} \, ,$$

where: $c = (p + k - p')$ is the pion momenta.

In the Breit coordinate system the invariants R_i can be expanded in six invariants of the Breit system. The spacial parts of these take the form/9/ $i\vec{\sigma}$; $i(\vec{\sigma}\,\vec{p})\,\vec{p}$; $i(\vec{\sigma}\,\vec{p})\vec{\lambda}$; $i(\vec{\sigma}\,\vec{\lambda})\vec{\lambda}$; $i(\vec{\sigma}\,\vec{\lambda})\,\vec{p}$; $[\vec{\lambda} \times \vec{p}]$.

By simple calculations we obtain that the quantities $\vec{R}_1, \vec{R}_5$ depend on the invariant $i(\vec{\sigma}\,\vec{p})\,\vec{p}$ only, and $R_{2,3,4,6}$ depend on the invariants $i\vec{\sigma}$; $i(\vec{\sigma}\,\vec{\lambda})\,\vec{\lambda}$. Taking it into account we get for the forward photoproduction as follows:

$$\vec{R}_1(t=0) = \vec{R}_5(t=0) = 0$$

$$R_{2,3,4,6}(t=0) \neq 0.$$

In account of that we get as follows. Consider the quantity $T(t=0)$. From the above discussion this quantity contains the invariant amplitudes L_2, L_3, L_4, L_6 only. Then we assume a high-energy behaviour for the quantity $T(t=0)$ which enables us to write down the unsubtracted dispersion relations in the variable ν both for the quantities $T(\nu, t=0)$ and $\nu \cdot T(\nu, t=0)$. Analogously to the above discussion we obtain in the resonance model the relations of the type (1.8), (1.9), (1.10), (1.11), (1.15) and (1.16) only.

As it was shown all these relations are in a reasonable agreement with experiment.

Consider the relations (1.8), (1.9), (1.10), (1.15) and (1.16) from the point of view of the SU(6)-symmetry.

In the static SU(6) - limit the relations (1.8), (1.10) and (1.16) coincide and take the form:

$$M^2 \to m^2 \gg m_\pi^2, -t:$$

$$\mu'^V(N) = \frac{4m}{3\sqrt{2}} \cdot \frac{g^* \mu(N^* \to N\gamma)}{g}, \qquad (1.17)$$

where

$$\frac{g^*}{g} = \lim_{M^2 \to m^2 \gg m_\pi^2} \frac{g_{N^*N\pi}}{g_{NN\pi}}.$$

In the SU(6) -symmetry we get [7]:

$$\mu(N^* \to N\gamma) = \frac{2\sqrt{2}}{3} \mu(P),$$

where $\mu(P)$ is the total proton magnetic moment.

From the sum rules for the forward πN - scattering (1.12) we obtain in the static SU(6) -limit as follows:

$$\frac{g^2}{g^{*2}} = \frac{16}{9} m^2 \quad . \qquad (1.18)$$

From the eqs. (1.17), (1.18) we have:

$$\mu'^{V}(N) = \frac{2}{3}\mu(P). \tag{1.19}$$

The SU(6) relation for the total nucleon moments reads:

$$\frac{\mu(p)}{\mu(n)} = -\frac{3}{2}\ ; \tag{1.20}$$

For the anomalous isovector nucleon magnetic moment from eq. (1.20) we get:

$$\mu'^{V}(N) = \frac{\mu(p) - 1 - \mu(n)}{2} = \frac{5\mu(p)}{6} - \frac{1}{2}\ ; \tag{1.21}$$

Making use of the eqs. (1.19)-(1.21) in the static SU(6) -limit we get as follows:

$$\mu(P) = 3\left(\frac{e}{2m_p}\right.$$
$$\mu(n) = -2\left(\frac{e}{2m_p}\right. , \tag{1.22}$$

The relations (1.9) and (1.15) coincide and take the form:

$$\mu'^{V}(N) = \frac{\mu(P) - 1 + \mu(n)}{2} = 0 . \tag{1.9}$$

Making use of the eq. (1.20) in the SU(6) -limit from eq. (1.9) we have the relations (1,22) too.

Finally, in the static SU(6) limit the eq. (1.11) takes the form:

$$M^2 \to m^2 \gg m_\pi^2\ ,\ -t:$$

$$\mu^{V}(N) = \frac{4m}{3\sqrt{2}} \cdot \frac{\mu(N^* \to N\gamma)\, g^*}{g} \tag{1.23}$$

If we compare eq. (1.23) with eq. (1.17) we see that the sum rules for the amplitudes L_2, L_3, L_4, L_6 give for the total isovector magnetic moment of nucleon the same value as this for the anomalous isovector magnetic moment of nucleon.

In the SU(6) -symmetry we get:

$$\mu^V(N)|_{SU(6)} = \frac{\mu(P)-\mu(n)}{2} = \frac{5\mu(P)}{6}. \qquad (1.25)$$

Making use of eqs. (1.24), (1.25) yields:

$$\mu^V(N)|_{\text{sum rules},SU(6)} = \frac{4}{5}\,\mu^V(N)|_{SU(6)} \quad . \qquad (1.26)$$

Thus, we have shown that the relations obtained from the sum rules for the invariant amplitudes L_2, L_3, L_6 are in agreement with SU(6) and the relation obtained from the sum rules for the amplitude L_4 gives for the quantity $\mu^V(N)$ the same value as for the quantity $\mu'^V(N)$ in the static SU(6) -limit. The all above relations show that (for $t = 0$) the sum rules for the amplitudes L_2, L_3, L_4, L_6 are saturated (with a certain accuracy) by the contributions coming from the members of the unitary octet and decuplet. These are joint as a barion 56-plet of SU(6).

For the forward photoproduction the all obtained sum rules in the resonance model give a reasonable agreement with the experimental data.

Note that the "saturation" by nucleon and the N_{33} -isobar of the relations obtained for the amplitudes L_2, L_3, L_4, L_6, show as follows. Perhaps, it testifies about the validity of the unsubtracted dispersion relations for the amplitudes $L_{2,3,4,6}$ and of the assumption about "decuplet dominance". But at the same time this "saturation" cannot be an unambiguous evidence of the validity of the unsubtracted dispersion relations in the variable ν for the quantities $L_{2,3,4,6}(\nu)$ and $\nu L_{2,3,4,6}(\nu)$. It only indicates on mutual compensation of the nucleon contribution with the N_{33} -isobar contribution, while the contributions of the mid-energy and high-energy regions to the quantity $\int \mathrm{Im}\, L_i(\nu)\, d\nu$ can be important and can give as follows:

$$\int \text{Im } L_1 (\nu) \, d\nu \neq 0.$$

To investigate this question one needs to take into account the contributions coming from the mid-energy and high-energy regions at the invariant amplitudes. Make the following assume that from any calculations the high-energy behaviour of an amplitude L_1 is unambiguously known and we have as follows:

$$\text{Im } L_1 (\nu, \; t = \text{const}, \; k^2 = \text{const}) \underset{\nu \to \infty}{\leq} \frac{\text{const}}{\nu^{1+a}},$$

where: $a > 0$.

In this case the following relation

$$\int_{-\infty}^{\infty} \text{Im } L_1 (\nu, \; t, \; k^2) \, d\nu = 0 \qquad (1.25)$$

is valid. In the opposite case the relation (1.25) is an assumption only.

2. The Equal-Time Commutator Relations and the Sum Rules for Photoproduction

The assumptions about the unsubtracted dispersion relations both for the quantities $L_1(\nu)$ and $\nu L_1(\nu)$ give the sum rules as follows:

$$\int_{-\infty}^{\infty} \text{Im } L_1 (\nu') \, d\nu' = 0 \; .$$

This relation is obtained without any current algebra postulated. In the method of Fubini, Furlan, Rosetti /2,3/ the sum rules are obtained from the assumptions about the validity of the commutator relations and about the validity of the unsubtracted dispersion relations for an amplitude L_1. These sum rules for photoproduction take the form:

$$\int_{-\infty}^{\infty} \frac{\mathrm{Im}\, L_i\ (\nu')}{\nu'}\, d\nu' = 0$$

and

$$\int_{-\infty}^{\infty} \mathrm{Im}\ L_i\ (\nu')\, d\nu' = 0\ .$$

It is interesting to compare in detail the sum rules obtained from the unsubtracted dispersion relations only with the sum rules obtained from postulated current algebra for various amplitudes. Consider the photoproduction problem. We shall deal with the same commutator relations as in refs./2,4/.

Consider the commutator relations which are used in ref./2/. These read :

$$[\bar{Q}_\rho\ ,\ j_\mu^{(3)}(\vec{x}, 0)] = 0 \tag{2.1}$$

$$[\bar{Q}_\rho\ ,\ j_\mu^{(8)}(\vec{x}, 0)] = 0\ , \quad \rho = 1,2,3, \tag{2.2}$$

where $j_\mu^{(3,8)}(x)$ are isovector and isoscalar parts of the electromagnetic current, $\bar{Q}_\rho$ is the axial "charge":

$$\bar{Q}_\rho = \int J^A_{\rho 0}(\vec{x}, 0)\, d^3x = \lim_{q_\mu \to 0} \int e^{-i\vec{q}\vec{x}}\, J^A_{\rho 0}(\vec{x}, 0)\, d^3x = \tag{2.3}$$

$$= \lim_{q_\mu \to 0} \int \Theta(-x_0)\, \partial^\mu \{ J^A_{\rho\mu}(x)\, e^{iqx} \}\, d^4x\ ,$$

$J^A_{\rho\mu}(x)$ is the axial current, and ρ is the SU(3) -index.

Take the matrix element of the commutators (2.1) and (2.2) between the one-nucleon states. Then following Fubini/2/, let us consider the quantity:

$$< N(p')|[\bar{Q}_\rho , j^{(3,8)}_\mu(0)]|N(P)> \epsilon_\mu = \lim_{q_\mu \to 0} F^{(3,8)}(q), \tag{2.4}$$

where ϵ_μ is an arbitrary covariant vector, and define:

$$\bar{D}_\rho(x) = \partial^\mu J^A_{\mu\rho}(x) \equiv \mathrm{div}\, J^A_\rho(x). \tag{2.5}$$

The quantity $\bar{D}_\rho(x)$ has the all quantum numbers of a pion.

Making use of eqs. (1.4) and (1.3) after partial integration we obtain for the quantities $F^{/3,8/}$ as follows:

$$\lim_{q_\mu \to 0} F^{(3,8)}(q) = \lim_{q_\mu \to 0} \{ \epsilon^\mu \int < N(p')|[\bar{D}_\rho(x), j_\mu(0)]|N(p)> \times \tag{2.6}$$

$$\times e^{iqx}\Theta(-x_0)\, d^4x +$$

$$+ iq_\nu \int < N(p')|[J^{\nu A}_\rho(x), j_\mu(0)]|N(p)> e^{iqx}\Theta(-x_0)\, d^4x \},$$

$$\mathrm{Im}\, q^0 = 0 \quad \text{for} \quad |q^0| < \delta, \quad \text{where} \quad \delta > 0.$$

Taking into account the arbitrariness of the vector ϵ_μ we shall choose in following: $(k \cdot \epsilon) = 0$ and take $t = k^2 = q^2 = 0$. We define the invariant variables s, u, t, ν as in the previous section. The quantities $F^{(3,8)}$ have the all transformation properties of the πN - photoproduction amplitude. In the virtue of it the quantity $F^{(3,8)}$ can be expanded on the invariant quantities as follows:

$$F = \bar{u}(p')[\alpha \cdot M_\alpha + \beta M_\beta + \gamma \cdot M_\gamma + \delta \cdot M_\delta]\, u(p). \tag{2.7}$$

In eq. (2.7) the invariant quantities M_α, M_β, M_γ, M_δ are defined as in refs.[1,2] and these have the form:

$$M_\alpha = \gamma_5 \hat{\epsilon} \hat{k}$$

$$M_\beta = 2\gamma_5 [(p'\cdot k)(p\cdot\epsilon) - (p\cdot k)(p'\cdot\epsilon)] \tag{2.8}$$

$$M_\gamma = \gamma_5 [((p-p')\cdot k)\hat{\epsilon} - ((p-p')\cdot\epsilon)\hat{k}]$$

$$M_\delta = \gamma_5 [((p+p')\cdot k)\hat{\epsilon} - ((p+p')\epsilon] - 2m\gamma_5 \hat{\epsilon}\hat{k},$$

where m is the nucleon mass, α, β, γ, δ are the invariant functions of the variables ν, t.

As it is easy seen[2] in the limit $q_\mu \to 0$ only M_α survives while M_β, M_γ, M_δ vanish in this limit. In the virtue of it we consider in this limit the invariant amplitude $\alpha(\nu,t)$ only. We use in eq.(1.6) the complete set of the states $|n>$ and take $E \neq p^0$, $E \neq p^{0'}$. From these we obtain that in the limit $q_\mu \to 0$ only the first term in r.h.s. of the eq. (2.6) survives and eq.(2.6) read:

$$\lim_{q_\mu \to 0} F_\rho^{(3,8)}(q) = \lim_{q_\mu \to 0} \epsilon^\mu \int < N(p')|[\bar{D}_\rho(x), j_\mu(0)]| N(p)> e^{iqx}\Theta(-x_0)d^4x. \tag{2.9}$$

Following Fubini[2,3] assume now that for the invariant amplitudes $\alpha_\rho^{(3,8)}$ the unsubtracted dispersion relations in the variable ν are valid for fixed $t = 0$. These dispersion relations take the form:

$$\alpha_\rho^{(3,8)}(\nu) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\mathrm{Im}\,\alpha_\rho^{(3,8)}(\nu')}{\nu' - \nu} d\nu'. \tag{2.10}$$

In the virtue of the commutator relations (2.1) and (2.2) corresponding sum rules read:

$$\lim_{q_\mu \to 0} \alpha_\rho^{(3)}(\nu) = 0, \tag{2.11}$$

$$\lim_{q_\mu \to 0} \alpha_\rho^{(8)}(\nu) = 0. \tag{2.12}$$

We make use of the Goldberger-Treiman relation and define the coupling constants as in the Sec. 1 by eqs. (1.6). From these in the resonance model, taking into account nucleon and the N_{33} -isobar, we obtain from eq. (2.12) as follows:

$$F_2^s(0)\, g_{NN\pi} = 0. \tag{2.14}$$

From eq. (2.11) we get as follows $(q^2 = 0)$:[x]

$$F_2^V(0)\, g_{NN\pi} - \frac{2}{9}\, \frac{(m+M)^2}{M^2}\, \frac{3\, g_{N^*N\pi}\, \mu(N^* \to N\gamma)}{2\sqrt{2}} = 0 \;. \tag{2.15}$$

As we see, eq. (2.15) coincides with eq. (1.16) for $t = q^2 = 0$.

From eq. (2.14) we obtain:

$$F_2^s(0) = \mu'^s(N) = \frac{\mu(P) + \mu(n)}{2} = 0 \;. \tag{2.16}$$

Make use of the experimental value of the anomalous isovector magnetic moment of nucleon:

$$F_2^V(0) = \mu'^V(N) = 1{,}85\, \frac{e}{2m_p} \;.$$

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[x] The relations (2.14), (2.15) as the corresponding results obtained by Fubini for photoproduction have been got in the following way. We took $q_\mu = 0$ and then singled out different structures in the sum rules for the quantity $F|_{q=0} = [\alpha M_\alpha + \beta M_\beta + \gamma M_\gamma + \delta M_\delta]_{q=0} = \alpha M\, |_{q=0}$.

There is in principle another way of treatment. We put $q_\mu \neq 0$ and make use of the mutual independence of the different invariant amplitudes. From them we obtain from the sum rules for the quantity $F = \alpha M_\alpha + \beta M_\beta + \gamma M_\gamma + \delta M_\delta$ the sum rules for each invariant amplitude. After that we take infinitly small values for q_μ everywhere. In this way the second term in r.h.s. of eq. (2.6) for $q_\mu \neq 0$ gives the contributions to the parts of the amplitudes, which are antisymmetric in the $3,\rho$ or $8,\rho$ indeces, and does not to symmetric parts of amplitudes (i.e. to $\alpha^{(+)}$, $\beta^{(+)}$, $\gamma^{(+)}$, $\delta^{(+)}$,). The sum rule for the quantity $\gamma^{(+)}$ is trivial. The one-nucleon contributions in the sum rules for the amplitudes $\alpha^{(+)}$, $\beta^{(+)}$ and $\delta^{(+)}$ are given in ref.[1] The calculations show as follows. If in this treatment we should take into account also the N_{33} -isobar contribution to these sum rules we obtained from eqs. (2.1) (2.2) the relations which contradicted to experiment.

From this and from eq. (2.15) we obtain for the forward photoproduction ($t = 0$) the value of the magnetic moment of the transition $N^* \to N\gamma$ as follows[x/]

$$\mu(N^* \to N\gamma) = \frac{2\sqrt{2}}{3}\,\mu(P)\,1{,}35 \quad .$$

While experiment gives:

$$\mu_{exp}(N^* \to N\gamma) = \frac{2\sqrt{2}}{3}\,\mu(P)\,(1{,}25 \pm 0{,}02).$$

The eq. (2.16) have been obtained in the resonance model from the one-dimensional dispersion relation for the virtual photoproduction if we take into account only nucleon and the N_{33} -isobar.

To obtain the sum rules for the other invariant amplitudes (2.8) we consider now other commutator relations. So, we take the commutator relations used in ref.[/4/], which take the form:

$$[\, j_0^{\alpha}(\vec{x},0), \phi^{\beta}(0)\,] = c_{\gamma}^{\alpha\beta}\,\phi^{\gamma}(0)\,\delta^{(3)}(\vec{x})\,, \tag{2.17}$$

where $j_0^{\alpha}(x)$ is the vector current component,
$\phi^{\beta}(x)$ is the pseudoscalar field operator,
α , γ and β are the SU (3)-indices.

For the matrix elements of eq. (2.1) between the one-nucleon states we get:

$$\langle p'|\,[\, j_0^{\alpha}(\vec{x},0), \phi^{\beta}(0)\,]\,|p\rangle = \langle p'|\, C_{\gamma}^{\alpha\beta}\,\phi^{\gamma}(0)|\,p\rangle \times$$
$$\times\,\delta^{(3)}(\vec{x}) = C_{\gamma}^{\alpha\beta}\,F^{\gamma}(t)\,\delta^{(3)}(\vec{x}) \quad . \tag{2.18}$$

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x/ In ref.[/2/] use of the following interaction

$$\langle P|\, j_{\mu}^{(3)}(0)\,|N^{+}_{33\nu}\rangle = i\,\frac{C}{m_{\pi}}\,\bar{\psi}_N\,\gamma_5\,\gamma_{\mu}\,u_{\nu}$$

for the $N^*N\gamma$ -vertex with the value C obtained in ref.[/10/] gives from eq. (2.11) as follows:

$$\mu'^{V}(N) = 1{,}99\,\frac{e}{2\,m_p} \quad .$$

Following Fubini[3] let us define the quantities:

$$T_{\mu}^{\alpha\beta} = \int \langle p' | [j_{\mu}^{\alpha}(x), \phi^{\beta}(0)] p \rangle \Theta(x_0) e^{ikx} d^4x \qquad (2.19)$$

and

$$t_{\mu}^{\alpha\beta} = \frac{-i}{2} \int \langle p' | [j_{\mu}^{\alpha}(x), \phi^{\beta}(0)] | p \rangle e^{ikx} d^4x . \qquad (2.20)$$

After the partial integration we obtain by a simple way as follows:

$$k^{\mu} T_{\mu}^{\alpha\beta} = W^{\alpha\beta} + c_{\gamma}^{\alpha\beta} F^{\gamma}(t) \qquad (2.21)$$

$$k^{\mu} t_{\mu}^{\alpha\beta} = w^{\alpha\beta} , \qquad (2.22)$$

where:

$$W^{\alpha\beta} = \int \langle p' | [D^{\alpha}(x), \phi^{\beta}(0)] | p \rangle \Theta(x^0) e^{ikx} d^4x \qquad (2.23)$$

$$w^{\alpha\beta} = \frac{-i}{2} \int \langle p' | [D^{\alpha}(x), \phi^{\beta}(0)] | p \rangle e^{ikx} d^4x . \qquad (2.24)$$

The quantity $(T^{\alpha\beta} \epsilon)$ can be expanded in the set of invariant quantities as follows:

$$T_{\mu}^{\alpha\beta} \epsilon^{\mu} = \bar{u}(p') \gamma_5 \{ A_1^{\alpha\beta} (P\epsilon) + A_2^{\alpha\beta} (k\epsilon) + A_3^{\alpha\beta} (q\epsilon) + A_4^{\alpha\beta} \hat{\epsilon} + \hat{k} [A_5^{\alpha\beta} (P\epsilon) + A_6^{\alpha\beta} (k\epsilon) + A_7^{\alpha\beta} (q\epsilon) + A_8^{\alpha\beta} \hat{\epsilon}] \} u(p) \qquad (2.25)$$

and the similar expantion for the quntity $(t^{\alpha\beta} \cdot \epsilon)$. If we choose α, β as isospin indices we get:

$$A^{\alpha\beta} = \delta^{\alpha\beta} A^{(+)} + \frac{1}{2} [\tau^{\alpha}, \tau^{\beta}]_{-} A^{(-)} . \qquad (2.26)$$

We assume now the unsabtracted dispersion relations in the variable ν for fixed t for the quantities $A_i(\nu)$ and $W(\nu)$.

These relations take the form:

$$A_1(\nu) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{a_1(\nu')}{\nu' - \nu} d\nu' \tag{2.27}$$

$$W(\nu) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{w(\nu')}{\nu' - \nu} d\nu' .$$

Making use of the eq. (2.25) and of the similar relation for the quantity $t^{\alpha\beta}\epsilon$ we obtain from eqs. (2.21), (2.27), (2.28) the sum rules which read:

$$m \int_{-\infty}^{\infty} a_1^{\alpha\beta}(\nu', k^2, q^2, t)\, d\nu' = c_\gamma^{\alpha\beta} F^\gamma(t) \tag{2.29}$$

and

$$\int_{-\infty}^{\infty} a_5^{\alpha\beta}(\nu', k^2, q^2, t)\, d\nu' = 0 . \tag{2.30}$$

The amplitudes $a_{1,5}^{(+)}(\nu)$ are the odd functions and $a_{1,5}^{(-)}(\nu)$ are the even functions of the variable ν. In the virtue of it the non-trivial relations coming from eqs. (2.29) and (2.30) can be written as follows/4/:

$$2m \int_0^{\infty} a_1^{(-)}(\nu', k^2, q^2, t)\, d\nu' = c_\gamma^{\alpha\beta} F^\gamma(t) \tag{2.31}$$

and:

$$\int_0^{\infty} a_5^{(-)}(\nu', k^2, q^2, t)\, d\nu' = 0. \tag{2.32}$$

Note, that eqs. (2.29) and (2.30) are the sole sum rules which follow from the commutator relations (2.17). For the other invariant virtual photoproduction amplitudes we get nothing from eq. (2.17).

Let us choose as follows: $\alpha = 3$, $\beta = 1,2,3$, i.e. we choose the quantity $\phi_\beta(x)$ as the pion field operator with isotopic index β.

Then we obtain as follows:

$$c_\gamma^{3\beta} F^\gamma(t) = \frac{2}{2} [\tau^3, \tau^\beta] F(t)$$

$$\partial^\mu j_\mu^3(x) = 0.$$

Thus, the quantity $\phi_\beta(x)$ is a pion field operator. In the virtue of it the l.h.s. of eq. (2.29) has the pole in the variable q^2 in the point $q^2 = m_\pi^2$, while the r.h.s. of eq. (2.29) does not depend on the variable q^2 quite. Therefore the residue of the l.h.s. of eq. (2.29) in the point $q^2 = m_\pi^2$ is equal to zero/4/:

$$2m \int_{0}^{\infty} \underset{(q^2=m_\pi^2)}{\text{residue}} \left[a_1^{(-)}(\nu', k^2, q^2, t) \right] d\nu' = 0 . \tag{2.33}$$

Now we get:

$$\underset{(q^2=m_\pi^2)}{\text{residue}} \left[a_1^{(-)}(\nu, k^2, q^2, t) \right] = (t + k^2 - m_\pi^2)\, \text{Im}\, \beta^{(-)}(\nu, k^2, t)$$

and similarly:

$$\underset{(q^2=m_\pi^2)}{\text{residue}} \left[a_5^{(-)}(\nu, k^2, q^2, t) \right] = \frac{2\text{Im}\, \delta^{(-)}(\nu, k^2, t)}{1},$$

where the invariant amplitudes $\beta(\nu, k^2 t)$ and $\delta(\nu, k^2, t)$ are defined as in eq. (2.8). From these using eq. (2.33) we have/4/ as follows:

$$(t + k^2 - m_\pi^2) \int_0^\infty \text{Im}\, \beta^{(-)}(\nu', k^2, t)\, d\nu' = 0 \tag{2.34}$$

and similarly from eq. (2.30) we get:

$$\int_0^\infty \text{Im}\, \delta^{(-)}(\nu', k^2, t)\, d\nu' = 0. \tag{2.35}$$

Consider the sum rule (2.35) in the resonance model taking into account nucleon and the N_{33} -isobar x/.

From this we have:

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x/ In ref./4/ use of the following interaction $\langle p | j_\mu^{(3)}(0) | N^+_{33\nu} \rangle = i \frac{C}{m_\pi} \bar{\psi}_N \gamma_5 \gamma_\mu u_\nu$ for the $N^* N \gamma$ vertex gives the value of the anomalous isovector magnetic moment of nucleon as follows:

$$\mu'^{V}(N) = 3,1 \frac{e}{2m_p} .$$

$$F_2^V(0)\, g_{NN\pi} + \frac{3\, g_{N^*N\pi}\, \mu(N^* \to N\gamma)}{2\sqrt{2}} \times$$

$$\times \left(-\frac{2}{3}\right)\left[\frac{(M+m)^2\,(5M-m) + m_\pi^2\,(m-2M)}{12\,M^2} + \frac{t}{4M}\right] = 0 \;. \qquad (2.36)$$

For the forward photoproduction we have from eq. (2.36) as follows:

$$\mu(N^* \to N\gamma) = \frac{2\sqrt{2}}{3}\, \mu(p)\; 0{,}96.$$

The sum rule (2.34) in the resonance model yields:

$$4F_1^V(0)\, g_{NN\pi} + \left(-\frac{2}{3}\right)\frac{(m+M)}{M}\, \frac{3 g_{N^*N\pi}\, \mu(N^* \to N\gamma)}{2\sqrt{2}} \times \qquad (2.37)$$

$$\times (m_\pi^2 - t) = 0 \,,$$

where $F_1^V(0) = \frac{e}{2}$.

Eq.(2.37) coincides with the relation (1.7) obtained from the one-dimensional dispersion relations for the invariant photoproduction amplitude $(R_1 \epsilon)$.

The relation (2.36) coincides with eq. (1.8) obtained from the sum rules coming from the one-dimensional dispersion relations for the invariant photoproduction amplitude ($R_2 \epsilon$) in the resonance model.

Finally the conclusions can be drown that use of the commutator relations together with the assumption about the unsubtracted dispersion relations for the photoproduction amplitude gives sum rules for some invariant amplitudes. These sum rules give rise to the same results as these coming only from the one-dimensional dispersion relations for the photoproduction amplitudes.

It is interesting to note, that the one-dimensional dispersion relations method allows us to obtain the sum rules for the amplitudes for which from the current algebra method (in the Fubini technique) this information can not be obtained.

The relation (2.37) and the coinciding with it eq. (1.7) do not hold for the experiment values of the quantities which are contained in eqs.(1.7), (2.37) for $t = 0$.

As in the Sec. I we conclude from this that the unsubtracted dispersion relations in the variable ν do not hold for the invariant amplitudes $\beta(\nu)$ and for the amplitudes $(R_1 \epsilon)_\nu$ coinciding with it. Perhaps, for these amplitudes the contribution of the high-energy region coming from the Regge-poles exchange in the t -channel is important. As it was pointed in the Sec. I the invariant multiplier $\bar{u}(p')\gamma_5 (P\epsilon)\, u(p)$ of the invariant amplitude $a_1(\nu)$ in eq. (2.25) vanishes for the forward photoproduction. In the virtue of it if analogously to the discussion in the Sec. I we consider the relation (2.21) and (2.22) for $t = 0$ we obtain from this the sum rules in the kind of (2.32) only. As it was shown these sum rules in the resonance model are in agreement with experiment.

Finally we conclude as follows. As it was shown in this paper, there are two alternative possibilities to obtain the sum rules for photoproduction problem. The first one consists in the making use of the assumptions about high-energy behaviour of the invariant amplitudes which enable us to write down the unsabtracted dispersion relations in the variable ν for the quantities $L_j(\nu)$ and $\nu L_j(\nu)$ (assumptions about "superconversion" of the amplitudes).

The second one consists in the making use of the equal-time commutator relations together with the assumptions about the unsabtracted dispersion relations in the variable ν for the invariant amplitudes $L_j(\nu)$.

We saw that both methods give the same results. Namely, the sum rules for a group of the invariant amplitudes in the resonance model are in agreement with experiment while these for another group of the invariant amplitudes are not.

Perhaps the last circumstance show that dispersion relations without subtractions do not hold for the invariant amplitudes of the second group.

Generally speaking, it is not clear for the time being both for the dispersion sum rules and for the sum rules obtained from the equal-time commutator relations, why used sum rules in the resonance model are valid for some amplitudes and do not hold for the other. This problem is being studied.

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References

1. Chew, Goldberger, Low, Namby. Phys.Rev., 106 , 1345 (1957).
2. Fubini, Furlan, Rozetti. Nuovo Cim., 40, 1171 (1965); 43, 161,(1966).
3. Fubini. Nuovo Cimento, 43, 474 (1966).
4. Mukuda, Radha. Nuovo Cimento, 44, 726 (1966).
5. I.G.Aznauryan, L.D.Soloviev. Preprint E-2544, Dubna (1966).
6. A.A.Logunov, L.D.Soloviev. Nucl. Phys., 10, 60 (1959).
7. B.Beg, V.Singh, Phys.Rev.Lett., 13, N 16 (1964).
8. V.A.Matveev, V.G.Pisarenko, B.V.Struminsky. Preprint E-2822, Dubna (1966).
9. A.A.Logunov, A.N.Tavkhelidze, L.D.Soloviev. Nucl.Phys., 4, 427 (1957).
7. Gourdin, Salin. Nuovo Cimento, 27, 193 (1963).

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