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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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Статические скалярное и электрическое поля в теории тяготения Эйнштейна

В работе рассмотрена модель сферически-симметричного тела (сферы) при наличии гравитационного, электромагнитного и скалярного полей. Найденные статические решения отличаются от решений проблемы, данных Фишером, Бергманом и Лейпником, в отличие от последних удовлетворяют общим требованиям, которые должны предъявляться к статическим решениям в рамках общей теории относительности, и естественно интерпретируются физически.

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Static Scalar and Electric Fields in the Einstein Theory of Gravitation

The model of a spherically symmetric body (sphere) was considered in the presence of gravitational, electromagnetic and scalar fields. The static solutions derived differ from those given by Fisher, Bergmann and Leipnik by that the former satisfy the general requirements imposed on static solutions in the framework of the general relativeity and are naturally interpreted from the physical point of view.

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The static spherically symmetric model of a body (sphere) of restricted dimensions is considered in the presence of gravitational, electromagnetic and scalar fields. A similar problem was repeatedly solved in the presence of only gravitational and electromagnetic fields (see for example, papers by Papapetrou[1] Bonnor etc.[2]). It was shown that such static solutions are possible. Such models were subjected to a criticism even by Einstein[3] because of some ambiguity which permits to choose various distributions of the electric charge density.

An advanced investigation of the problem in the presence of only gravitational and scalar fields with a point source was made by I.Fisher in 1948[1]. Note however that this solution in itself has no physical meaning since it does not satisfy the requirement for the metric to be Euclidean in the infinitely small region near the origin[5] (i.e. at this point the ratio of the length of an infinitely small circle to its diameter is not π, what follows from the equality $e^{\lambda(0)}=0$). Moreover, in ref.[4] the error in the calculations has led to a wrong conclusion about the non-Schwarzschild behaviour of the function $e^{\nu(r)}$ at $r\to\infty$. The total energy was also found to be wrong (the infinity) due to a strongly singular transformation of the coordinates. Thus, in reality, the described system can not be realized. Moreover, as it will be shown below, the static model of a particle (body) consisting of a "dust-like" matter can not also be realized in the presence of only gravitational and scalar fields. (It is impossible to fulfil the condition like (15) $\varkappa^2 m^2 + \varkappa G^2 = 0$, here G is the scalar constant, $\varkappa$ is the gravitational constant. This conclusion can be understood from the physical point of view since both gravitational and scalar forces are attractive and, consequently, can not be a basis for the construction of a steady system.

The problem of a point source for gravitational and scalar massless fields was also considered by O.Bergmann and R.Leipnik[6] in 1957. They gave, in addition to the Fisher's solution, some other ones. However, a fraction of them, as the authors point out, does not obey the condition for the metric to be Galilean at infinity. The remaining one does not satisfy the natural conditions of positiveness (>0) of the quantities $\varkappa^2 m^2$ and $\varkappa G^2$ (or the sum $\varkappa^2 m^2 + \varkappa G^2$). In particular, the solution for which $m = 0$, $G \neq 0$ does not satisfy the Bianchi identity $\frac{\nu'}{2}\rho + j V' = 0$ (comp. below eq. (6)) at the point $r = 0$ since at this point we would have $\rho = 0$ but $jV' \neq 0$.

It is clear from the physical point of view that if, in addition to the gravitational and scalar fields, we introduce the electrostatic one then we may hope to construct a steady model since the electrostatic forces are repulsive. We first consider the case of a massless scalar field. We do not introduce phenomenological external forces inside the body. This picture corresponds to a "dust-like" body structure.

For the Einstein equations $G_\mu^\nu = 8\pi\varkappa T_\mu^\nu$ the material tensor consists in the mentioned case of three parts. The tensor of the matter is

$$\overset{(m)}{T}{}_\mu^\nu = -\rho g_{\mu\sigma} \frac{dx^\sigma}{ds}\frac{dx^\nu}{ds},$$

here ρ is the invariant mass density. The electromagnetic field tensor (in Gaussian units) is

$$\overset{(elm)}{T}{}_\mu^\nu = \frac{1}{4\pi}\left(F_{\mu\sigma}F^{\nu\sigma} - \frac{1}{4}\delta_\mu^\nu F_{\sigma\lambda}F^{\sigma\lambda}\right),$$

here

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

The tensor of the scalar massless field is

$$\overset{(sc)}{T}{}_\mu^\nu = \frac{1}{4\pi}\left(-\nabla_\mu V \nabla^\nu V + \frac{1}{2}\delta_\mu^\nu \nabla_\sigma V \nabla^\sigma V\right).$$

The equations for electromagnetic and scalar fields are

$$\nabla_\mu F^{\nu\mu} = 4\pi J^\nu, \quad \nabla_\sigma \nabla^\sigma V = -4\pi j,$$

here j is the invariant density for the scalar field source. In the considered static spherically symmetric case we have

$$(ds)^2 = g_{\mu\nu} d\xi^\mu d\xi^\nu = -e^\lambda (dr)^2 - r^2 (d\theta)^2 - r^2 \sin^2\theta (d\varphi)^2 + e^\nu (dt)^2,$$
$$\lambda = \lambda(r),\ \nu = \nu(r),\ \det g_{\mu\nu} = g = -e^{\lambda+\nu} r^4 \sin^2\theta,$$
$$(\xi^1, \xi^2, \xi^3, \xi^4 = r, \theta, \varphi, t).$$

The Einstein equations become

$$-\frac{\lambda'}{r}+\frac{1-e^{\lambda}}{r^2}=-\varkappa(V')^2-\varkappa e^{-\nu}(\varphi')^2-8\pi\varkappa\rho e^{\lambda}, \quad (1)$$

$$\frac{\nu'}{r}+\frac{1-e^{\lambda}}{r^2}=\varkappa(V')^2-\varkappa e^{-\nu}(\varphi')^2, \quad (2)$$

$$\frac{\nu''}{2}+\frac{\nu'-\lambda'}{2r}+\frac{\nu'(\nu'-\lambda')}{4}=-\varkappa(V')^2+\varkappa e^{-\nu}(\varphi')^2, \quad (3)$$

here φ is the scalar potential of the electromagnetic field and the prime means the derivative d/dr.

The equation for the electromagnetic field takes on the form

$$\varphi''+\left(\frac{2}{r}-\frac{\nu'+\lambda'}{2}\right)\varphi'=-4\pi J_4 e^{\lambda}, \quad (4)$$

here J_4 is the electric charge density, and the equation for the scalar field is

$$V''+\left(\frac{2}{r}+\frac{\nu'-\lambda'}{2}\right)V'=4\pi j e^{\lambda}. \quad (5)$$

The contracted Bianchi identity $(\nabla_\sigma T_1^\sigma=0)$ gives

$$\frac{\nu'}{2}\rho+jV'+\varphi' J_4=0. \quad (6)$$

It retains in the set of equations (1)-(5) only four independent. Note that the sum of the equations (2) and (3) leads to

$$\frac{\nu''}{2}+\frac{3}{2}\frac{\nu'}{r}+\frac{(\nu')^2}{4}+\frac{1}{r^2}-\lambda'\left(\frac{1}{2r}+\frac{\nu'}{4}\right)-\frac{e^{\lambda}}{r^2}=0, \quad (7)$$

which on integration gives

$$e^{-\lambda}=\frac{1}{\left(1+\frac{r\nu'}{2}\right)^2}+\mathcal{D}\frac{e^{-\nu}}{r^2\left(1+\frac{r\nu'}{2}\right)^2}, \quad (8)$$

$\mathcal{D}$ is a constant of integration.

We first find the asymptotics of the solutions for $r\to\infty$ then we use it as an "external" solution. In order to construct the solution inside the body we use well-known receipts of connecting the internal and external solutions at the boundary.

Outside the body (i.e. for $\rho=j=J_4=0$) eqs. (4) and (5) have the following solutions vanishing at infinity

$$\varphi' = -\frac{\varepsilon}{r^2} e^{\frac{\lambda+\nu}{2}} , \quad V' = -\frac{G}{r^2} e^{\frac{\lambda-\nu}{2}} , \tag{9}$$

here ε is the electric charge.

In this case the problem reduces to a completely definite system of equations

$$-\frac{\lambda'}{r} + \frac{1-e^{\lambda}}{r^2} = -\frac{\varkappa G^2}{r^4} e^{\lambda-\nu} - \frac{\varkappa \varepsilon^2}{r^4} e^{\lambda} , \tag{10}$$

$$\frac{\nu'}{r} + \frac{1-e^{\lambda}}{r^2} = \frac{\varkappa G^2}{r^4} e^{\lambda-\nu} - \frac{\varkappa \varepsilon^2}{r^4} e^{\lambda} . \tag{11}$$

To find the solution for this system at $r \to \infty$ we use the requirement for the metric to be Galilean at infinity $(e^{\nu} \to 1, e^{\lambda} \to 1)$ and the condition of correspondance with the Newtonian approximation (for this the integration constant is chosen so that $e^{\nu} \to 1 - \frac{2\varkappa m}{r} + \ldots$). As a result we get the expansions

$$e^{\nu} = 1 - \frac{2\varkappa m}{r} + \frac{\varkappa \varepsilon^2}{r^2} + \frac{1}{3} \frac{(\varkappa m)\varkappa G^2}{r^3} + \ldots , \tag{12}$$

$$e^{\lambda} = 1 - \frac{2\varkappa m}{r} + \frac{\varkappa G^2 + \varkappa \varepsilon^2}{r^2} + \frac{(\varkappa m)\varkappa G^2}{r^3} + \ldots \tag{13}$$

These expansions if $\varepsilon = 0$ naturally give the asymptotics for the Fisher's solutions, if $G = 0$ transform to the solution for the electrostatic problem (Nordström-Reissner /7/, $e^{\nu} = e^{-\lambda} = 1 - \frac{2\varkappa m}{r} + \frac{\varkappa \varepsilon^2}{r^2}$) and if $\varepsilon = G = 0$ transform to the Schwarzschild solution.

Using the asymptotics found it is easy to determine the constant $\mathcal{D}$ in the integral (8) of the initial system of equations

$$\mathcal{D} = \varkappa^2 m^2 + \varkappa G^2 - \varkappa \varepsilon^2 . \tag{14}$$

To fulfil the requirement for the metric to be Euclidean if $r \to 0$ it is necessary to have $e^{\lambda(0)} = 1$. If the function e^{ν} is not too singular in the origin then from (8) it follows that the solution of interest can be derived only if

$$\mathcal{D} = \varkappa^2 m^2 + \varkappa G^2 - \varkappa \varepsilon^2 = 0 . \tag{15}$$

Note that if $G = 0$ this gives the condition $\varkappa^2 m^2 - \varkappa \varepsilon^2 = 0$ for electrostatic problems obtained in the Papapetrou and Bonnor's models. Thus, we have the solution for the problem outside the body in the form (12), (13) which is used as an "external" in constructing the model of the sphere of large radius

$$R \gg \varkappa m , \varkappa \varepsilon^2, \varkappa G^2. \tag{16}$$

We have also the exact relation which is valid both outside and inside the sphere

$$e^\lambda = \left(1 + \frac{r\nu'}{2}\right)^2 . \tag{17}$$

Now we look for the internal solution for $\nu(r)$ in the form

$$\nu(r) = Ar + B , \qquad A \text{ and } B \text{ are constants.} \tag{18}$$

The boundary conditions on ν and ν' (see Synge/8/) require the continuity of them at $r = R$ and give:

$$A = \frac{2\varkappa m}{R^2} + \frac{4\varkappa^2 m^2 - 2\varkappa \varepsilon^2}{R^3} + \ldots , \tag{19}$$

$$B = -\frac{4\varkappa m}{R} + \frac{-4\varkappa^2 m^2 + \varkappa \varepsilon^2 + 2\varkappa G^2}{R^2} + \ldots \tag{20}$$

The quantity e^λ is connected automatically owing to (17). The derivative λ' has a discontinuity at $r = R$ (simultaneously with ν'' and ρ). Then we have to connect the derivatives of the fields (V' and φ'). Due to some ambiguity we choose for $\varphi'(r)$ elsewhere inside the sphere the value which it takes on at the boundary :

$$\varphi' = -\frac{\varepsilon}{R^2} e^{\frac{\lambda(R) + \nu(R)}{2}} \cong -\frac{\varepsilon}{R^2} . \tag{21}$$

Then the scalar field is connected automatically due to (2), which gives inside the sphere

$$\varkappa (V')^2 = \varkappa e^{-\nu} (\varphi')^2 - \frac{A^2}{4} \cong \frac{\varkappa \varepsilon^2}{R^4} - \frac{\varkappa^2 m^2}{R^4} . \tag{22}$$

Thus, the natural requirement for the quantity $(V')^2$ to be positive is satisfied owing to the condition (15). Using (17), (18) and (21) from the Einstein equation(1) we obtain for the mass density

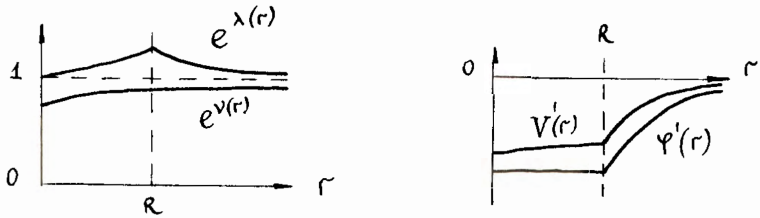
$$8\pi\varkappa r^2 e^{2\lambda}\rho = Ar\left(1+\frac{Ar}{2}\right) + Ar\left(1+\frac{Ar}{2}\right)^3 - 2\varkappa r^2 e^{\lambda-\nu}(\varphi')^2 \cong \tag{23}$$

$$\cong \frac{4\varkappa m r}{R^2} - \frac{2\varkappa r^2 \varepsilon^2}{R^4}.$$

To provide for the quantity $\rho(r)$ to be positive the following restriction

$$\frac{2\varkappa m}{R} > \frac{\varkappa\varepsilon^2}{R^2}, \text{ or } R > \frac{\varkappa\varepsilon^2}{2\varkappa m} \tag{24}$$

should be fulfilled elsewhere inside the sphere which does not contradict the conditions (15) and (16). The behaviour of the functions e^λ, e^ν, V' and φ' is given by the graphs of Fig. 1.



Let us consider the behaviour of the covariant densities of the electric and scalar charges in this model. Taking $\varphi' = \text{const}, \nu' = A, e^\lambda = \left(1+\frac{rA}{2}\right)^2$ from (4) for the first one we have

$$-8\pi r e^{2\lambda} J_y = \left(1+\frac{Ar}{2}\right)\left(4-\frac{A^2r^2}{2}\right)\varphi' \cong 4\varphi' < 0, \tag{25}$$

since $Ar \cong \frac{2\varkappa m}{R}\cdot\frac{r}{R} \ll 1$ and $\varphi' = \text{const} < 0$.

Consequently, $J_y(r)$ has elsewhere the same sign (and is positive). For the density of the scalar field source we have from eqs. (5) and (2)

$$8\pi\varkappa r e^{\nu+\lambda} V'_j = -\varkappa r\nu'(\varphi')^2 + (4+r\nu'-r\lambda')\left[\varkappa(\varphi')^2 - \frac{A^2}{4}e^\nu\right] \cong \tag{26}$$

$$\cong \frac{\varkappa\varepsilon^2}{R^4} - \frac{\varkappa^2 m^2}{R^4} = 4\frac{\varkappa G^2}{R^4}.$$

The last equality follows from eq. (15). Consequently, j has elsewhere the same sign (and $j<0$ since $V'<0$). Note that from these formulas it follows that the densities j and $\mathcal{J}_\gamma$ have (integrable) singularities at the origin ($\sim 1/r$). It is known that this singularity is specific and due to our simplest choice $\nu' =$ const. If we assume that at least $\nu' \sim r$ at the origin and the quntities φ' and V' vanish at $r \to 0$ the singularity disappears. The same is valid for $\rho(r)$. (From eq. (23) it follows that at $r \to 0$, $8\pi\varkappa\rho \sim \frac{2A}{r}$).

The expression for the total electric charge of the system in our metrics

$$I = 4\pi \int_0^\infty \sqrt{\mathcal{J}_\mu \mathcal{J}^\mu}\, r^2 e^{\frac{\lambda}{2}} dr = -\int_0^\infty \left(\varphi' r^2 e^{-\frac{\nu+\lambda}{2}}\right)' dr = = \left[-\varphi' r^2 e^{-\frac{\lambda+\nu}{2}}\right]\Big|_0^\infty \tag{27}$$

for the solution obtained gives

$$I = \left[-\varphi' r^2 e^{-\frac{\lambda+\nu}{2}}\right]\Big|_0^R = \frac{\varepsilon}{R^2} e^{\frac{\lambda(R)+\nu(R)}{2}} \left(R^2 e^{-\frac{\lambda(R)+\nu(R)}{2}} - \lim_{r\to 0} r^2 e^{-\frac{\nu+\lambda}{2}}\right) = \tag{28}$$

$$= \varepsilon .$$

This is in agreement with the electric charge conservation law.

The total mass of the system according to the Landau formula [5]

$$M = \frac{1}{2\varkappa} \left[r(e^\lambda - 1)\right]\Big|_0^\infty \tag{29}$$

is in this case

$$M = \frac{1}{2\varkappa} \underset{(r\to\infty)}{\left[r\left(\frac{2\varkappa m}{r} + \ldots\right)\right]} - \frac{1}{2\varkappa} \underset{(r\to 0)}{\left[r(1 + \ldots - 1)\right]} = m . \tag{30}$$

Thus the construction of the model is completed.

We consider now a possible construction of an analogous model of the sphere in the presence of scalar field of mass μ. The Einstein equations and those of the scalar field are

$$-\frac{\lambda'}{r} + \frac{1-e^\lambda}{r^2} = -\varkappa(V')^2 - \varkappa\mu^2 V^2 e^\lambda - \varkappa e^{-\nu}(\varphi')^2 - 8\pi\varkappa\rho e^\lambda , \tag{31}$$

$$\frac{\nu'}{r} + \frac{1-e^\lambda}{r^2} = \varkappa(V')^2 - \varkappa\mu^2 V^2 e^\lambda - \varkappa e^{\nu}(\varphi')^2 , \tag{32}$$

$$\frac{\nu''}{2}+\frac{\nu'-\lambda'}{2r}+\frac{\nu'(\nu'-\lambda')}{4}=-\varkappa(V')^2-\varkappa\mu^2V^2e^{\lambda}+\varkappa e^{-\nu}(\varphi')^2 \tag{33}$$

$$V''+\left(\frac{2}{r}+\frac{\nu'-\lambda'}{2}\right)V-\mu^2Ve^{\lambda}=4\pi j e^{\lambda}. \tag{34}$$

The electrostatic field equation (4) and the Bianchi identity (6) conserve the previous form. The sum of eqs. (32) and (33) leads to the equation for e^{λ}, which integrates into

$$e^{-\lambda}=\left(1+\frac{r\nu'}{2}\right)^{-2}+r^{-2}e^{-\nu}\left(1+\frac{r\nu'}{2}\right)^{-2}\left[\bar{D}+2\varkappa\mu^2\int_r^{\infty}V^2r^{-2}(r^2e^{\nu})'dr\right], \tag{35}$$

$\bar{D}$ is a constant of integration.

Outside the body (i.e. for $\rho=j=J_y=0$) the system of equations under consideration becomes completely definite. We find the asymptotic expansions of its solutions for $r\to\infty$ taking into account the condition for the metric to be Galilean at infinity and the correspondance with the Newtonian approximation. For the electric field we have as before the exact expression $\varphi'=-\frac{\varepsilon}{r^2}e^{\frac{\nu+\lambda}{2}}$. For the scalar field represented as

$$V(r)\sim\frac{1}{r}e^{-\int W dr}, \tag{36}$$

from eq. (34) we get

$$-W'+W^2-\frac{\nu'-\lambda'}{2}\left(W+\frac{1}{r}\right)=\mu^2e^{\lambda}. \tag{37}$$

It is natural to look for the asymptotics of e^{ν} and $e^{-\lambda}$ in the form

$$1-\frac{2\varkappa m}{r}+\frac{\varkappa\varepsilon^2}{r^2}+O\left(e^{-\mu r}\right), \tag{38}$$

assuming that $e^{-\mu r}\ll 1$

Then from eq. (37) for the scalar field we obtain

$$W=\mu+\frac{\mu\varkappa m}{r}+\frac{\alpha}{r^2}+\frac{\beta}{r^3}+\dots,$$

$$2\alpha=\varkappa m+3\varkappa^2m^2\mu-\varkappa\varepsilon^2\mu,$$

$$2\beta\mu=\varkappa m+2\varkappa^2m^2\mu+5\varkappa^3m^3\mu^2-\varkappa\varepsilon^2\mu-3(\varkappa m)\varkappa\varepsilon^2\mu^2.$$

Substituting this expression into eq.(36) and choosing the integration constant from correspondance with the usual solution without gravitation we get for the scalar field

$$V=\frac{G}{r}e^{-\mu r-\varkappa m\mu \ln \mu r-\frac{d}{r}+\dots}=\frac{G}{r(\mu r)^{\varkappa m\mu}}e^{-\mu r-\frac{d}{r}+\dots}. \quad (39)$$

Note the pecularity of this expansion in comparison with the flat-space case (factor $(\mu r)^{-\varkappa m\mu}$) which was indicated by Zastavenko. Now it is possible to find the first terms of the expansions with the exponents for $e^{-\lambda}$ and e^{ν}. They are

$$e^{-\lambda}=1-\frac{2\varkappa m}{r}+\frac{\varkappa\varepsilon^2}{r^2}+\varkappa G^2\left(\frac{\mu}{r}+\frac{1-\varkappa m\mu}{r^2}+\dots\right)e^{-2\mu r-2\varkappa\mu m\ln\mu r+\dots}+\dots, \quad (40)$$

$$e^{\nu}=1-\frac{2\varkappa m}{r}+\frac{\varkappa\varepsilon^2}{r^2}-\frac{\varkappa G^2}{2}\left(\frac{1}{r^2}+\dots\right)e^{-2\mu r-2\varkappa m\mu\ln\mu r+\dots}+\dots \quad (41)$$

where the last dots are followed by the terms with the higher power exponents. The expansions (39)-(41) are easily compared with the Stephenson /10/ results, who has obtained the expansions of the same quantities in a power series in $\varkappa$.

Using the asymptotics obtained we find the constant in the exact integral (35)

$$\bar{D}=\varkappa^2m^2-\varkappa\varepsilon^2. \quad (42)$$

It is now obvious that for $e^{-\lambda}$ not to be singular at the origin when the functions V and e^{ν} are not singular there it is necessary to have

$$\varkappa^2m^2-\varkappa\varepsilon^2+2\varkappa\mu^2\int_0^\infty V^2r^2(r^2e^{\nu})'dr=0. \quad (43)$$

Then for $e^{-\lambda}$ we get

$$e^{-\lambda}=\left(1+\frac{r\nu'}{2}\right)^{-2}-r^{-2}\left(1+\frac{r\nu'}{2}\right)^{-2}e^{-\nu}2\varkappa\mu^2\int_0^r V^2r^2(r^2e^{\nu})'dr. \quad (44)$$

Further in order to construct the model we use the asymptotics as an "external" solution. This means that the following restrictions $\frac{\varkappa m}{R},\frac{\varkappa\varepsilon^2}{R^2},\frac{\varkappa G^2}{R^2},e^{-\mu R}\ll 1$ are to be imposed on the body radius and, consequently, outside the body the scalar field will give an exponentially small contribution. Assuming that inside the sphere the scalar field remains small we obtain a model which in its physical meaning is very close to the Bonnor models involving the gravitational and electrical fields only. This is also seen from eqs. (43) and (44) which give immediately relations close to the Papapetrou and Bonnor formulas $\varkappa^2 m^2 - \varkappa \varepsilon^2 \cong 0,\ e^{\lambda} \cong \left(1+\frac{r\nu'}{2}\right)^2$.

We take again for the internal solution

$$\nu(r) = Ar + B\,, \qquad A \text{ and } B \text{ are constants.}$$

Imposing on ν and ν' the boundary conditions, we obtain

$$AR = \frac{2\varkappa m}{R} + \frac{4\varkappa^2 m^2 - 2\varkappa \varepsilon^2}{R^2} + \ldots,\ B = -\frac{4\varkappa m}{R} + \frac{3\varkappa\varepsilon^2 - 6\varkappa^2 m^2}{R^2} + \ldots$$

Then we have to connect on the boundary the quantity $\overset{(sc)}{T}{}_1^1 = \frac{1}{8\pi}\left[(V')^2 e^{-\lambda} - \mu^2 V^2\right]$. After letting V and V' continuously go inside the sphere by a simple choice of $V = v_1 + v_2 r$ (v_1 and v_2 are constants) we provide simultaneously the continuity both of the function e^{λ} (according to eq. (44)) and the expression $\overset{(sc)}{T}{}_1^1$. This procedure leads to the expression V inside the body

$$V = \frac{G}{R}\left(\mu R + \mu\varkappa m + 2 + \ldots\right)e^{-\mu R + \ldots} + \frac{G}{R}\left(-\mu R - \mu\varkappa m + 1 + \ldots\right)e^{-\mu R + \ldots}\frac{r}{R}\,.$$

Finally, the function φ' turns out to be connected automatically due to eq. (32). To simplify the analysis of the mass density behaviour we subtract eq. (32) from (31) and get

$$8\pi\varkappa r e^{\lambda}\rho = \lambda' + \nu' - 2\varkappa r (V')^2. \tag{45}$$

Using the values

$$\nu' = A \cong \frac{2\varkappa m}{R^2}\,,\ \lambda' \cong \frac{r\nu'' + \nu'}{1 + \frac{r\nu'}{2}} \cong \nu' = A\,,\ V' \cong -\frac{\mu G}{R}e^{-\mu R}\,,$$

we get for the right-hand side of (45) an approximate expression

$$2A - 2\frac{\varkappa G^2}{R^2}\mu^2 r e^{-2\mu R} > 0\ ,$$

i.e. ρ is surely positive. For the electromagnetic field eq. (32) gives everywhere inside the sphere the value

$$\varkappa(\varphi')^2 \cong \frac{\varkappa\varepsilon^2}{R^4} \cong \frac{\varkappa^2 m^2}{R^4}\,,$$

which is almost constant and close to that on the boundary.

Now it can be shown that the density of the electric charge J_y has the same sign and the total charge and the total mass of the system are $\mathcal{E}$ and m respectively.

Using the obtained expression for the scalar field we can interpret the condition (43) approximately as follows

$$\varkappa^2 m^2 - \varkappa \mathcal{E}^2 + \frac{1}{15}(\varkappa G^2)(\mu R^\gamma)\, e^{-2\mu R + \ldots \atop + \ldots} = 0 .$$

This equation is really very close to the Papapetrou condition $\varkappa^2 m^2 - \varkappa \mathcal{E}^2 = 0$ since $e^{-\mu R} \ll 1$.

The problem of a possible role of the gravitation in the nature of elementary particles was considered even by Einstein. Its various quantum and classic aspects are being discussed /10/. Such models may be interesting as applied to elementary particle theory. We hope to come back to this problem later on.

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