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189_W and Meson Decays

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I

The relativistic generalization of $SU(6)$ for collinear processes leads to the $SU(6)_W$ group (Lipkin and Meshkow [1], see also [2]). One definition of the W-spin group is based on the quarks:

$$W_{\pm} q = J_{\pm} q \qquad W_{\pm} \bar{q} = -J_{\pm} \bar{q}$$

$$W_3 q = J_3 q \qquad W_3 \bar{q} = J_3 \bar{q}$$

$W_{\pm}$, W_3, are W-spin operators and $J_{\pm}$, J_3 are J-spin operators. The calculation of the W-spin multiplets for the 35-and 56-plets of $SU(6)$ is very easy and well-known. To obtain the $SU(6)_W$-plet as a function of the $SU(6)_J$ states for a complicated representation, we define the representation in question in an abstract way. Taking the vector with the hihgest weight, we can generate the full representation if the $SU(6)$ generators act on this state in all possible ways. In our case we need only the operators $N_{\pm}$, $S_{\pm}$, $J_{\pm}$ the $SU(3)$ generators $I_{\pm}$, $K_{\pm}$ and the Casimir operators J^2, N^2, S^2, $C_2^{(3)}$, $C_2^{(4)}$, $C_2^{(6)}$ to select the states. Then we can express the $SU(6)_W$ opreators in terms of $U(6)$ operators and let they act on the state with the highest weight. As result we get the $SU(6)_W$ states as functions of $SU(6)_J$ states. It is well-known that the 189_W-plet mixes [1] the 1_J-, 35_J-and 189_J-plets. To obtain the remaining 35_W and 1_W as functions of $SU(6)_J$ states, we have to consider in the same way $U(6)_W$ operators. The results are listed in Table I and II. Another method for calculating the W-spin multiplets is given by [3].

II

If we consider the decays of the 189-plet mesons into two negative parity mesons, then we have to take into account

$$g_1 \quad 189^+ \times (35^- \times 35^-)_{189} \qquad g_4 \quad 1^+ \times (1^- \times 1^-)$$

$$g_2 \quad 35^+ \times (35^- \times 35^-)_{35} \qquad g_5 \quad 35^+ \times (35^- \times 1^- + 1^- \times 35^-)$$

$$g_3 \quad 1^+ \times (35^- \times 35^-)$$

as possible couplings invariant by $SU(6)$ and charge conjugation. From these cuoplings, written down for the W-spin multiplets we calculate the coupling constants for the $SU(3)$ couplings for each helicity state separately. We have used the Clebsch Gordan Coefficient table of C.L.Cook and G.Murtaza/4/ and the relation

$$(189 \times 189)_1 = \frac{1}{\sqrt{189}} \Big[\sqrt{5}\,(1.5)(1.5) - \sqrt{40}\,(8.5)(8.5) + \sqrt{24}\big((8.3)_2(8.3)_2 - (8.3)_1(8.3)_1\big)$$
$$+ \sqrt{30}\big((10.3)(10^*.3) - (10^*.3)(10.3)\big) + (1.1)(1.1) - \sqrt{8}\,(8.1)(8.1)$$
$$+ \sqrt{27}\,(27.1)(27.1) \Big]$$

whereby the usual sign convention is also fullfilled. The results are collected in Table III, IV and V. Their mean feature are/6/:

2^+ -Mesons

VV, PV and PP decays are possible, but the VV decay is mass forbidden for the existing 2^+ -mesons. The remaining couplings are

$$\left(-\frac{3}{2}\sqrt{\frac{1}{6}}\,g_1 + \frac{3}{4}\sqrt{\frac{1}{35}}\,g_2\right)\sqrt{2} \quad 8^{2+}(8^{1-}8^{0-})_{8_a} \qquad \text{for } h = \pm 1$$

$$\left.\begin{aligned} &\left(\frac{1}{3}\sqrt{\frac{5}{2}}\,g_1 - \frac{1}{2}\frac{1}{\sqrt{21}}\,g_2\right) \quad 8^{2+}(8^{0-}8^{0-})_{8_s} \\ &\left(-\frac{4}{15}\,g_1 + \frac{8}{5}\sqrt{\frac{1}{21}}\,g_3\right) \quad 1^{2+}(8^{0-}8^{0-})_1 \end{aligned}\right\} \text{for } h = 0$$

We see $\dfrac{g_{8^{2+}\to 10^-}}{g_{8^{2+}\to 0^-0^-}} = -\dfrac{3}{2}\sqrt{\dfrac{3}{5}}$ which is in agreement with/5/.

1^+ - Mesons

Here only the PV and VV decays are possible. For the PV decay we have

$h=0$	$h=\pm 1$	
$\left(-\frac{\sqrt{18}}{4}g_1 + \frac{1}{4}\sqrt{\frac{3}{7}}\,g_2\right)\sqrt{2}$	$\sqrt{5}\,g_1$	$8_2^{1+}(8^{1-}8^{0-})_s$
$2g_1$	$\left(\frac{1}{2}\sqrt{2}\,g_1 + \frac{3}{4}\sqrt{\frac{3}{35}}\,g_2\right)\sqrt{2}$	$8_1^{1+}(8^{1-}8^{0-})_a$
$-\sqrt{5}\,g_1$	$\sqrt{5}\,g_1$	$10\,(8^{1-}8^{0-})$
$-\sqrt{5}\,g_1$	$\sqrt{5}\,g_1$	$10^*(8^{1-}8^{0-})$
$-2\sqrt{\frac{3}{35}}\,g_5$		$8_2^{1+}(8^{0-}1^{1-})$

In agreement with the invariance by charge conjugation the first octet has only the decay in $(8^{1-}8^{0-})_{8_a}$ the second in $(8^{1-}8^{1-})_{8_s}$. We do not list the VV decay here, because they are mass forbidden for the known 1^+ mesons.

0^+ - Mesons

Beside the VV decay we have only the PP decay with the couplings

$$\frac{3}{2}\sqrt{3}\,g_1 \quad 27^{0+}(8^{0-}8^{0-})_{27}$$

$$\left(-\frac{13}{12}\sqrt{2}\,g_1 - \frac{1}{4}\sqrt{\frac{5}{21}}\,g_2\right) \quad 8^{0+}(8^{0-}8^{0-})_{8_s}$$

$$\left(\frac{23}{30}\sqrt{\frac{1}{5}}\,g_1 + \frac{16}{5}\frac{1}{\sqrt{105}}\,g_3\right) \quad 1^{0+}(8^{0-}8^{0-})_1$$

III

The decay probability is related to the invariant S -matrix element by

$$\frac{1}{\tau} = \frac{2\bar{k}}{M^2}\,\frac{1}{(2J+1)}\sum_{\text{Spin states}} |\langle p \mid k_1 k_2 \rangle|^2$$

$$\bar{k} = \frac{M}{2}\sqrt{1 - 2\frac{m_1^2 + m_2^2}{M^2} + \frac{(m_1^2 - m_2^2)^2}{M^4}}$$

M positive parity meson, m_1 0^- or 1^- meson
m_2 0^- meson

whereby $p \parallel k_1 \parallel k_2$. So it seems to be that the decay is a collinear process and it is possible to apply the $SU(6)_W$ group. For this reason we develop the lorentz invariant matrix element in helicity state amplitudes of Jacob and Wick /20/

$$\langle p | k_1 k_2 \rangle = \sum_h h^h(M, m_1, m_2)\,(a|bc)^h_J$$

This expression is invariant under rotations $SU(2)_J$ and lorentz transformations in z-direction. On the other hand our $SU(6)_W$ model gives us relations between different helicity state amplitudes (reduced matrix elements) $(abc)^h_W$ invariant under the rotation $SU(2)_W$ and the same special lorentz transformation. If we restrict our space transformations to rotations in the xy-plane, both expressions would have the same transformation properties and we may connect

$$(a|bc)^h_J = (a|bc)^h_W = g^h_{abc} \cdot f$$

so that different matrix elements are connected with the help of their helicity state amplitudes

$$\begin{tikzcd}[column sep=huge]
\langle p | k_1 k_2 \rangle \arrow[r] & h^h(M, m_1, m_2)\,(a|bc)^h \arrow[d, leftrightarrow, "SU(6)_W"] \arrow[d, phantom, swap, "\begin{array}{c}\text{Lorentz transf.}\\ \text{in } z\text{-direction}\end{array}"] \\
\langle p | k_1 k_2 \rangle' \arrow[r] \arrow[u, phantom, "SU(3) \times \text{Lorentz group}"] & h'^h(M', m_1', m_2')\,(a|bc)'^h
\end{tikzcd}$$

Unfortunately, two further assumptions are necessary: a) The considerations of couplings with a different number of derivatives leads to different dimensions of the matrix elements $\langle p | k_1 k_2 \rangle$. From the assumption, that the helicity state amplitudes have always the same dimensions follows that the factor $h(M, m_1, m_2)$ has a mass dimension. We get reasonable results, if we assume that all masses are measured in units of the mass of the decaying particle or in units of the average mass of the corresponding $SU(3)$ or $SU(6)$ plets. Another possibility is to choose the mass unit as a free parameter. b) To $(a|bc)$ there correspond many matrix-elements which differs from the simplest one by 2n derivatives ($n = 1,2,\ldots$) in the coupling.

For simplicity we take only the simplest possible matrix element into account. Practically this means: For suitably chosen couplings (with the right helicity behaviour) we can use for G our $SU(6)_W$ coupling constants.

$\underline{2^+ \to 1^- 0^-}$

$$\langle p | k_1 k_2 \rangle = G\, \varepsilon_{\alpha\mu\nu\sigma}\, p^\mu (k_1 - k_2)_\beta (k_1 - k_2)^\nu \left(T^{\alpha\beta}{}_{(p)} \,|\, V^\sigma_{(k_1)}\, P_{(k_2)} \right)$$

$$= \frac{4i}{\sqrt{2}}\, G\, \bar{k}^2 M \left[(2^+_1 | 1^-_{-1} 0^-) - (2^+_{-1} | 1^-_1 0^-) \right]$$

$$\text{Phase space} \sim G^2 \frac{2}{5\pi} \bar{k}^5$$

$\underline{2^+ \to 0^- 0^-}$

$$\langle p | k_1 k_2 \rangle = G\, (k_1 - k_2)_\nu (k_1 - k_2)_\mu \left(T^{\mu\nu}{}_{(p)} \,|\, P_{(k_1)}\, P_{(k_2)} \right)$$

$$= 4\sqrt{\tfrac{2}{3}}\, G\, \bar{k}^2\, (2^+_0 | 0^- 0^-)$$

$$\text{Phase space} \sim G^2 \frac{4}{15\pi} \frac{\bar{k}^5}{M^2}$$

$\underline{1^+ \to 1^- 0^-}$

$$\langle p | k_1 k_2 \rangle = G\, (k_1 - k_2)_\mu \left(A^\mu_{(p)} \,|\, V^\nu_{(k_1)}\, P_{(k_2)} \right) (k_1 - k_2)_\nu$$

$$= 2 \bar{k}^2 \frac{M}{m_1}\, G\, (1^+_0 | 1^-_0 0^-)$$

$$\text{Phase space} \sim G^2 \frac{1}{6\pi m_1^2} \bar{k}^5$$

$\underline{1^+ \to 1^- 0^-}$

$$\langle p | k_1 k_2 \rangle = G \left[\left(A_{\mu(p)} \,|\, V^\mu_{(k_1)}\, P_{(k_2)} \right) - \frac{M^2 + m_1^2 - m_2^2}{4 \bar{k}^2 M^2} (k_1 - k_2)_\mu (k_1 - k_2)_\nu \left(A^\mu_{(p)} \,|\, V^\nu_{(k_1)}\, P_{(k_2)} \right) \right.$$

$$= -G \left[(1^+_1 | 1^-_{-1} 0^-) + (1^+_{-1} | 1^-_1 0^-) \right.$$

$$\text{Phase space} \sim G^2 \frac{1}{12\pi M^2} \bar{k}$$

$\underline{0^+ \to 0^- 0^-}$

$$\langle p | k_1 k_2 \rangle = G\, (0^+ | 0^- 0^-) \qquad \text{Phase space} \sim G^2 \frac{1}{8\pi M^2} \bar{k}$$

In the case 1^+ we have chosen such complicated couplings because they are easely related to the helicity states. In spite of the difficulties under **a** and **b** we think that this is a natural method to handle $SU(6)_W$ calculations. Difficulties as anounced by Bokow[7] do not ocur here in agreement with Ruegg[8].

We have tried to compare the results with the paper of Kao Ti et al.[6] which calculated the same decay probabilities with $U(6,6)$ techniques. In general the results differ by some numerical factors (of order one), esspecially the relation between coupling constants of different dimensions (for example $\frac{g^1_{210}}{g^1_{200}} = \frac{M+m_1+m_2}{M+2m_1}\,\frac{m_2}{Mm_1}$) are others. Also we have 5 parameters and they only 3.

IV

The 189-plet or the corresponding plets of $U(6,6)$ and $SL(6)$ are discussed by several authors[6,9,11,12]. At present it is not clear if the 189-plet, the 405-plet or the kinetic supermultiplets[10] are the right description of the positive parity mesons. Some authors believe in the 405-plet[5,21] while other have given arguments in favour of the 189-plet[13]. Now the question arises: what are the particle states? Knowing that this is an approximation, we assume that the particles belong to the states of the unphysical chain, the mixing relations[14,11]. (Table VI, the signs are choosen in agreement with the conventions of 4 and 15) and the $SU(3)$ Clebsch Gordan Coefficients (P.Mc.Namee, Frank Chilton[15]) allows us to obtain the coupling constants for each decaying particle[12] separately.

2^+ – Mesons

As well-known the 2^+ nonet may be fitted into the 189 plet. Pure $SU(3)$ considerations similar to Glashow and Socolow[16] Tichonin and Nguen Van Hieu[17] (see also G. Goldhaber[18]) give

		α $(G=2\sqrt{2}F)$	β $\frac{\bar{k}^5}{M^2}$ $[10^6 (MeV)^3]$	ε Observed rate [%]	Γ (MeV)	$\frac{\varepsilon\Gamma}{\alpha\cdot\beta}$
$f \to \pi\pi$	$(2F+\sqrt{2}G)^2$	$36F^2$	54,7	~ 100		$5{,}8 \cdot 10^{-8}$
$\to K\bar{K}$	$\frac{4}{3}(F-\sqrt{2}G)^2$	$12F^2$	5,4	< 4	112 ± 8	6,9
$\to \eta\eta$	$\frac{1}{3}(2F-\sqrt{2}G)^2$	$\frac{4}{3}F^2$	1.6			–
$A_2 \to \pi\eta$	$8F^2$	$8F^2$	23,0	4 ± 4	84 ± 7	1,8
$\to K\bar{K}$	$12F^2$	$12F^2$	7,8	4,6 ± 1,5		4.1
$K^{**} \to K\pi$	$18F^2$	$18F^2$	43,0	50 ± 10	96 ± 7	6,2
$\to K\eta$	$2F^2$	$2F^2$	13,7	2 ± 1		7,0
$f' \to \pi\pi$	$(2\sqrt{2}F-G)^2$	0	97,0	small		
$\to K\bar{K}$	$\frac{4}{3}(\sqrt{2}F+G)$	$24F^2$	25,0	~ 60	80	8.0
$\to \eta\eta$	$\frac{1}{3}(2\sqrt{2}F+G)^2$	$\frac{8}{3}F^2$	15,5			
$A_2 \to \rho\pi$	$4H^2$	$4H^2$	12.3	~ 90	84 ± 7	$1{,}5 \cdot 10^{-12}$
$K^{**} \to K^*\pi$	$1{,}5H^2$	$1{,}5H^2$	11,8	50 ± 10		2.7
$\to \rho K$	$1{,}5H^2$	$1{,}5H^2$	3,5	< 10	96 ± 7	< 1.8
$\to \omega K$	$0{,}5H^2$	$0{,}5H^2$	2,3	1 ± 1		0.8
$f' \to K^*\bar{K} + \bar{K}^*K$	$4H^2$	$4H^2$	1.6	~ 40	80	5
			$\bar{k}^5$ $[10^{12} (MeV)^5]$			

This is essentially the table of Glashow and Socolow, we have chosen $G = 2\sqrt{2}F$ (Goldberg $G = (3{,}9 \pm 1)F$) some mass values and the mixing angles are changed (unphysical chain). The experimental data are taken from A.H.Rosenfeld[19]. Possible average values are

$$\left(\frac{\varepsilon\Gamma}{\alpha\beta}\right)_{2^+\to 0^-0^-} = 6{,}0\cdot 10^{-8} = \frac{4}{15\pi}F^2 \qquad F^2_{2^+\to 0^-0^-} = 7.1\cdot 10^{-7}\ (MeV)^{-2}$$

$$\left(\frac{\varepsilon\Gamma}{\alpha\beta}\right)_{2^+\to 1^-0^-} = 2{,}3\cdot 10^{-12} = \frac{2}{5\pi}H^2 \qquad F^2_{2^+\to 1^-0^-} = \frac{1}{9}H^2 = 2{,}0\cdot 10^{-12}\ (MeV)^{-4}$$

$SU(6)_W$ gives us

$$F = \frac{1}{4\sqrt{10}}\left(\frac{1}{3}\sqrt{\frac{5}{2}}\,g_1 - \frac{1}{2}\frac{1}{\sqrt{21}}\,g_2\right), \quad G = \frac{1}{2\sqrt{2}}\left(-\frac{4}{15}g_1 + \frac{8}{5}\sqrt{\frac{1}{21}}\,g_3\right), \quad H = -3F$$

The ratio of $F_{2^+\to 0^-0^-}$ to $F_{2^+\to 1^-0^-}$ has the dimension of a mass

$$m_0^2 = \frac{F^2_{2^+\to 0^-0^-}}{F^2_{2^+\to 1^-0^-}} = 3{,}5\cdot 10^5\ (\mathrm{MeV})^2$$

We use m_0 to compare couplings of different dimensions. With F and $G = 2\sqrt{2}\,F$ we obtain

$$g_2 = -8\sqrt{210}\,\left(\varepsilon F - \frac{1}{24}g_1\right) \qquad \varepsilon = \pm 1$$

$$g_3 = -5\sqrt{21}\,\left(F + \frac{1}{30}g_1\right)$$

Free parameters are now g_1, g_4, g_5.

1^+ - Mesons

Possible particles are A_1, B, D, E, C, H, $K_{\pi\pi}(1330)$ and $K_{\pi\pi}(1327)$. If we choose the D -meson as a member of the 189-plet, then the E -meson must be excluded. For all states we calculate the individual coupling constants (Table A). The mixing of the $8_2^{1^+}(8^{1^-}8^{0^-})_s$ and $8_1^{1^+}(8^{1^-}8^{0^-})_a$ couplings destroys usually fullfilled equalities between coupling constants, for example $|g_{\tilde K\to K^*\pi}| = |g_{\tilde K\to \rho K}|$ We use the decay rates to calculate the remaining parameters. If we look at the table, we see that the g_1 values show important differences. But taking into account that the g_1 values are strongly dependent on the mass unit m_0 in a complicated way (especially for the 1^+-mesons), we conclude that the particles do not contradict this plet. (Remark that in 11 the G -parity for the I=1 states $(1,0)^{15}$, $(1,0)^{20}$ is opposite to the values given there).

		$h=\pm1$	$h=0$	Particles	result $g_1\left[\frac{1}{\mathrm{MeV}}\right]$
$I=1$	$(1,1)\to\rho\pi$	$\frac14 g_1+\frac18\sqrt{\frac{2}{35}}g_2$	0	$A_1\to 3\pi$, $\Gamma=125$	$g_1=\pm2{,}6\cdot10^{-3}$
	$(0,1)^{10}\to\rho\pi$	$\frac{1}{6\sqrt2}g_1+\frac14\sqrt{\frac{2}{35}}g_2$	$\frac{1}{\sqrt2}g_1$		$g_1=\pm2{,}4\cdot10^{-3}$
	$(1,0)\to\omega\pi$, $\to\varphi\pi$	0, 0	$\left.\begin{matrix}-\sqrt{\frac13}\\ \sqrt{\frac23}\end{matrix}\right\}\left(\frac14\sqrt{\frac13}g_1+\frac14\sqrt{\frac{2}{14}}g_2\right)-\frac{\sqrt2}{1}\sqrt{\frac{1}{14}}g_5$	$B\to\omega\pi$, $\Gamma=125$	$g_1=\mp3{,}6\cdot10^{-3}$ if $B\not\to\varphi\pi$
	$(0,1)^{15}\to\omega\pi$, $\to\varphi\pi$	$\left.\begin{matrix}-\sqrt{\frac13}\\ \sqrt{\frac23}\end{matrix}\right\}\left(\frac32\frac{1}{\sqrt6}g_1\right)$	$\left.\begin{matrix}-\sqrt{\frac13}\\ \sqrt{\frac23}\end{matrix}\right\}\left(-\frac34\frac{1}{\sqrt6}g_1+\frac18\sqrt{\frac{2}{35}}g_2\right)-\frac{\sqrt2}{4}\sqrt{\frac{1}{14}}g_5$		$g_1=1{,}5\cdot10^{-3}$ $B\to\varphi\pi$ (?)
$I=0$	$(1,1)\to K^*K$	$-\frac14\left(\sqrt{\frac12}g_1+\frac32\sqrt{\frac{2}{35}}g_2\right)$	$-\frac{1}{\sqrt2}g_1$	D, $\Gamma=40$	below threshold
	$(0,1)\to K^*K$, $\to\rho\pi$, $\to\omega\eta$	$\left.\begin{matrix}-1\\ \sqrt3\\ -\frac{1}{\sqrt3}\end{matrix}\right\}\sqrt{\frac12}g_1$	$\left.\begin{matrix}-1\\ \sqrt3\\ -\frac{1}{\sqrt3}\end{matrix}\right\}\left(-\frac{\sqrt{11}}{4}g_1+\frac14\sqrt{\frac{2}{7}}g_2\right)+\sqrt{\frac{2}{35}}g_5$	$H\to3\pi$? $\Gamma=120$	$\lvert g_1\rvert\le1{,}2\cdot10^{-3}$
$I=\frac12$	$C\to K^*\pi$	$\frac12 g_1$	$-\frac12 g_1$	$C\to K^*\pi$ 25%, $3K$ 75% ?, $\Gamma=60$	$g_1=\pm0{,}9\cdot10^{-3}$ below threshold
	$\to\rho K$	$\frac12 g_1$	$-\frac12 g_1$		
$I=\frac12$	$(\frac12,\frac32)\to\omega K$, $\to K^*\eta$	$\left.\begin{matrix}-\sqrt{\frac13}\\ 1\end{matrix}\right\}\left(\frac{1}{24\sqrt2}(4\pm9)g_1\pm\frac{1}{16}\sqrt{\frac{2}{35}}g_2\right)$	$\left.\begin{matrix}-\sqrt{\frac13}\\ 1\end{matrix}\right\}\left(\frac{\sqrt2}{24}(1\pm10)g_1+\frac{1}{24}\sqrt{\frac{2}{35}}g_2+\frac23\sqrt{\frac{1}{35}}g_5\right)$	$K_{\pi\pi}(1320)$? $K_{\pi\pi}(1327)$?	
	$\to\rho K$, $\to K^*\pi$	$\frac{1}{24\sqrt2}(12\mp7)g_1\pm\frac{1}{16}\sqrt{\frac{2}{35}}g_2$	$\frac13\sqrt2(-1\pm2)g_1-\frac18\sqrt{\frac{2}{35}}g_2$		
	$(\frac12,\frac12)^{10}\to\omega K$, $\to K^*\eta$	$\left.\begin{matrix}-\sqrt{\frac13}\\ 1\end{matrix}\right\}\left(\frac{1}{12\sqrt2}(2\mp3)g_1\mp\frac18\sqrt{\frac{2}{35}}g_2\right)$	$\left.\begin{matrix}-\sqrt{\frac13}\\ 1\end{matrix}\right\}\left(\frac{\sqrt2}{24}(1\mp2)g_1+\frac{1}{24}\sqrt{\frac{2}{35}}g_2\right)-\frac23\sqrt{\frac{1}{35}}g_5$		
	$\to\rho K$, $\to K^*\pi$	$\frac{1}{12\sqrt2}(6\pm1)g_1\mp\frac13\sqrt{\frac{2}{35}}g_2$	$+\frac13\sqrt2(-1\mp2)g_1-\frac13\sqrt{\frac{2}{35}}g_2$		
	$(\frac12,\frac12)^{\bar{10}}\to\omega K$, $\to K^*\eta$	$\left.\begin{matrix}-\sqrt{\frac13}\\ 1\end{matrix}\right\}\left(\frac{1}{12\sqrt2}(1\mp3)g_1\pm\frac13\sqrt{\frac{2}{35}}g_2\right)$	$\left.\begin{matrix}-\sqrt{\frac13}\\ 1\end{matrix}\right\}\left(\frac{\sqrt2}{48}(1\pm16)g_1+\frac{1}{48}\sqrt{\frac{2}{35}}g_2+\frac23\sqrt{\frac{1}{35}}g_5\right)$		
	$\to\rho K$, $\to K^*\pi$	$\frac{1}{12\sqrt2}(-1\pm5)g_1\pm\frac13\sqrt{\frac{2}{35}}g_2$	$-\frac{1}{16}\sqrt{\frac{2}{35}}g_2$		

J = 1 Particles Table A

			Particles	Results g_1 [$\frac{10}{MeV}$]
$I=0$	$(1,1) \to K\bar{K}$	$\frac{33}{20}\frac{1}{\sqrt{6}}g_1 - \frac{2}{15}\frac{1}{\sqrt{14}}g_3 - \frac{1}{24}\frac{1}{\sqrt{35}}g_2$	$K_0\bar{K}_0 \to K\bar{K}$ >30%, $\pi\pi$ <70% ?, $\Gamma = 80$	$K_0\bar{K}_0$? $\to |g_1| > 0.67 \cdot 10^{-3}$
	$\to \pi\pi$	$\frac{1}{20\sqrt{2}}g_1 - \frac{4}{5}\frac{1}{\sqrt{42}}g_3 + \frac{1}{3}\frac{1}{\sqrt{105}}g_2$	$\sigma \to \pi\pi$?	$\pi\pi$ small
	$(0,0)^1 \to K\bar{K}$	$-\frac{41}{40}\frac{1}{\sqrt{15}}g_1 - \frac{2}{15}\frac{1}{\sqrt{35}}g_3 - \frac{1}{24}\frac{1}{\sqrt{14}}g_2$	$S_0 \to \pi\pi$?, $\Gamma = 50$	$K_0\bar{K}_0$ $|g_1| = \frac{2.7}{7.2} \cdot 10^{-3}$ $K\bar{K}$ $\pi\pi$
	$\to \pi\pi$	$\frac{7}{40}\frac{1}{\sqrt{5}}g_1 - \frac{4}{5}\frac{1}{\sqrt{105}}g_3 + \frac{1}{3}\sqrt{\frac{1}{42}}g_2$		S_0 $|g_1| = 3.7 \cdot 10^{-3}$
	$(0,0)^2 \to K\bar{K}$	$-\frac{1}{10\sqrt{6}}g_1 + \frac{2}{15}\frac{1}{\sqrt{14}}g_3 - \frac{1}{12}\frac{1}{\sqrt{35}}g_2$		S_0 $|g_1| = 0.5 \cdot 10^{-3}$
	$\to \pi\pi$	$\frac{33}{40}\frac{1}{\sqrt{2}}g_1 + \frac{2}{5}\sqrt{\frac{1}{42}}g_3 + \frac{1}{4}\frac{1}{\sqrt{105}}g_2$		
$I=1$	$(1,1) \to K\bar{K}$	$\frac{5}{24}\sqrt{6}\,g_1 + \frac{1}{40}\sqrt{\frac{5}{7}}g_2$	$K_1\bar{K}_1 \to K\bar{K}$, $\Gamma = 57$	$|g_1| \le 1.9 \cdot 10^{-3}$, $\pi\eta$ small
	$\to \pi\eta$	$\frac{1}{12}g_1 - \frac{1}{4}\frac{1}{\sqrt{210}}g_2$		
	$(0,1) \to K\bar{K}$	$-\frac{1}{8}g_1 - \frac{1}{3}\sqrt{\frac{1}{70}}g_2$		
	$\to \pi\eta$	$\frac{5}{12}\sqrt{\frac{3}{2}}g_1 + \frac{1}{6}\sqrt{\frac{2}{35}}g_2$		
$I=\frac{1}{2}$	$(\frac{1}{2},\frac{1}{2})^1 \to K\pi$	$-\frac{\sqrt{2}}{16}g_1 - \frac{1}{4}\sqrt{\frac{1}{35}}g_2$	? $\kappa \to K\pi$, $\Gamma < 12$	$|g_1| < 0.5 \cdot 10^{-3}$
	$(\frac{1}{2},\frac{1}{2})^2 \to K\pi$	$-\frac{3}{8}\sqrt{2}\,g_1 - \frac{1}{8}\sqrt{\frac{2}{35}}g_2$		
	$(J=2, Y=0) \to \pi\pi$	$\frac{1}{2}g_1$		
	$(J=\frac{3}{2}, Y=1) \to K\pi$	$\frac{1}{2}g_1$		
	$(J=1, Y=2) \to KK$	$\frac{1}{2}g_1$	M_1	

J=0 Particles Table B

0^+ – Mesons

Here the situation is more complicated. Possible particles are (if they exist at all) σ , S_0 , $K_0\bar{K}_0$, $K_1\bar{K}_1$, κ . A fit into the representation is very douptful (look at the Table B).

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TABLE I $189_w(225)$

	$W=2$ (octet)	$W=2$ (singlet)
$W_z=2$	$(J=2)$	$(J=2)$
$W_z=1$	$\frac{1}{2}(J=1)_{35}-\frac{\sqrt{3}}{2}(J=1)_{(2)}$	$(J=1)_{35}$
$W_z=0$	$-\frac{1}{3}(J=2)+\frac{\sqrt{5}}{3}(J=0)_{189}-\frac{1}{\sqrt{3}}(J=0)_{35}$	$-\frac{1}{3}(J=2)+\frac{4}{3}\frac{1}{\sqrt{5}}(J=0)_{189}-\sqrt{\frac{8}{15}}(J=0)_{1}$
$W_z=-1$	$-\frac{1}{2}(J=1)_{35}+\frac{\sqrt{3}}{2}(J=1)_{(2)}$	$-(J=1)_{35}$
$W_z=-2$	$(J=2)$	$(J=2)$

	$W=1$ (octet)$_1$	$W=1$ (octet)$_2$	$W=1$ (10-plet)	$W=1$ (10^*-plet)
$W_z=1$	$\frac{1}{2}(J=1)_{(2)}+\frac{\sqrt{3}}{2}(J=1)_{35}$	$\frac{1}{2}(J=1)_{(1)}-\frac{\sqrt{3}}{2}(J=2)$	$(J=1)_{189}$	$-(J=1)_{189}$
$W_z=0$	$-(J=1)_{(1)}$	$\frac{\sqrt{3}}{2}(J=1)_{35}+\frac{1}{2}(J=1)_{(2)}$	$-(J=1)_{189}$	$-(J=1)_{189}$
$W_z=-1$	$\frac{1}{2}(J=1)_{(2)}+\frac{\sqrt{3}}{2}(J=1)_{35}$	$\frac{1}{2}(J=1)_{(1)}+\frac{\sqrt{3}}{2}(J=2)$	$(J=1)_{189}$	$-(J=1)_{189}$

	$W=0$ (octet)	$W=0$ (singlet)	$W=0$ (27-plet)
$W_z=0$	$-\frac{1}{6}(J=0)_{189}-\sqrt{\frac{5}{12}}(J=0)_{35}+\frac{\sqrt{5}}{3}(J=2)$	$\frac{4}{3}\frac{1}{\sqrt{5}}(J=2)-\frac{4}{5}\sqrt{\frac{2}{3}}(J=0)_{1}-\frac{7}{15}(J=0)_{189}$	$-(J=0)_{189}$

TABLE II

$35_w\ (225)$

	$W=1$ (octet)	$W=0$ (octet)	$W=1$ (singlet)
$W_z=1$	$\frac{1}{2}(J=2)+\frac{\sqrt{3}}{2}(J=1)_{①}$		$(J=2)$
$W_z=0$	$-\frac{1}{2}(J=1)_{35}+\frac{\sqrt{3}}{2}(J=1)_{②}$	$-\frac{1}{2}(J=0)_{35}-\sqrt{\frac{5}{12}}(J=0)_{189}-\frac{1}{\sqrt{3}}(J=2)$	$(J=1)_{35}$
$W_z=-1$	$-\frac{1}{2}(J=2)+\frac{\sqrt{3}}{2}(J=1)_{①}$		$-(J=2)$

$1_w\ (225)$

$W=0$ (singlet)

$$-\sqrt{\frac{2}{15}}(J=2)-\frac{4}{5}\sqrt{\frac{2}{3}}(J=0)_{189}-\frac{1}{5}(J=0)_1$$

$35_w\ (36)$

	$W=1$ (octet)	$W=0$ (octet)	$W=1$ (singlet)
$W_z=1$	$(J=1)_{35}$		$(J=1)_{35}$
$W_z=0$	$(J=0)_{35}$	$(J=1)_{35}$	$(J=0)_1$
$W_z=-1$	$-(J=1)_{35}$		$-(J=1)_{35}$

$1_w\ (36)$

$W=0$ (singlet)

$(J=1)_{35}$

TABLE III SU(3) COUPLING CONSTANTS ($J=0$ PARTICLES)

	$(8^{1-}8^{1-}+8^{1-}8^{1-})_s$	$8^{1-}_{0}8^{1-}_{0}$	$8^{1-}_{1}1^{1-}+1^{1-}_{1}8^{1-}$	$1^{1-}_{1}8^{1-}+8^{1-}1^{1-}$	$8^{0-}8^{0-}$	$8^{0-}1^{0-}+1^{0-}8^{0-}$	$1^{0-}1^{0-}$	$8^{1-}1^{1-}+1^{1-}8^{1-}$	$8^{1-}_{0}1^{1-}_{0}+1^{1-}8^{1-}$
1^{0+}_{165}	$-\frac{33}{90}\sqrt{\frac{1}{5}}g_1$ $\frac{16}{5}\frac{1}{\sqrt{105}}g_3$	$-\frac{21}{30}\sqrt{\frac{1}{5}}g_1$ $\frac{16}{5}\frac{1}{\sqrt{105}}g_3$			$\frac{87}{90}\sqrt{\frac{1}{5}}g_1$ $\frac{16}{5}\frac{1}{\sqrt{405}}g_3$		$\frac{12}{45}\sqrt{\frac{1}{10}}g_1$ $-\frac{4}{5}\sqrt{\frac{2}{105}}g_3'$ $\frac{4}{15}\sqrt{2}g_4$	$-\frac{43}{45}\frac{1}{\sqrt{10}}g_1$ $-\frac{4}{5}\sqrt{\frac{2}{105}}g_3$ $\frac{4}{15}\sqrt{2}g_4$	
8^{0+}_{189}	$\frac{12}{36}\sqrt{2}g_1$ $-\frac{1}{4}\sqrt{\frac{5}{21}}g_2$	$-\frac{1}{4}\sqrt{2}g_1$ $-\frac{1}{4}\sqrt{\frac{5}{21}}g_2$	$\frac{2}{36}\sqrt{5}g_1$ $-\frac{1}{2}\sqrt{\frac{1}{42}}g_2$	$\frac{2}{36}\sqrt{5}g_1$ $-\frac{1}{2}\sqrt{\frac{1}{42}}g_2$	$-\frac{33}{36}\sqrt{2}g_1$ $-\frac{1}{4}\sqrt{\frac{5}{21}}g_2$	$-\frac{10}{36}\sqrt{5}g_1$ $-\sqrt{\frac{1}{42}}\frac{1}{4}g_2$			$\sqrt{\frac{2}{21}}g_5$
27^{0+}_{189}	$\frac{3}{2}\sqrt{3}g_1$	$-\frac{3}{2}\sqrt{3}$			$+\frac{3}{2}\sqrt{3}$				
8^{0+}_{35}	$\frac{5}{12}\sqrt{\frac{10}{3}}g_1$ $-\frac{1}{4}\frac{1}{\sqrt{7}}g_2$	$-\frac{3}{4}\sqrt{\frac{10}{3}}g_1$ $-\frac{1}{4}\frac{1}{\sqrt{7}}g_2$	$-\frac{7}{6}\sqrt{\frac{1}{3}}g_1$ $-\frac{1}{2}\sqrt{\frac{1}{70}}g_2$	$-\frac{7}{6}\sqrt{\frac{1}{3}}g_1$ $-\frac{1}{2}\sqrt{\frac{1}{70}}g_2$	$\frac{3}{4}\sqrt{\frac{10}{3}}g_1$ $-\frac{1}{4}\sqrt{\frac{1}{7}}g_2$	$-\frac{1}{6}\sqrt{\frac{1}{3}}g_1$ $-\frac{1}{2}\sqrt{\frac{1}{70}}g_2$			$\sqrt{\frac{2}{35}}g_5$
1^{0+}_{1}	$\frac{8}{5}\sqrt{\frac{1}{30}}g_1$ $\frac{2}{5}\sqrt{\frac{2}{35}}g_3$	$-\frac{12}{5}\sqrt{\frac{1}{30}}g_1$ $\frac{2}{5}\sqrt{\frac{2}{35}}g_3$			$\frac{12}{5}\sqrt{\frac{1}{30}}g_1$ $\frac{2}{5}\sqrt{\frac{2}{35}}g_3$		$-\frac{12}{5}\sqrt{\frac{1}{15}}g_1$ $-\frac{1}{5}\frac{1}{\sqrt{35}}g_3$ $\frac{1}{5}\sqrt{\frac{1}{3}}g_4$	$-\frac{16}{15}\sqrt{\frac{1}{15}}g_1$ $-\frac{1}{5}\sqrt{\frac{1}{35}}g_3$ $+\frac{1}{5}\sqrt{\frac{1}{3}}g_4$	

TABLE IV SU(3) COUPLING CONSTANTS ($J=1$ PARTICLES)

	$(8^{1-}8^{1-}+8^{1-}8^{1-})$	$(8^{1-}1^{1-}+1^{1-}8^{1-})$	$(8^{1-}1^{1-}+1^{1-}8^{1-})$	$(8^{0-}8^{1-}-8^{1-}8^{0-})_a$	$(8^{0-}8^{1-}+8^{1-}8^{0-})_s$	$(8^{1-}8^{1-}-8^{1-}8^{1-})_a$	$(8^{0-}1^{1-}+1^{1-}8^{0-})$	$(8^{1-}1^{0-}+1^{0-}8^{1-})$	$(1^{0-}1^{1-}+1^{1-}1^{0-})$
$8_{(2)}$				$\sqrt{\frac{5}{2}}\,g_1$	$\sqrt{\frac{5}{2}}\,g_1$	$\mp\frac{1}{\sqrt{12}}\,g_1$		g_1	
$8_{(1)}$	$\mp\frac{1}{4}\sqrt{10}\,g_1$ $\pm\frac{1}{4}\sqrt{\frac{3}{7}}\,g_2$	$\mp\frac{1}{2}g_1$ $\mp\frac{1}{4}\sqrt{\frac{6}{35}}\,g_2$	$\mp\sqrt{\frac{6}{35}}\,g_5$	$-\frac{3}{4}\sqrt{\frac{2}{35}}\,g_2$ $-\frac{1}{2}\sqrt{\frac{1}{2}}\,g_1$					
10				$-\sqrt{\frac{3}{2}}\,g_1$		$\pm\sqrt{\frac{3}{2}}\,g_1$			
10^*				$-\sqrt{\frac{3}{2}}\,g_1$		$\pm\sqrt{\frac{3}{2}}\,g_1$			
8^{1+}_{35}					$-\sqrt{\frac{1}{6}}\,g_1$	$\mp\sqrt{\frac{3}{2}}\,g_1$	$-2\sqrt{\frac{1}{3}}\,g_1$	$\sqrt{\frac{1}{3}}\,g_1$	
1^{1+}_{35}					$\sqrt{\frac{1}{3}}\,g_1$				$\sqrt{\frac{1}{6}}\,g_1$

	$(8^{1-}1^{1-}+1^{1-}8^{1-})$	$(1^{1-}8^{1-}+8^{1-}1^{1-})$	$(8^{0-}8^{1-}-8^{1-}8^{0-})_a$	$(8^{0-}8^{1-}+8^{1-}8^{0-})_s$	$(8^{1-}8^{1-}-8^{1-}8^{1-})_a$	$(8^{0-}1^{1-}+1^{1-}8^{0-})$	$(8^{1-}1^{0-}+1^{0-}8^{1-})$
$8_{(2)}$				$-\frac{\sqrt{10}}{4}g_1+\frac{1}{4}\sqrt{\frac{3}{7}}\,g_2$	$\frac{1}{2}\sqrt{\frac{1}{2}}\,g_1+\frac{3}{4}\sqrt{\frac{1}{35}}\,g_2$	$-\sqrt{\frac{6}{35}}\,g_5$	$-\frac{1}{2}g_1+\frac{1}{4}\sqrt{\frac{6}{35}}\,g_2$
$8_{(1)}$	g_1	$-g_1$	$-\sqrt{2}\,g_1$				
10			$\sqrt{\frac{3}{2}}\,g_1$		$\sqrt{\frac{3}{2}}\,g_1$		
10^*			$\sqrt{\frac{3}{2}}\,g_1$		$-\sqrt{\frac{3}{2}}\,g_1$		
8^{1+}_{35}				$-\sqrt{\frac{1}{3}}\,g_1-\frac{1}{4}\sqrt{\frac{1}{7}}\,g_2$	$\frac{1}{2}\sqrt{\frac{3}{2}}\,g_1-\frac{1}{4}\sqrt{\frac{1}{35}}\,g_2$		$-\frac{\sqrt{3}}{2}g_1-\frac{1}{4}\sqrt{\frac{2}{35}}\,g_2$
1^{1+}_{35}				$\sqrt{\frac{1}{30}}\,g_2$			

TABLE V SU(3) COUPLING CONSTANTS ($J=2$ PARTICLES)

$H=\pm 2$

	$1^{1-}1^{1-}$	$8^{1-}8^{1-}$	$8^{1-}1^{1-}+1^{1-}8^{1-}$
1^{2+}	$\sqrt{\frac{1}{3}}\,g_2$	$\sqrt{\frac{1}{3}}\,g_2$	
8^{2+}		$-2\sqrt{\frac{5}{3}}\,g_1$	$-\sqrt{\frac{2}{3}}\,g_2$

$H=\pm 1$

	$(8^{0-}8^{1-}-8^{1-}8^{0-})_a$	$(8^{1-}8^{1-}+8^{1-}8^{1-})_s$	$(8^{1-}1^{1-}+1^{1-}8^{1-})$
1^{2+}		$\frac{1}{\sqrt{70}}\,g_2$	$\sqrt{\frac{1}{35}}\,g_5\,(1^{1-}1^{1-}+1^{1-}1^{1-})$
8^{2+}	$\pm\frac{3}{2}\sqrt{\frac{1}{6}}\,g_1$ $\mp\frac{3}{4}\sqrt{\frac{1}{35}}\,g_2$	$\frac{3}{4}\sqrt{\frac{10}{3}}\,g_1$ $\frac{1}{4}\sqrt{\frac{1}{7}}\,g_2$	$\frac{3}{2}\sqrt{\frac{1}{3}}\,g_1$ $-\frac{1}{4}\sqrt{\frac{2}{35}}\,g_2$ $-\sqrt{\frac{6}{35}}\,g_5\,(8^{1-}1^{1-}+1^{1-}8^{1-})$

$H=0$

	$8^{1-}8^{1-}+8^{1-}8^{1-}$	$8^{1-}8^{1-}$	$8^{1-}1^{1-}+1^{1-}8^{1-}$	$1^{1-}8^{1-}+8^{1-}1^{1-}$	$8^{0-}8^{0-}$	$8^{0-}1^{0-}+1^{0-}8^{0-}$	$1^{0-}1^{0-}$	$1^{1-}1^{1-}+1^{1-}1^{1-}$	$8^{1-}1^{1-}+1^{1-}8^{1-}$
1^{2+}	$\frac{3}{45}\,g_1$ $\frac{8}{5}\sqrt{\frac{1}{21}}\,g_3$	$\frac{2}{5}\,g_1$ $\frac{8}{5}\sqrt{\frac{1}{21}}\,g_3$			$-\frac{12}{45}\,g_2$ $\frac{8}{5}\sqrt{\frac{1}{21}}\,g_3$		$\frac{2}{15}\frac{1}{\sqrt{2}}\,g_1$ $-\frac{2}{5}\sqrt{\frac{2}{21}}\,g_3$ $\frac{2}{3}\sqrt{\frac{2}{5}}\,g_4$	$\frac{7}{15}\frac{1}{\sqrt{2}}\,g_1$ $-\frac{2}{5}\sqrt{\frac{2}{21}}\,g_3$ $\frac{1}{3}\sqrt{\frac{2}{5}}\,g_4$	
8^{2+}	$-\frac{1}{9}\sqrt{\frac{5}{2}}\,g_1$ $-\frac{1}{2}\frac{1}{\sqrt{21}}\,g_2$	$\sqrt{\frac{5}{2}}\,g_1$ $-\frac{1}{2}\frac{1}{\sqrt{21}}\,g_2$	$\frac{4}{9}\,g_1$ $-\frac{1}{\sqrt{210}}\,g_2$	$\frac{4}{9}\,g_2$ $-\frac{1}{\sqrt{210}}\,g_1$	$\frac{3}{9}\sqrt{\frac{5}{2}}\,g_1$ $-\frac{1}{2}\sqrt{\frac{1}{21}}\,g_2$	$\frac{7}{9}\,g_1$ $-\frac{1}{\sqrt{210}}\,g_2$			$2\sqrt{\frac{3}{105}}\,g_5$

TABLE VI Mixing Relations

$J = 2$

$$\begin{pmatrix} 2,0 \\ 1,1 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \end{pmatrix} \qquad J = 0 \quad Y = 0$$

$J = 1$

$$\begin{pmatrix} 1,1 \\ 0,1 \\ (1,0)^{20} \\ (1,0)^{15} \end{pmatrix} = \begin{pmatrix} 0 & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} & +\sqrt{\frac{1}{6}} & 0 & -\sqrt{\frac{1}{6}} \\ 0 & +\sqrt{\frac{1}{6}} & \sqrt{\frac{2}{3}} & +\sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} & 0 & +\sqrt{\frac{1}{3}} \end{pmatrix} \begin{pmatrix} 8_{2} \\ 10^{*} \\ 8_{1} \\ 10 \end{pmatrix}$$

$J = 1 \quad Y = 0$

$$\begin{pmatrix} (\frac{3}{2},\frac{1}{2}) \\ (\frac{1}{2},\frac{1}{2})^{20} \\ (\frac{1}{2},\frac{1}{2})^{4} \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} & \pm\frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \mp\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & \pm\frac{2}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 8_{2} \\ 8_{1} \\ \binom{10^{*}}{10} \end{pmatrix}$$

$J = \frac{1}{2} \quad Y = \pm 1$

$(1,1) = 8$

$(1,0) = 8$ $\qquad J = 0 \quad Y = 0$

$J = 0$

$$\begin{pmatrix} (1,1) \\ (0,0) \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} \sqrt{2} & \sqrt{3} \\ -\sqrt{3} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 8 \\ 27 \end{pmatrix} \qquad J = 1 \quad Y = 0$$

$$\begin{pmatrix} (\frac{3}{2},\frac{1}{2})^{4} \\ (\frac{1}{2},\frac{1}{2})^{20} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} \frac{1}{\sqrt{5}} \begin{pmatrix} 8 \\ 27 \end{pmatrix} \qquad J = \frac{1}{2} \quad Y = \pm 1$$

$$\begin{pmatrix} (1,1) \\ (0,0)^{1} \\ (0,0)^{20} \end{pmatrix} = \begin{pmatrix} -\frac{5}{\sqrt{60}} & \sqrt{\frac{2}{15}} & -\frac{3}{2}\frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ +\frac{5}{\sqrt{60}} & \sqrt{\frac{8}{15}} & -\frac{1}{\sqrt{20}} \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix} \qquad J = 0 \quad Y = 0$$

Notation:

$(NS)^{d} = \mathfrak{R}\ (D)$

d dimension of the SU(4) repr.

D dimension of the SU(3) repr.