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SU/3/ - BREAKING AS CONSEQUENCE OF INTERACTION OF HADRON CURRENTS

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The important role of currents in describing the interactions of particles was early recognized. But the real value of current relations became clear only in recent years, when a number of sum rules was derived on the basis of current-commutation-relations /1/. One of the most surprising successes of current algebra is that it established the universality of weak interactions in the nonleptonic decays too /2/ describing them by Cabibbo's current /3/. In the nonleptonic decays only strongly interacting particles take place and one can still handle with them by the use of current-algebraic methods. Thus it is not hopeless to try to treat the SU(3)-breaking part of the strong interactions if we assume its current-current form. An early attempt to describe the SU(3) breaking medium-strong interaction by means of the strangeness current was done by Y. Ne'eman /4/ and recently Y. T. Chiu and J. Schechter /5/ have investigated the possibility of describibg the mass-splitting by means of a current×current effective Hamiltonian. The purpose of this note is twofold: first, to show how adequate the divergence conditions /6, 7/ (or current algebras /8/) are for deaiing with medium-strong interactions, if nature happens to accept its current×current form; secondly, to derive some relations which perhaps can be used for the determination of the relevant coupling constants.

Let us denote the vector and axial-vector currents of strongly interacting particles by $V_\mu(x)^a_b$ and $A_\mu(x)^a_b$ respectively (a, b =1, 2, 3). Then the most general form of an $SU(2)_I \otimes U(Y)$, C and P invariant current×current Hamiltonian is the following:

$$H_{ms} = \int d^3x \, \mathcal{H}_{ms}(x) \quad ;$$

$$\mathcal{H}_{ms}(x) = g_I^V V^i_j V^j_i + g_C^V \left(V^r_3 V^3_r + V^3_r V^r_3 \right) + g_Y^V V^3_3 V^3_3 + \\ + g_I^A A^i_j A^j_i + g_C^A \left(A^r_3 A^3_r + A^3_r A^r_3 \right) + g_Y^A A^3_3 A^3_3 \qquad (1)$$

(Lorentz indices are omitted, $i, j, k = 1, 2;$ $r = 1, 2, 3$.)

The case $g_I^V = g_c^V = -g_Y^V$, $g_I^A = g_c^A = -g_Y^A$ clearly corresponds to SU(3) symmetry. From a "naive" quark model with Lagrangian etc. one easily derives /7/ the divergence conditions. Thus we have e.g.

$$-i\partial^\mu V_\mu(x)^a_b = g_I^V\left\{\delta^a_i V^{ik}_{kb} - \delta^k_b V^{ia}_{ki}\right\} + g_c^V\left\{\delta^a_3 V^{r3}_{br} - \delta^3_b V^{ar}_{r3}\right\} + g_Y^V\left\{\delta^a_3 V^{33}_{b3} - \delta^3_b V^{a3}_{33}\right\} + (V\to A) , \tag{2}$$

where

$$V^{ab}_{cd} = \left[V^a_c, V^b_d\right]_+ , \qquad A^{ab}_{cd} = \left[A^a_c, A^b_d\right]_+ . \tag{3}$$

Our calculations are based on eq. (2) and the analogous expressions for $\partial^\mu A_\mu$. For brevity eq. (2) can be written also in the form

$$-i\partial^\mu V_\mu(x)^a_b = \left[H_{ms}, V_0(x)^a_b\right] , \tag{4}$$

if one assumes the current commutation relations /1/ without Schwinger terms. Taking into account the PCAC in the SU(3) symmetric ($g_I^V = \dots = g_Y^A = 0$) case, the analogous relation for the axial current is (ϕ^a_b=pseudoscalar meson octet):

$$-i\partial^\mu A_\mu(x)^a_b = C\phi^a_b + \left[H_{ms}, A_0(x)\right] . \tag{5}$$

The decays of $3/2^+$ baryon decuplet (D) into $1/2^+$ baryon octet (B) + 0^- meson octet (M). Reducing the decay amplitude

$$T_{D\to BM} = {}_{out}\langle BM^a_b|D\rangle_{in} = i\int d^4x \frac{e^{ikx}}{\sqrt{2(2\pi)^3k_0}}\left(\Box_x+\mu^2\right)\langle B|\phi^a_b(x)|D\rangle , \tag{6}$$

eq. (5) immediately gives the on mass shell relation

$$T_{D\to BM} = T^{SU(3)}_{D\to BM} + iC^{-1}\int d^4x \frac{e^{ikx}}{\sqrt{2(2\pi)^3k_0}}\left(\Box_x+\mu^2\right)\langle B|\left[A_0(x)^a_b, H_{ms}\right]|D\rangle . \tag{7}$$

Here k and μ are the meson momentum and mass respectively. As a first approximation, we can evaluate the right hand side of eq. (7) in the exact SU(3) limit. Using only the SU(3) transformation properties of the terms we can relate the different decay amplitudes. The results are summarized in Table I. The essential point is, that we remained on the mass shell thus our results do not suffer from off mass shell uncertainities.

Using Table I. we can write down 8 relations between the different decay amplitudes. Experimentally the ones concerning π decay-modes can be checked. These are

$$(\Xi^*|\Xi\pi) = 3(Y_1^*|\Lambda\pi) - \sqrt{2}(N^*|N\pi) ; \qquad \sqrt{2}(\Xi^*|\Xi\pi) = \sqrt{3}(Y_1^*|\Sigma\pi) . \tag{8}$$

Taking into account phase-space corrections, both relations in eq. (8) are well satisfied. It is noteworthy that for π decay-modes the analogous relations derived from the assumption of a symmetry breaking term transforming like the 8-th component of an octet coincide with the SU(3) predictions /9/. One the other hand SU(3) in this case is not satisfied by the data /10/ which can be taken as an argument in favour of the Hamiltonian in (1).

The problem of mass splitting within unitary multiplets was already treated in Ref. /5/, where the $1/2^+$ baryon octet was considered. Here we proceed in a slightly different way, namely instead of calculating the diagonal matrix elements of the mass operator, we evaluate the mass differences directly from the divergence condition for $V_\mu(x)^a_b$. Let us take the matrix element of eq. (2) or (4) e.g. between $1/2^+$ baryon states at rest. Then we get

$$(M_{B_2} - M_{B_1})\langle B_2|V_0(x)^a_b|B_1\rangle = g_I^V\left\{\delta^a_i\langle V\rangle^{ij}_{jb} - \delta^j_b\langle V\rangle^{ia}_{ji}\right\} + g_c^V\left\{\delta^a_3\langle V\rangle^{r3}_{br} - \delta^3_b\langle V\rangle^{ar}_{r3}\right\} + g_Y^V\left\{\delta^a_3\langle V\rangle^{33}_{b3} - \delta^3_b\langle V\rangle^{a3}_{33}\right\} + (V\to A) ; \tag{9}$$

where

$$\langle V\rangle^{ab}_{cd} = \langle B_2|V^{ab}_{cd}|B_1\rangle . \tag{10}$$

Evaluating the matrix elements in the SU(3) limit, and supposing – as usual – the vector current to be of F-type, it is easy to get the following expressions:

$$M_\Lambda - M_N = \frac{6}{5}G_t + \frac{1}{3}G_d + G_f \,, \qquad M_\Xi - M_\Lambda = -\frac{6}{5}G_t - \frac{1}{3}G_d + G_f \,,$$

$$M_\Sigma - M_N = \frac{2}{5}G_t - G_d + G_f \,, \qquad M_\Xi - M_\Sigma = -\frac{2}{5}G_t + G_d + G_f \,. \tag{11}$$

Here we introduced the notations

$$G_t = \tau^V\left(g_I^V + g_Y^V\right) + \tau^A\left(g_I^A + g_Y^A\right) \,,$$

$$G_d = \delta^V\left(-\frac{4}{3}g_I^V + \frac{5}{3}g_c^V + \frac{1}{3}g_Y^V\right) + \delta^A\left(-\frac{4}{3}g_I^A + \frac{5}{3}g_c^A + \frac{1}{3}g_Y^A\right) \,, \tag{12}$$

$$G_f = \phi^V\left(-\frac{4}{3}g_I^V + \frac{5}{3}g_c^V + \frac{1}{3}g_Y^V\right) + \phi^A\left(-\frac{4}{3}g_I^A + \frac{5}{3}g_c^A + \frac{1}{3}g_Y^A\right) \,;$$

where $\tau^{V(A)}$, and $\delta^{V(A)}$, $\phi^{V(A)}$, are the $\underline{27}$, $\underline{8}_D$, $\underline{8}_F$, contributions in $\langle V(A)\rangle^{ab}_{cd}$ respectively. (See Ref. /5/.). Of course, this procedure can be used in an arbitrary SU(3) multiplet, the only problem is the evaluation of the quantities τ, δ, ϕ. For the $1/2^+$ octet τ, δ, ϕ was estimated in Ref. /5/ taking octet and decuplet baryon intermediate states. With the results there: ($\tau^V = 8{,}6$; $\tau^A = -52{,}8$; $\delta^V = -172{,}9$; $\delta^A = -30{,}8$; $\phi^V = 225$; $\phi^A = 157$) $\times 10^{-4} M_p^3$,

we have from eqs. (11,12)

$$-\frac{4}{3}g_I^V + \frac{5}{3}g_c^V + \frac{1}{3}g_Y^V = 2M_p^{-2} \,,$$

$$-\frac{4}{3}g_I^A + \frac{5}{3}g_c^A + \frac{1}{3}g_Y^A = 9M_p^{-2} \,, \tag{13}$$

$$g_I^V + g_Y^V = 6\left(g_I^A + g_Y^A\right) + 8M_p^{-2} \,.$$

(M_p is the mass of the proton.) As eqs. (13) do not determine the values of $g_I^V \ldots g_Y^A$, further investigations are needed for the determination of the exact form of the Hamiltonian (1).

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Table I.

Notations:

$$g_i = \frac{1}{2}\left(g_i^V + g_i^A\right) \,, \quad h_i = \frac{1}{2}\left(g_i^V - g_i^A\right) \,, \quad i = I, c, Y \,,$$

$$_{out}\langle BM|D\rangle_{in} = \sum_{k=1}^{4} \gamma_k R_k + {}_{out}\langle BM|D\rangle_{in}^{SU(3)} \,.$$

Decay \ γ_k	γ_1	γ_2	γ_3	γ_4
$(N^*\|N\pi)$	$2\sqrt{6}\,(2g_I+g_c)$	$-2\sqrt{3}\,(g_c-g_I)$	–	–
$(Y_1^*\|\Sigma\pi)$	$2\sqrt{2}\,(2g_I+g_c)$	$4\,(g_c-g_I)$	–	–
$(Y_1^*\|\Lambda\pi)$	$2\sqrt{3}\,(2g_I+g_c)$	–	–	–
$(\Xi^*\|\Xi\pi)$	$2\sqrt{3}\,(2g_I+g_c)$	$2\sqrt{6}\,(g_c-g_I)$	–	–
$(Y_1^*\|\Sigma\eta)$	$-6\sqrt{2}\,g_c$	–	–	–
$(\Xi^*\|\Xi\eta)$	$-6\sqrt{2}\,g_c$	–	–	–
$(N^*\|\Sigma K)$	$\sqrt{6}\,(2g_I+5g_c+g_Y)$	–	$-2\sqrt{6}\,(h_I+h_Y)$	$-\sqrt{6}\,(4h_I-5h_c-h_Y)$
$(Y_1^*\|\Xi K)$	$\sqrt{2}\,(2g_I+5g_c+g_Y)$	–	$8\sqrt{2}\,(h_I+h_Y)$	$-\sqrt{2}\,(4h_I-5h_c-h_Y)$
$(Y_1^*\|N\bar{K})$	$\sqrt{2}\,(2g_I+5g_c+g_Y)$	$\sqrt{6}\,(g_I-2g_c-g_Y)$	$2\sqrt{2}\,(h_I+h_Y)$	$\sqrt{2}\,(4h_I-5h_c-h_Y)$
$(\Xi^*\|\Sigma\bar{K})$	$\sqrt{3}\,(2g_I+5g_c+g_Y)$	$-(g_I-2g_c-g_Y)$	$7\sqrt{3}\,(h_I+h_Y)$	$\sqrt{3}\,(4h_I-5h_c-h_Y)$
$(\Xi^*\|\Lambda\bar{K})$	$\sqrt{3}\,(2g_I+5g_c+g_Y)$	$\sqrt{3}\,(g_I-2g_c-g_Y)$	$-3\sqrt{3}\,(h_I+h_Y)$	$\sqrt{3}\,(4h_I-5h_c-h_Y)$
$(\Omega\|\Xi\bar{K})$	$2\sqrt{3}\,(2g_I+5g_c+g_Y)$	$-\sqrt{3}\,(g_I-2g_c-g_Y)$	$-6\sqrt{3}\,(h_I+h_Y)$	$2\sqrt{3}\,(4h_I-5h_c-h_Y)$

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