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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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ON THE CURRENT DIVERGENCES

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О дивергенциях токов

В рамках модели кварков рассмотрены дивергенции аксиального и векторного адронных токов с учетом в первом порядке электромагнитных и слабых (полулептонных и нелептонных) взаимодействий. На основе этих выражений для дивергенций токов можно получить многие следствия алгебры токов, касающиеся процессов электромагнитных и слабых взаимодействий.

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On the Current Divergences

In the framework of the quark model the divergences of axial and vector hadronic currents are considered taking into account, in the first order, the electromagnetic and weak interactions (semileptonic and nonleptonic).

On the basis of these relations for current divergence many consequences of current algebra dealing with the electromagnetic and weak interactions can be derived.

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ON THE CURRENT DIVERGENCES

Lately the possibility to check the equal-time commutation relations of vector and axial vector currents $\mathcal{V}_\mu^i$ and $\mathcal{A}_\mu^i$ [I] (where i is the unitary index of the octets) has been widely discussed. Let us assume for simplicity that unitary symmetry is not violated by the strong interactions. Then, if electromagnetic and weak interactions are neglected, the vector currents are conserved:

$$\partial_\mu \mathcal{V}_\mu^i = 0. \tag{I}$$

For axial currents, on the other hand, we assume, following Gell-Mann and Lévy[2], that their divergences are proportional to the pseudoscalar fields

$$\partial_\mu \mathcal{A}_\mu^i = i\alpha\varphi^i, \tag{2}$$

however this is not necessary for the derivation of the consequences of current algebra[3]. Relations (I) and (2) are to be modified when the corrections due to the electromagnetic and weak interactions are taken into account. If we assume the minimal electromagnetic interactions (i.e. we replace ∂_μ by $\partial_\mu \mp ieA_\mu$, ∂_μ for the particles with electric charge $\pm$ I,0, respectively)then e.g. for the charged currents

$$\mathcal{V}^\pm = \frac{1}{\sqrt{2}}\left(\mathcal{V}^1 \pm i\mathcal{V}^2\right),$$

instead of the relation (I) we have [4]

$$\partial_\mu \mathcal{V}_\mu^\pm = ie A_\mu \mathcal{V}_\mu^\pm . \tag{3}$$

In a recent paper of Veltman [5] relations for the divergence of the vector and axial-vector currents were assumed, which generalize equation (3) to the case when both electromagnetic and semileptonic weak interactions are taken into account. It has been also shown there that from the hypothesis on the divergence of the currents a lot of sum rules obtained earlier on the basis of equal-time commutation relations follow.

In the present paper we show that both the equal-time commutation relations of Gell-Mann and the divergence conditions of Veltman result from the same dynamical model - from the quark model. We also present a generalization of Veltman's results for the case when also the nonleptonic weak interations are taken into account. From our hypothesis on the divergence of the currents it is possible to derive all the relations between the nonleptonic decay amplitudes of baryons and mesons obtained earlier from current algebra.

Deriving equal-time commutation relations Gell-Mann assumed that the currents $\mathcal{V}_\mu^i$ and $\mathcal{A}_\mu^i$ are of the form

$$\mathcal{V}_\mu^i = \bar{\Psi}\gamma_\mu \lambda^i \Psi, \qquad \mathcal{A}_\mu^i = \bar{\Psi}\gamma_\mu\gamma_5\lambda^i\Psi, \tag{4}$$

where Ψ stands for the quark field operators. As usual between the quarks we assume minimal electromagnetic and universal $V-A$ weak interactions (see [6]). If neutral hadronic currents exist [7], then they are described by the neutral components of the currents (4). Consequently, the Lagrangian of the electromagnetic and weak interactions of hadrons can be written as *):

$$\mathcal{L}_e = ie A_\mu^i \mathcal{V}_\mu^i \tag{5}$$

$$\mathcal{L}_W = \frac{iG}{\sqrt{2}}(\mathcal{V}_\mu^i + \mathcal{A}_\mu^i) W_\mu^i + \frac{G}{\sqrt{2}}\alpha_{ij}(\mathcal{V}_\mu^i + \mathcal{A}_\mu^i)(\mathcal{V}_\mu^j + \mathcal{A}_\mu^j). \tag{6}$$

Here for the sake of convenience we introduced the electromagnetic field as an octet having only the appropriate nonvanishing components. The W_μ^i are the products of the leptonic currents and some constants (containing $\sin\theta$ and $\cos\theta$, where θ is the Cabibbo angle) and the α_{ij} are constants. Further, we assume that in the strong interactions of the quarks the vector currents of the form (4) are conserved:

$$\partial_\mu(\bar{\Psi}\gamma_\mu\lambda^i\Psi) = 0.$$

Let us denote by (div $\mathcal{A}^i$ $)_0$ the divergence of the axial vector currents in the absence of the electromagnetic and weak interactions. Then PCAC means:

$$(\operatorname{div}\mathcal{A}^i)_0 = i\alpha\varphi^i. \tag{7}$$

Now using the Euler-Lagrange equations derived from the Lagrangians (5) and (6) together with the PCAC hypothesis (7), we get the expressions for $\partial_\mu \mathcal{V}_\mu^i$ and $\partial_\mu \mathcal{A}_\mu^i$ in which the electromagnetic and weak interactions are already taken into account:

$$\begin{aligned}\partial_\mu \mathcal{V}_\mu^i = {} & ie f_{ijk} A_\mu^j \mathcal{V}_\mu^k + \frac{iG}{\sqrt{2}} f_{ijk} W_\mu^j (\mathcal{V}_\mu^k + \mathcal{A}_\mu^k) + \\ & \frac{G}{\sqrt{2}}\alpha_{jk}\left[f_{ijl}(\mathcal{V}_\mu^l + \mathcal{A}_\mu^l)(\mathcal{V}_\mu^k + \mathcal{A}_\mu^k) + f_{ikl}(\mathcal{V}_\mu^l + \mathcal{A}_\mu^l)(\mathcal{V}_\mu^j + \mathcal{A}_\mu^j)\right]\end{aligned} \tag{8}$$

*) Summation on repeated indices is understood.

$$\begin{aligned}\partial_\mu \mathcal{A}_\mu^i = {} & ic\varphi^i + ie f_{ijk} A_\mu^j \mathcal{A}_\mu^k + \frac{G}{\sqrt{2}}\alpha_{jk}\left[f_{ijl}(\mathcal{V}_\mu^l + \mathcal{A}_\mu^l)(\mathcal{V}_\mu^k + \mathcal{A}_\mu^k) + \right. \\ & \left. + f_{ikl}(\mathcal{V}_\mu^l + \mathcal{A}_\mu^l)(\mathcal{V}_\mu^j + \mathcal{A}_\mu^j)\right]\end{aligned} \tag{9}$$

These formulae present the generalization of the Veltman's relations. They contain terms corresponding to the nonleptonic weak interactions.

The hypothesis of Veltman on the divergences of currents is, thus, a consequence of the quark model with the minimal electromagnetic interactions (5) and the universal weak interactions (6). The equal-time commutation relations of Gell-Mann were also derived in the framework of this model. Therefore it is not surprising that many of the results of current algebra can be derived also from the Veltman's divergence conditions. The Veltman's method and the algebra of currents are just two different approaches in the study of the same model.

In our treatment we assumed that the strong interactions do not violate SU(3). If the breaking of the unitary symmetry is taken into account then relations (8), (9) are to be modified: e.g., to the r.h.s. of (8) we must add a term (div $\mathcal{V}^i$$)_0$ giving the divergence of the vector current (in the absence of electromagnetic and weak interactions). But assuming, as usual, that the vector and axial vector currents are of the form (4), even if the SU(3) breaking is not neglected, the contribution of electromagnetic and weak interactions remains unchanged.

We note that from the expression (9) for the divergence of the axial-vector current all known results of current algebra concerning the nonleptonic decays of baryons and mesons can be derived. In fact, the off-mass-shell amplitude of a nonleptonic decay $B_1 \to B_2 + \varphi^i$ (where φ^i is the octet of pseudoscalar mesons) is of the form:

$$M(B_1 \to B_2\varphi^i) = (m_\pi^2 + k^2)\langle B_2|\varphi^i(0)|B_1\rangle, \tag{10}$$

here k is the momentum of the virtual meson and φ^i denotes the Heisenberg field of the meson if the weak interactions are taken into account to the first order. Because of

$$\lim_{k\to 0}\langle B_2|\partial_\mu \mathcal{A}_\mu^i|B_1\rangle = 0,$$

we get from (9)

$$\lim_{k\to 0} \langle B_2 | \varphi^i(0) | B_1 \rangle - \frac{1}{c} \frac{G}{\sqrt{2}} \alpha_{jk} \lim_{k\to 0} \langle B_2 | [f_{ijl} (V^l_\mu + A^l_\mu)(V^k_\mu + A^k_\mu) + \\ + f_{ikl} (V^l_\mu + A^l_\mu)(V^j_\mu + A^j_\mu)] | B_1 \rangle . \tag{II}$$

Similar expressions were derived earlier on the basis of the currents algebra[8].

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