ОБЪЕДИНЕННЫЙ
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E2 - 3006

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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# CURRENT - FIELD EQUAL - TIME COMMUTATION RELATIONS

1966

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Submitted to Jad. Phys.

x) On leave of absence from Warsaw University.

In the paper by Feynman, Gell-Mann and Zweig [1] the current commutation relations have been proposed to give a relativistic generalization of the SU(6) group. Equal-time commutation relations between field operators have been discussed by Delbourgo, Salam and Strathdee [2]. In this note, we propose to introduce the U(6) ⊗ U(6) group through the equal-time commutation relations between the currents and fields operators.

To simplify the derivation we start from the quark model. Vector and axial currents $V_\mu^i$ and $A_\mu^i$ , $i = 0,1,2, .... 8$, are constructed from the quark field $\psi(x)$ in the usual manner

$$V_\mu^i(x) = \bar{\psi}(x)\lambda_i \gamma_\mu \psi(x) = \bar{\psi}^A(x)(\lambda_i \gamma_\mu)_A^B \psi_B(x) , \qquad (1)$$

$$A_\mu^i(x) = \bar{\psi}(x)\lambda_i \gamma_\mu \gamma_5 \psi(x) = \bar{\psi}^A(x)(\lambda_i \gamma_\mu \gamma_5)_A^B \psi_B(x) . \qquad (2)$$

Capital indices $A$ denote both bispinor and SU(3) indices $\alpha$ and $a$ , $\alpha =$ 1,2,3,4, $a =$ 1,2,3. From the equal-time canonical commutation relations for the quark field we obtain

$$[\bar{\psi}(x)\Gamma\psi(x), \psi_A(y)]_{x^0=y^0} = -(\gamma_4 \Gamma)_A^B \psi_B(x)\delta(\vec{x}-\vec{y}) , \qquad (3)$$

$$[\bar{\psi}(x)\Gamma\psi(x), \bar{\psi}^A(y)]_{x^0=y^0} = \bar{\psi}^B(x)(\Gamma\gamma_4)_B^A \delta(\vec{x}-\vec{y}) , \qquad (4)$$

where $\Gamma$ are the tensor products of 16 Dirac matrices and 9 Gell-Mann matrices $\lambda_i$ which form the U(12) group. For the vector and axial currents $\Gamma = \lambda_i \gamma_\mu$ and $\Gamma = \lambda_i \gamma_\mu \gamma_5$ , respectively. From the relations (3) and (4) we can construct the U(12) group acting on the field operators. The generators $G_\Gamma$ of this group act in the following manner

$$G_\Gamma \psi_A(y) = [\int d^3x\, \bar{\psi}(x)\Gamma\psi(x),\, \psi_A(y)]_{x^0=y^0} = -(\gamma_4\Gamma)_A^B \psi_B(y),$$

$$G_\Gamma \bar{\psi}^A(y) = [\int d^3x\, \bar{\psi}(x)\Gamma\psi(x),\, \bar{\psi}^A(y)]_{x^0=y^0} = \psi^A(y)(\Gamma\gamma_4)_A^B . \tag{5}$$

As in the theory of Feynman, Gell-Mann and Zweig integrated vector and axial currents generate a $U(6) \otimes U(6)$ subgroup of U(12).

To describe arbitrary fields we introduce higher rank spinors $\psi_{A_1 \dots A_n}^{B_1 \dots B_m}(x)$. In particular, vector and pseudoscalar mesons are described by a second rank spinor $\phi_A(x)$. We assume that the physical components of the field can be extracted from the spinors $\psi_{A_1 \dots A_n}^{B_1 \dots B_m}(x)$ by the same formulae as that for the free field in the $\tilde{U}(12)$ symmetry of Delbourgo, Salam and Strathdee[3] (these formulas can be derived also in the broken SL(6,C) symmetry, as was shown by Nguyen van Hieu et al.[4,5]).

For example,

$$\Phi_A^B(x) = [(1 - \frac{1}{m_P}\gamma_\mu \frac{\partial}{\partial x_\mu})\gamma_5]_\alpha^\beta (\phi(x))_a^b + [(1 - \frac{1}{m_V}\gamma_\mu \frac{\partial}{\partial x_\mu})\gamma_\nu]_\alpha^\beta (\xi_\nu(x))_a^b , \tag{6}$$

where $(\phi(x))_a^b$ and $(\xi_\nu(x))_a^b$ are the pseudoscalar and vector meson field, $m_P$ and $m_V$ are their masses.

The equal-time commutation relations between the quantities $\bar{\psi}(x)\Gamma\psi(x)$ and arbitrary spinors $\psi_{A_1 \dots A_m}^{B_1 \dots B_m}$ will be assumed in the form

$$[\bar{\psi}(x)\Gamma\psi(x),\, \psi_{A_1 \dots A_n}^{B_1 \dots B_m}(y)]_{x^0=y^0} = \{ \sum_{i=1}^{n} (-\gamma_4\Gamma)_{A_i}^{A_i'} \psi_{A_1 \dots A_i' \dots A_n}^{B_1 \dots B_m}(x) + \sum_{i=1}^{m} \psi_{A_1 \dots A_n}^{B_1 \dots B_i' \dots B_m} (\Gamma\gamma_4)_{B_i'}^{b_i} \} \delta(\vec{x}-\vec{y}). \tag{7}$$

In particular, for mesons we have

$$[\bar{\psi}(x)\Gamma\psi(x),\, \phi_A^B(y)]_{x^0=y^0} = \{-(\gamma_4\Gamma)_A^{A'} \phi_{A'}^B(x) + \phi_A^{B'}(x)(\Gamma\gamma_4)_{B'}^B\} \cdot \delta(\vec{x}-\vec{y}). \tag{8}$$

In this way we get higher representations of the U(12) group, whose generators are defined through an equation similar to (5),

$$G_\Gamma \psi^{B_1 \dots B_m}_{A_1 \dots A_n}(y) \equiv [\int d^3 x \bar{\psi}(x)\Gamma\psi(x), \ \psi^{B_1 \dots B_m}_{A_1 \dots A_m}(y)]_{x^0 = y^0} \quad .$$

These representations of the U(12) group induce representations of the U(6) ⊗ U(6) current algebra. Since the generators of the corresponding group U(6) ⊗ U(6) act only on fields operators but not on the state vectors, the difficulty discussed by Coleman[6] and Okubo[7] will not show up here.

Formula (6) can now be used to derive, from the relation (8), the equal-time commutation relations for currents and physical meson fields. We have, for example,

$$[V_4^i(x), (\phi(y))_c^d]_{x^0 = y^0} = [\phi(x)\lambda_i - \lambda_i \phi(x)]_c^d \, \delta(\vec{x} - \vec{y}) \quad , \tag{9}$$

$$[A_j^i(x), (\phi(y))_c^d]_{x^0 = y^0} = \frac{1}{m_V}(\delta_{\mu j} \frac{\partial}{\partial x_4} - \delta_{\mu 4} \frac{\partial}{\partial x_j})(\xi_\mu(x)\lambda_i - \lambda_i \xi_\mu(x))_c^d \quad , \tag{10}$$

$$[V_j^i(x), (\phi(y))_c^d]_{x^0 = y^0} = \frac{1}{m_P} \epsilon_{4 i \nu \mu} \frac{\partial}{\partial y_\nu}(\xi_\mu(x)\lambda_i - \lambda_i \xi_\mu(x))_c^d \quad , \tag{11}$$

etc. We can use the same method to derive the commutation relation between currents and baryon fields.

Current-currents commutation relations led to many predictions which are in a good agreement with experiment[8-10]. We hope that some useful physical consequences can be also obtained from the commutation relations proposed in this note.

## References


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Received by Publishing Department
on November 4, 1966.