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NOTE ON THE PARASTATISTICS

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In connection with the problem of the saturation in triplet models of hadrons (see, for example[1]) it has been suggested to regard the quarks as the fundamental order three parafermions[1,3]. The purpose of this note is to show that this assumption is equivalent to the assumption of the three degenerated triplets of the usual Fermi-quarks[4,5]. Moreover, it is asserted that the theory of any parafield is equivalent to the theory of the canonical (Fermi- or Bose-) field with inner coordinate having the dimensions equal to the order of a parafield[6]. Our basic assumption is that a parafield can be represented by the Green ansatz[7], so that the Wightman functions of different, generally speaking, parafields can be written as the sum of the Wightman functions of Green components:

$$<\Phi_0 , \psi_1 (x_1) \dots \psi_n (x_n) \Phi_0 > = \sum_{\substack{A=1,\dots p_1 \\ B=1,\dots, p_2}} <\Phi_0 , \psi_1^{(A)} (x_1) \dots \psi_n^{(B)} (x_n) \Phi_0 > . \quad (1)$$

Here p_i is the order of the parafield $\psi_i(x)$ and for brevity we do not distinguish between $\psi(x)$ and conjugated field $\bar{\psi}(x)$. There is a gauge transformation for each Green component separately, which is allowed by the commutation relations for the Green ansatz. On the spacelike surface the Green components must satisfy the following relations:

$$\begin{aligned} &[\psi_i^{(A)}(x), \bar{\psi}_j^{(B)}(y)]_{(2\delta_{AB}-1)\epsilon_{ij}} = -iS(x-y)\delta_{AB}\delta_{ij} , \\ &[\psi_i^{(A)}(x), \psi_j^{(B)}(y)]_{(2\delta_{AB}-1)\epsilon_{ij}} = [\bar{\psi}_i^{(A)}(x), \bar{\psi}_j^{(B)}(y)]_{(2\delta_{AB}-1)\epsilon_{ij}} = 0, \end{aligned} \quad (2)$$

where $\epsilon_{ij} = +$ in the case of para-Fermi relations between the parafields $\psi_i(x)$ and $\psi_j(y)$ and $\epsilon_{ij} = -$ in the case of para-Bose relations between these fields. $S(x)$ is the usual commutation function for fermions or bosons, respectively. The assumption (1) have been strictly proved for the free fields[7], but it is an open question for the interacting fields[8]. The functions (1) must obey the usual requirements of Lorentz-invariance, spectrum-conditions (including the existence of the lowest nonzero mass) etc.[9]. Now we can use the Araki theorem[10]. If C_I and C_{II} are the products of field operators and if C_I and $L(\lambda a,1)\, C_{II}\, L(\lambda a,1)^{-1}$ anticommute for Lorentz-translation $L(\lambda a,1)$ with spacelike vector a and sufficiently large λ, then the vacuum expectation value of either C_I or C_{II} vanishes: $\langle \Phi_0, C_I \Phi_0\rangle \langle \Phi_0, C_{II}\Phi_0\rangle = 0$. This theorem imposes on the Wightman functions the same restrictions as the requirement of locality does on the interaction Hamiltonian[7]. The necessary conditions for the Wightman function not to vanish is that the total number of Fermi-like (fermion or parafermion) fields should be even and the total degree of any Green component have a parity independent of the index of this component. The latter implies that the nonvanishing Wightman function must have an even number of fields from the pair of any fixed Green components, say $\psi^{(A)}$ and $\psi^{(B)}$. Consequently, one can generate two subspaces from the unique vacuum by polynomials of Green components which are odd or even degrees with respect to fields from this pair of components. The whole space is splitted into two these subspaces orthogonal to each other. One can define a suitable operator $\hat{q}\,\underline{(A,B)}$ with eigenvalues equal to +1 and -1 on these subspaces, respectively. Now, we can define new fields $'\psi^{(A)}$ and $'\psi^{(B)}$ by means of the Klein transformation[10]:

$$\psi^{(A)} \to {'\psi}^{(A)} = \psi^{(A)}\ ;\ \psi^{(B)} \to {'\psi}^{(B)} = q(A,B)\,\psi^{(B)}\,. \tag{3}$$

The commutation relations for new fields have the same form irrespectively of either the indices are equal or not. In order to write the Wightman functions in terms of new components we introduce the operator $\hat{q}(A,B)$ in front of $\psi^{(B)}(x)$ and change the sign of the Wightman function if between this operator and the right vacuum there is an odd number of the fields $\psi^{(A)}$ and $\psi^{(B)}$. In the normal case of the mutual relations between different parafields[7] the abovementioned requirements are not only necessary but also sufficient in order that the operators C_I and $L(\lambda a,1)\, C_{II}\, L(\lambda a,1)^{-1}$ commute. In this widest case[7] the nonvanishing Wightman functions are:

$$\sum_{A=1}^{p} \langle \Phi_0, \ldots \psi_i^{(A)}(x)\psi_j^{(A)}(y)\ldots\Phi_0\rangle = \langle\Phi_0, \ldots \sum_{A=1}^{p} {'\psi}_i^{(A)}(x)\, {'\psi}_j^{(A)}(y)\ldots\Phi_0\rangle\,,$$

$$\sum_{A_1\neq\ldots\neq A_p=1}^{p} \langle \Phi_0, \ldots \psi_1^{(A_1)}(x_1)\ldots\psi_p^{(A_p)}(x_p)\ldots\Phi_0\rangle = \langle\Phi_0,\ldots \sum_{A_1\neq\ldots A_p=1}^{p}(-)^{\eta}\, {'\psi}_1^{(A_1)}(x_1)\ldots{'\psi}_p^{(A_p)}(x_p)\ldots\Phi_0\rangle\,, \tag{4}$$

where η is a parity of the permutation indices ($A_1 \ldots A_p$) with respect to ($1 \ldots p$). The equalities (4) and (5) are determined up to a common sign due to the Klein transformation. The function (4) possesses SU_p symmetry while the function (5) is only SO_p symmetrical. It is interesting that the inner degree of freedom appears as a consequence of a paraquantization of a field. Thus, the order three para-Fermi-field is equivalent to the usual Fermi-field with three inner dimensions. In this case the function (5) vanishes otherwise this function would include an odd number of Fermi-like fields. Hence we have only one non-vanishing function:

$$< \Phi_0 , \ldots \sum_{A=1}^{3} {}'\bar{\psi}^{(A)}(x)\, {}'\psi^{(A)}(y) \ldots \Phi_0 > . \tag{6}$$

Consequently this field theory is exactly SU_3 summetrycal. If in addition there exists an order three para-Bose-field then another function:

$$\sum_{A \neq B \neq C \neq A = 1}^{3} < \Phi_0 , \ldots \bar{\psi}^{(A)}(x)\, \psi^{(B)}(y)\, \phi^{(C)}(z) \ldots \Phi_0 > = < \Phi_0 , \ldots \sum_{A \neq B \neq C \neq A = 1}^{3} (-)^{\eta}\, {}'\bar{\psi}^{(A)}(x)\, {}'\psi^{(B)}(y)\, {}'\phi^{(C)}(z) \ldots \Phi_0 > \tag{7}$$

may also differ from zero. This function possesses the SO_3 (or SU_2) symmetry because the right-hand side of equality (7) is the "determinant" of the components and only under the real transformations this "determinant" is multiplied by the transformation determinant and is, consequently, an invariant. Thus, the theory of the order three para-Fermi- and para-Bose-fields is equivalent to the theory of the usual Fermi- and Bose-fields with broken SU_3 but exact $SO_3(SU_2)$ symmetry. Remark that the function (6) allows a gauge transformation for each component while the function (7) allows only a gauge transformation common for all components.

From our results it also follows the TCP-theorem and the connection of the spin with the type of parastatistics (para-Bose-statistics for integral spin and para-Fermi- statistics for half-integral spin)/9/, the selection rules for parafields when paraparticles are produced or decay/7/ as well as an increase p times of the cross section for photoproduction of a paraparticle pair as compared to the cross section for a usual pair/11-13/. Following this way one may hope to solve the paradoxe of Galindo and Indurain/14/ as far as the parafields is another form of writing for usual fermion and boson field have a certain inner degree of freedom.

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