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ON THE THEORY OF RELATIVISTIC SL(6,C) SYMMETRY

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§ 1. Introduction

In order to construct the relativistic scheme of the $SU(6)$ symmetry, Budini, Fronsdal[1] and Michel[2] proposed the symmetry group

$$G = P \cdot S$$

which is the semidirect product of the Poincaré group P and the group of internal symmetry S, P being the group of inner automorphisms of the group S. The noncompact groups $SL(6,C)$ and $U(6,6)$ containing a subgroup $SL(2,C)$ can be chosen as internal symmetry groups S. The elementary particles are classified according to unitary representations of the group S and thus form infinite multiplets. In the present paper we study some special unitary representations of the group $SL(6,C)$ which can be used for the classification of the known mesons and baryons. We investigate also the structure of vertices and scattering amplitudes.

These problems were already treated in[3-7]. But in all these papers vertices and the matrix elements of scattering processes are considered in the framework of the so-called S-matrix approach where the connections between matrix elements for states with definite values of momenta, spins etc. are investigated, the spin operators being generators of Wigner's little group of the Lorentz group and no use is made of the usual relativistic spinors, 4-vectors, i.e. finite dimensional representations of homogeneous Lorentz group. The wave functions transform according to certain finite dimensional representations of the homogeneous Lorentz group and in the framework of the quantum field theory the scattering amplitudes and vertices are usually expressed by means of the wave functions of particles and by scalar functions possessing definite analyticity properties and crossing symmetry. Therefore it is highly desirable to get expressions for the matrix elements in terms of the wave functions. This is intended to be done in our paper. In order to establish the connection between the symmetry and quantum field theory we shall follow a method proposed in our previous papers/8,9/.

Before considering the possibility of classifying particles and investigating the structure of vertices and scattering amplitudes it is necessary to study the irreducible unitary representations of $SL(6,C)$ and the splitting of these infinite dimensional representations into irreducible representations of $SU(6)$ as well as into multiplets of the little group $SU(6)_p$. This latter contains the little group $SU(2)_p$ of the Lorentz group (for the definition of the little group $SU(6)_p$ see/3,5,8/). Gelfand, Graev and Vilenkin/10/, Fronsdal/3/ and Rühl/5/ has shown, that the unitary representations of $SL(n,C)$ may be realized in Hilbert spaces of homogeneous functions. But, investigating the splitting of unitary representations of $SL(n,C)$ into i.r's of compact subgroup $SU(n)$ Fronsdal applied the method of analytical continuation of the nonunitary finite-dimensional representations in the number of indices. Following a paper of Rühl/5/ and our papers/11,12/, we apply homogeneous functions consistently, and we shall introduce generalized tensors for the description of infinite multiplets as it was proposed in/12/.

Section §2 has an introductory character. It contains a brief description of the technique of constructing unitary representations of $SL(6,C)$. In particular, the connection between the method of homogeneous functions and the method of Gelfand and Naimark is established. The baryon and the meson multiplets are studied in §3 and §4. In §5 the structure of the vertex is investigated.

§2. Unitary Representations of $SL(6,C)$

The unitary representations of $SL(6,C)$ will be realized in functional Hilbert spaces on some sets z. Gelfand and Naimark/13/ has shown that these sets z may always be identified with some subsets of the space L of all complex unimodulary matrices of 6-th order. More precisely, it is possible to realize z as the manyfold of cosets of the group $SL(6,C)$ with respect to certain subgroups K. Before studying these spaces let us consider the space L itself. Let η denote an element of this space. η is a complex unimodulary matrix of 6-th order:

$$\eta = \begin{pmatrix} \eta_{11} & \eta_{12} & \cdots & \eta_{16} \\ \cdot & \cdot & \cdot & \cdot \\ \eta_{61} & \cdots & & \eta_{66} \end{pmatrix}. \tag{1}$$

We shall denote the elements of the last row η_{6A} by $\Delta^{(1)}_{A}$,

$$\Delta^{(1)}_{A} = \eta_{6A}, \tag{2}$$

and the minors of order n with the elements from the last n rows we shall denote by $\Delta^{(n)}_{A_1 \ldots A_n}$:

$$\Delta^{(2)}_{AB} = \begin{vmatrix} \eta_{5A} & \eta_{5B} \\ \eta_{6A} & \eta_{6B} \end{vmatrix}, \quad \Delta^{(3)}_{ABC} = \begin{vmatrix} \eta_{4A} & \eta_{4B} & \eta_{4C} \\ \eta_{5A} & \eta_{5B} & \eta_{5C} \\ \eta_{6A} & \eta_{6B} & \eta_{6C} \end{vmatrix}, \ \ldots,$$

$$\Delta^{(5)}_{ABCDE} = \begin{vmatrix} \eta_{2A} & \cdots & \eta_{2E} \\ \vdots & & \vdots \\ \eta_{6A} & \cdots & \eta_{6E} \end{vmatrix}. \tag{3}$$

Performing the transformation

$$\eta \rightarrow \eta g, \tag{4}$$

$g \in SL(6,C)$, these polynomials transform like spinors of the representations $\Psi_{A}, \Psi_{[AB]}, \ldots, \Psi_{[ABCDE]}$ (for spinor representations of the group $SL(6,C)$ see /14/).

We shall realize the unitary representations of the group $SL(6,C)$ in Hilbert spaces of homogeneous functions of the variables $\Delta^{(1)}_{A}, \ldots, \Delta^{(5)}_{ABCDE}$. First we consider the case, when these functions depend on all the $\Delta^{(i)}_{A_1 \ldots A_i}$, $i = 1, \ldots, 5$. The corresponding representations form the so-called nondegenerate series. As it was shown by Gelfand and Naimark[13], almost all matrices η can be written in the form:

$$\eta = \xi \delta z , \tag{5}$$

where the matrices ξ , δ and z have the form:

$$\xi = \begin{pmatrix} 1 & \xi_{12} & \cdots & \xi_{16} \\ \vdots & 1 & & \vdots \\ & & 1 & \xi_{56} \\ & & & 1 \end{pmatrix}, \quad z = \begin{pmatrix} 1 & & & \\ z_{21} & 1 & & \\ z_{31} & z_{32} & 1 & \\ \vdots & & & \ddots \\ z_{61} & \cdots & & 1 \end{pmatrix}, \quad \delta = \begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_6 \end{pmatrix}. \tag{6}$$

It can be shown, that polynomials $\Delta^{(i)}_{A_1 \ldots A_i}$ do not depend on ξ and can be expressed by means of the matrix elements δ and z in the following way:

$$\Delta^{(1)}_{A} = d_6 z_{6A} , \qquad \Delta^{(2)}_{AB} = d_6 d_5 \begin{vmatrix} z_{5A} & z_{5B} \\ z_{6A} & z_{6B} \end{vmatrix} , \; \ldots ,$$

$$\Delta^{(5)}_{ABCDE} = d_6 d_5 d_4 d_3 d_2 \begin{vmatrix} z_{2A} & \cdots & z_{2E} \\ \vdots & & \vdots \\ z_{6A} & \cdots & z_{6E} \end{vmatrix} . \tag{7}$$

Let $f(\eta) = f(\Delta^{(1)} \ldots \Delta^{(5)})$ be a homogeneous function of degree λ_1 and μ_1 with respect to $\Delta^{(1)}$ and $\bar{\Delta}^{(1)}$ and λ_2 and μ_2 with respect to $\Delta^{(2)}$ and $\bar{\Delta}^{(2)}$ and so on. Then from (7) it follows, that

$$f(\Delta^{(1)} \ldots \Delta^{(5)}) = (d_6)^{\lambda_1} (\bar{d}_6)^{\mu_1} (d_6 d_5)^{\lambda_2} (\overline{d_6 d_5})^{\mu_2} \ldots (d_6 d_5 d_4 d_3 d_2)^{\lambda_5} (\overline{d_6 d_5 d_4 d_3 d_2})^{\mu_5} f'(z) =$$

$$= (d_6)^{\lambda_1 + \ldots + \lambda_5} (\bar{d}_6)^{\mu_1 + \ldots + \mu_5} (d_5)^{\lambda_2 + \ldots + \lambda_5} (\bar{d}_5)^{\mu_2 + \ldots \mu_5} \ldots (d_2)^{\lambda_5} (\bar{d}_2)^{\mu_5} f'(z), \tag{8}$$

where $f'(z)$ is a function of the matrix elements of thetriangular matrix z. Thus in this case the unitary representations of $SL(6,C)$ are realized in the Hilbert space of the functions $\varphi(z)$ on the manifold of matrices z. The scalar product for the space of homogeneous functions under consideration is defined in the following way:

$$(f_1, f_2) = \int \bar{f}_1(\eta) f_2(\eta)\, d\sigma(\eta), \qquad (9)$$

where $d\sigma(\eta)$ is an invariant measure. The functions $f(\eta)$ depend effectively only on z_{AB}, $A > B$, thus $d\sigma(\eta)$ can contain only the differentials dz_{AB}. It is possible to show, that $d\sigma(\eta)$ is of the form:

$$d\sigma(\eta) = |d_2|^4 |d_3|^8 |d_4|^{12} |d_5|^{16} |d_6|^{20} \prod_{A>B} \left(\frac{i}{2}\right) dz_{AB}\, d\bar{z}_{AB}\,. \qquad (10)$$

From formulae (8) and (10) it follows that the definition of the scalar product (9) makes sense only when the degrees of $f(\Delta)$ plus the degrees of $d\sigma(\Delta)$ give zero, i.e. when λ_i satisfies:

$$\lambda_i + \bar{\mu}_i + 2 = 0, \qquad i = 1, 2, \dots, 5.$$

On the other hand, (8) makes sense only if $\lambda_i - \mu_i$, $i = 1, 2, \dots, 5$, are integers. Thus we have:

$$\lambda_i = \frac{i\rho_i}{2} - 1 + \nu_i\,, \qquad \mu_i = \frac{i\rho_i}{2} - 1 - \nu_i\,, \qquad (11)$$

where the ν_i are integer or half integer numbers.

We define now the operators representing the internal symmetry group $S = SL(6,C)$.

Let g be an element of this group and let us introduce the corresponding operator T_g:

$$T_g f(\eta) = f(\eta g). \tag{12}$$

It can be easily proved that the correspondence

$$g \to T_g$$

gives a representation of the group S in the Hilbert space of homogeneous functions $f(\eta)$. Moreover, the invariance of the measure $d\sigma(\eta)$ guarantees that the operators T_g are unitary with respect to the **scalar** product (9). Thus we have unitary representations of $SL(6,C)$. They are irreducible, as it was shown by Gelfand and Naimark/13/, and form the so-called principal nondegenerate series.

Those cases, when the functions $f(\Delta)$ do not depend on all $\Delta^{(i)}$, $i = 1, \ldots, 5$, but only on subsets of them, can be considered in a similar way (the degenerate series). In the following paragraphs two series will be considered which can be applied to the classification of baryons and mesons.

§3. Baryon Multiplet

Following many authors we assume that the baryons belong to the maximal degenerate series. More exactly, we assume, that baryons are described by a unitary representation corresponding to the homogeneous functions of $\Delta^{(1)}_A$ only. In this case for the matrix η we make use of the decomposition (5) with matrices ξ, z and δ of the form:

$$\xi = \left(\begin{array}{c|c} I^5 & \begin{matrix}\xi_{16}\\ \vdots \\ \xi_{56}\end{matrix} \\ \hline 0 \ldots 0 & 1 \end{array}\right), \quad z = \left(\begin{array}{c|c} I^5 & \begin{matrix}0\\ \vdots \\ 0\end{matrix} \\ \hline z_{61} \ldots z_{65} & 1 \end{array}\right), \qquad \delta = \left(\begin{array}{c|c} D^5 & \begin{matrix}0\\ \vdots \\ 0\end{matrix} \\ \hline 0 \ldots 0 & d_6 \end{array}\right), \tag{13}$$

where D^5 is a matrix of the 5-th order with nonvanishing determinant and I^5 is a unity matrix of the same dimension. For the sake of convenience we put $\xi_A = \eta_{6A}$.

From the definition (2) we have:

$$\Delta_A^{(1)} = \xi_A = z_{6A} d_6 \quad . \tag{14}$$

Therefore the homogeneous functions of degrees λ and μ in $\Delta_A^{(1)}$ have the form:

$$f(\Delta^{(1)}) = f(\xi) = d_6^{\lambda} \bar{d}_6^{\mu} f^{1}(z). \tag{15}$$

It can be shown that the invariant measure for the product (9) equals [13]:

$$d\sigma(\xi) = |d_6|^{12} \prod_{A=1}^{5} \frac{i}{2} dz_{6A} d\bar{z}_{6A} . \tag{16}$$

Similarly to the case of nondegenerate series it follows from (9), (15) and (16) that λ and μ are equal to

$$\begin{aligned} \lambda &= \frac{i\rho}{2} - 3 + \nu , \\ \mu &= \frac{i\rho}{2} - 3 - \nu , \end{aligned} \tag{17}$$

where ν is an integer or half-integer and ρ is a real number. The given infinite-dimensional representation of the group $SL(6,C)$ splits into the following representations of the compact subgroup $SU(6)$:

$\Phi_{A_1\ldots A_{2\nu}}, \Phi^{B_1}_{A_1\ldots A_{2\nu+1}}, \ldots, \Phi^{B_1\ldots B_{\tau-\nu}}_{A_1\ldots A_{\tau+\nu}} \ldots$; we note that these generalized tensors can be explicitly constructed. To fix the idea we assume $\nu > 0$. Then

$$\Phi^{B_1\ldots B_{\tau-\nu}}_{A_1\ldots A_{\tau+\nu}} = C_\tau (\xi_A \bar{\xi}^A)^{\frac{i\rho}{2}-3-\tau} Z^{B_1\ldots B_{\tau-\nu}}_{A_1\ldots A_{\tau+\nu}}, \qquad \tau \geq \nu, \tag{18}$$

where $Z^{B_1\ldots B_{\tau-\nu}}_{A_1\ldots A_{\tau+\nu}}$ can be obtained from the products $\xi_{A_1}\ldots\xi_{A_{\tau+\nu}}\bar{\xi}^{B_1}\ldots\bar{\xi}^{B_{\tau-\nu}}$ by substracting traces :

$$Z^{B_1\ldots B_{\tau-\nu}}_{A_1\ldots A_{\tau+\nu}} = \sum_{s=0}^{\tau-\nu} \alpha(\tau-\nu, s, 2\nu)(\xi_A\bar{\xi}^A) \sum_{(A,B)} \delta^{B_1}_{A_1}\ldots\delta^{B_s}_{A_s}\xi_{A_{s+1}}\ldots\xi_{A_{\tau+\nu}}\bar{\xi}^{B_{s+1}}\ldots\bar{\xi}^{B_{\tau-\nu}}. \tag{19}$$

Here
$$\alpha(\tau-\nu, s, 2\nu) = (-1)^s \frac{(\tau-\nu)!(\tau+\nu)!(2\tau+4-s)!}{s!(\tau-\nu-s)!(\tau+\nu-s)!(2\tau+4)!},$$

and C_τ is a normalization constant (see also /12/). We note that the spinors (18) transform among themselves in the transformations from $SU(6)$ only. With respect to this group dotted and undotted indices do not differ, therefore upper indices can be written without dots.

Putting $\nu = \frac{3}{2}$ we find that the first $SU(6)$ multiplet $(\tau = \nu)$ contained in the infinite multiplet of $SL(6,C)$ under consideration is just the 56-plet. Since we intend to classify the well-known baryons forming the 56-plet of $SU(6)$, we assume that $\nu = \frac{3}{2}$.

Let us now turn to the transformation properties of the considered multiplet of $SL(6,C)$ with respect to the space reflection P. For this purpose the transformation properties of the $SL(6,C)$ generators under space reflection should be utilized. Let us introduce the matrices ℓ^A_B and $\tilde{\ell}^A_B$:

$$(\ell^A_B)^C_D = \delta^C_B\delta^A_D - \frac{1}{6}\delta^A_B\delta^C_D, \qquad (\tilde{\ell}^A_B)^C_D = i\left(\delta^C_B\delta^A_D - \frac{1}{6}\delta^A_B\delta^C_D\right). \tag{20}$$

The elements of the group can be written in the form

$$g = e^{i\left(\alpha^{\pm}_{AB} m^{\pm}_{AB} + \beta^{\pm}_{AB} n^{\pm}_{AB}\right)}, \tag{21}$$

where $\alpha^{\pm}_{AB}$ and $\beta^{\pm}_{AB}$ are real parameters and $m^{\pm}_{AB}$ and $n^{\pm}_{AB}$ are:

$$m^{+}_{AB} = \ell^{A}_{B} + \ell^{B}_{A}, \qquad m^{-}_{AB} = \tilde{\ell}^{A}_{B} - \tilde{\ell}^{B}_{A};$$

$$n^{+}_{AB} = \tilde{\ell}^{A}_{B} + \tilde{\ell}^{B}_{A}, \qquad n^{-}_{AB} = \ell^{A}_{B} - \ell^{B}_{A}. \tag{22}$$

The matrices $m^{\pm}_{AB}$ are hermitian; they are the generators of the compact subgroup $SU(6)$. On the other hand, the matrices $n^{\pm}_{AB}$ are antihermitian and are the noncompact generators. Let us denote the corresponding generators in the representations by $M^{\pm}_{AB}$ and $N^{\pm}_{AB}$. (For the sake of simplicity the indices A and B will be omitted sometimes.) The former commute, the latter (like in the case of the Lorentz group) anti-commute with the inversion

$$PM - MP = 0, \tag{23}$$

$$PN + NP = 0. \tag{24}$$

As P commutes with the generators of $SU(6)$, we can put

$$P\,\Phi^{B_1 \dots B_{\tau-\nu}}_{A_1 \dots A_{\tau+\nu}} = \eta_{\tau}\,\Phi^{B_1 \dots B_{\tau-\nu}}_{A_1 \dots A_{\tau+\nu}}, \tag{25}$$

where $\eta_\tau = \pm 1$ is a common constant for the whole $SU(6)$ multiplet. Let us now consider the consequences of the condition (24). Using the explicit expressions (18) and (19) for $\Phi^{B_1 \ldots B_{\tau-\nu}}_{A_1 \ldots A_{\tau+\nu}}$ and the transformation rule for the homogeneous functions in the given Hilbert space

$$T_g f(\xi) = f(\xi g),$$

one can show that the generators $N^{\pm}_{AB}$ act on the spinors $\Phi^{B_1 \ldots B_{\tau-\nu}}_{A_1 \ldots A_{\tau+\nu}}$ in the following way:

$$(N^{\pm})\Phi^{B_1 \ldots B_{\tau-\nu}}_{A_1 \ldots A_{\tau+\nu}} = \sum_{\tau'=\tau-1}^{\tau+1} \Phi^{D_1 \ldots D_{\tau'-\nu}}_{C_1 \ldots C_{\tau'+\nu}} (N^{\pm})^{C_1 \ldots C_{\tau'+\nu};\, B_1 \ldots B_{\tau-\nu}}_{D_1 \ldots D_{\tau'-\nu};\, A_1 \ldots A_{\tau+\nu}} , \tag{26}$$

where

$$(N^{\pm})^{C_1 \ldots C_{\tau'+\nu};\, B_1 \ldots B_{\tau-\nu}}_{D_1 \ldots D_{\tau'-\nu};\, A_1 \ldots A_{\tau+\nu}} =$$

$$\begin{aligned}
&= \lambda_{CD}(\tau,\tau') \mathop{S}_{(ABCD)} \delta^{B_1}_{A_1} \ldots \delta^{B_{\tau-\tau'+1}}_{A_{\tau-\tau'+1}} (n^{\pm})^{C_1}_{D_1} \delta^{C_2}_{A_{\tau-\tau'+2}} \ldots \delta^{C_{\tau'+\nu}}_{A_{\tau+\nu}} \delta^{B_{\tau-\tau'+2}}_{D_2} \ldots \delta^{B_{\tau-\nu}}_{D_{\tau'-\nu}} + \\
&+ \lambda_{BD}(\tau,\tau') \mathop{S}_{(ABCD)} \delta^{B_1}_{A_1} \ldots \delta^{B_{\tau-\tau'}}_{A_{\tau-\tau'}} \delta^{C_1}_{A_{\tau-\tau'+1}} \ldots \delta^{C_{\tau'+\nu}}_{A_{\tau+\nu}} (n^{\pm})^{B_{\tau-\tau'+1}}_{D_1} \delta^{B_{\tau-\tau'+2}}_{D_2} \ldots \delta^{B_{\tau-\nu}}_{D_{\tau'-\nu}} + \\
&+ \lambda_{CA}(\tau,\tau') \mathop{S}_{(ABCD)} \delta^{B_1}_{A_1} \ldots \delta^{B_{\tau-\tau'}}_{A_{\tau-\tau'}} (n^{\pm})^{C_1}_{A_{\tau-\tau'+1}} \delta^{C_2}_{A_{\tau-\tau'+2}} \ldots \delta^{C_{\tau'+\nu}}_{A_{\tau+\nu}} \delta^{B_{\tau-\tau'+1}}_{D_1} \ldots \delta^{B_{\tau-\nu}}_{D_{\tau'-\nu}} + \\
&+ \lambda_{BA}(\tau,\tau') \mathop{S}_{(ABCD)} \delta^{B_1}_{A_1} \ldots \delta^{B_{\tau-\tau'-1}}_{A_{\tau-\tau'-1}} (n^{\pm})^{B_{\tau-\tau'}}_{A_{\tau-\tau'+1}} \delta^{C_1}_{A_{\tau-\tau'+1}} \ldots \delta^{C_{\tau+\nu}}_{A_{\tau+\nu}} \delta^{B_{\tau-\tau'+1}}_{D_1} \ldots \delta^{B_{\tau-\nu}}_{D_{\tau'-\nu}} .
\end{aligned} \tag{27}$$

The constants λ in (27) vanish for $|\tau-\tau'|>1$ and for $|\tau-\tau'|\leq 1$ they are equal to:

$\tau'=\tau+1$:

$$\lambda_{CD}(\tau,\tau+1)=(i\rho-6-2\tau),$$
$$\lambda_{BD}(\tau,\tau+1)=\lambda_{CA}(\tau,\tau+1)=\lambda_{BA}(\tau,\tau+1)=0\,; \tag{28}$$

$\tau'=\tau$:

$$\lambda_{CD}(\tau,\tau)=-\frac{2i\rho(\tau+\nu)(\tau-\nu)}{(2\tau+6)(2\tau+4)},$$
$$\lambda_{BD}(\tau,\tau)=(\tau-\nu)\frac{i\rho}{2\tau+6}, \tag{29}$$
$$\lambda_{CA}(\tau,\tau)=(\tau+\nu)\frac{i\rho}{2\tau+6}\,;$$

$\tau'=\tau-1$:

$$\lambda_{CD}(\tau,\tau-1)=(i\rho+4+2\tau)\frac{(\tau-\nu)(\tau+\nu)(\tau-\nu-1)(\tau+\nu-1)}{(2\tau+5)(2\tau+4)^2(2\tau+3)},$$
$$\lambda_{BD}(\tau,\tau-1)=(i\rho+4+2\tau)\frac{(\nu-\tau)(\tau+\nu)(\tau-\nu-1)}{(2\tau+4)^2(2\tau+5)}, \tag{30}$$
$$\lambda_{CD}(\tau,\tau-1)=(i\rho+4+2\tau)\frac{(\nu-\tau)(\tau+\nu)(\tau+\nu-1)}{(2\tau+4)^2(2\tau+5)},$$
$$\lambda_{BA}(\tau,\tau-1)=(i\rho+4+2\tau)\frac{(\tau-\nu)(\tau+\nu)}{(2\tau+5)(2\tau+4)}.$$

It should be noted that

$$\left(N^{\pm}\right)^{C_1\ldots C_{\tau+\nu};\, B_1\ldots B_{\tau-\nu}}_{D_1\ldots D_{\tau-\nu};\, A_1\ldots A_{\tau+\nu}} = 0, \qquad \text{if } \rho = 0. \tag{31}$$

From (25) and (26) it can easily be seen that the condition (24) is equivalent to the equation:

$$\sum \left(\eta_\tau + \eta_{\tau'}\right) \Phi^{D_1\ldots D_{\tau'-\nu}}_{C_1\ldots C_{\tau'+\nu}} N^{C_1\ldots C_{\tau'+\nu};\, B_1\ldots B_{\tau-\nu}}_{D_1\ldots D_{\tau'-\nu};\, A_1\ldots A_{\tau+\nu}} = 0. \tag{32}$$

If $\rho \neq 0$, then from (32) we get the relations:

$$2\eta_\tau = 0, \qquad \eta_{\tau\pm 1} + \eta_\tau = 0.$$

The first one can not be satisfied since $\eta_\tau = \pm 1$. It means that for $\rho \neq 0$ the relations (25) are not satisfied and the given representation of $SL(6,C)$ does not transform into itself under space reflexion. There are two possibilities to obtain states with definite parity: either to introduce two equivalent representations transforming mutually one into the other under P, or to put $\rho = 0$. In the first case there will exist always pairs of multiplets with opposite parity. In what follows we shall assume that the second possibility occurs, namely we shall put $\rho = 0$. In this case the equation (32) is equivalent to the condition

$$\eta_{\tau\pm 1} + \eta_\tau = 0.$$

This means that the adjacent $SU(6)$ multiplets contained in the infinite multiplet of $SL(6,C)$ under consideration have opposite parity. Therefore the spinors $\Phi^{B_1\ldots B_{\tau-\nu}}_{A_1\ldots A_{\tau+\nu}}$ of the $SL(6,C)$ multiplet containing the baryon 56-plet have the following parity:

$$P_\tau = (-1)^{\tau-\nu}.$$

This result was obtained first by Fronsdal,[/3/] who used another method.

For the unitary representation of the group $SL(6,C)$ under consideration formulae (18) and (19) give the canonical basis corresponding to the reduction $SL(6,C) \supset SU(6)$. However, as it was noted in a number of papers /3,5,8,9/, elementary particles are classified according to the i.r's of $SU(6)_\mu$ and not of $SU(6)$. Thus for applications in the symmetry of elementary particles it is necessary to consider the splitting of the given representation of $SL(6,C)$ into i.r's of the little group $SU(6)_p$, i.e. we must construct a basis corresponding to the reduction $SL(6,C) \supset SU(6)_p$.

It was shown in a series of papers /3,8,9,15/ that it is possible to introduce for each particle from the given infinite multiplet the corresponding quantized field which transforms according to some spinor (nonunitary) representation of the homogeneous Lorentz group. For the sake of convenience we shall introduce an auxiliary (following the terminology of Feldman and Matthews /16/) group $S' = SL(6,C)$ containing the homogeneous Lorentz group. This group is isomorphic to the group of internal symmetry S, but it is not identical with S. It may be identified with the group $SL(6,C)$ proposed earlier in a number of papers /14,17-21/. We stress that the new auxiliary group is introduced only to establish the connection between the symmetry and the quantum field theory. We do not require invariance under this new group. For the description of particles we introduce in each multiplet of the internal symmetry group S an infinite number of spinor representations of the auxiliary group S'. As it was shown in /3,8/ for these spinor fields there exists the usual connection between spin and statistics.

Thus, constructing the basis from $SU(6)_p$ spinors it is convenient to supply them with definite transformation properties with respect to the auxiliary group S'. The variables ξ_A transform under Lorentz transformations like the spinors with a lower undotted index, therefore we assume that they form a spinor with a lower undotted index also under the auxiliary group S'. In this case the complex conjugate quantities are spinors with an upper dotted index and will be denote by $\bar{\xi}^{\dot{A}}$.

Constracting the basis for the reduction $SL(6,C) \supset SU(6)_p$ we shall use the quantity

$$\bar{\xi}^{\dot{A}} \left(-\frac{i\hat{p}}{m}\right)_{\dot{A}}^{B} \xi_B$$

instead of the $SU(6)$ invariant

$$\bar{\xi}^{\dot{A}} \xi_A \ .$$

Here $A=(a,\alpha)$, $a=1,2$ is the spin index, $\alpha=1,2,3$ is the unitary index and

$$\left(-\frac{i\hat{p}}{m}\right)_{\dot{B}}^{A} = \left(-\frac{i\hat{p}}{m}\right)_{\dot{b}}^{a} \delta_{\beta}^{\alpha} .$$

Thus the above expression is an $SU(6)_p$ invariant. In addition it is invariant also under Lorentz transformations. Instead of the basis defined in (18) and (19) we have thus the following relativistically invariant basis

$$\tilde{\Phi}^{\dot{B}_1 \dots \dot{B}_{\tau-\nu}}_{A_1 \dots A_{\tau+\nu}} = C_\tau \left(\bar{\xi}^{\dot{A}} \left(-\frac{i\hat{p}}{m}\right)_{\dot{A}}^{B} \xi_B\right)^{\frac{i\varsigma}{2}-3-\tau} Y^{\dot{B}_1 \dots \dot{B}_{\tau-\nu}}_{A_1 \dots A_{\tau+\nu}} . \qquad (33)$$

Here

$$Y^{\dot{B}_1\dots\dot{B}_{\tau-\nu}}_{A_1\dots A_{\tau+\nu}}=\sum_{s=0}^{\tau-\nu}\alpha(\tau-\nu,s,2\nu)\left[\bar{\xi}^{\dot{A}}\left(-\frac{i\hat{p}}{m}\right)^{B}_{\dot{A}}\xi_B\right]^s\underset{(A,B)}{S}\left(-\frac{i\hat{p}}{m}\right)^{\dot{B}_1}_{A_1}\cdots\left(-\frac{i\hat{p}}{m}\right)^{\dot{B}_s}_{A_s}\xi_{A_{s+1}}\cdots\xi_{A_{\tau+\nu}}\bar{\xi}^{\dot{B}_{s+1}}\cdots\bar{\xi}^{\dot{B}_{\tau-\nu}} \tag{34}$$

satisfy the relation:

$$\left(-\frac{i\hat{p}}{m}\right)^{A_i}_{\dot{B}_j}Y^{\dot{B}_1\dots\dot{B}_j\dots\dot{B}_{\tau-\nu}}_{A_1\dots A_i\dots A_{\tau+\nu}}=0. \tag{35}$$

It is easy to see that the $SU(6)_p$ spinors defined by (33) and (34) can be obtained from (18) and (19) by putting $\left(-\frac{i\hat{p}}{m}\right)^{A}_{\dot{B}}$ instead of δ^A_B. Analogously the relation (35) is a generalization of the relation:

$$\delta^{A_i}_{B_j}Z^{B_1\dots B_j\dots B_{\tau-\nu}}_{A_1\dots A_i\dots A_{\tau+\nu}}=0.$$

Every vector in the Hilbert space of the given representation can be represented in the form:

$$\Psi=\sum_{\tau=0}^{\infty}\Psi^{A_1\dots A_{\tau+\nu}}_{\dot{B}_1\dots\dot{B}_{\tau-\nu}}\tilde{\Phi}^{\dot{B}_1\dots\dot{B}_{\tau-\nu}}_{A_1\dots A_{\tau+\nu}}.$$

In what follows, the matrix elements of processes and vertices will be represented explicitly in terms of the components $\Psi^{A_1\dots A_{\tau+\nu}}_{\dot{B}_1\dots\dot{B}_{\tau-\nu}}$.

§4. Meson Multiplet

Also for the description of mesons we can in principle use a unitary representation from the maximally degenerate series. But the meson and the baryon multiplets cannot belong simultaneously to the maximally degenerate series for in such a case there exists no invariant meson-baryon vertex. Therefore we assume that the meson multiplet belongs to the degenerate series which we realized in the Hilbert spaces of homogeneous functions of the variables $\Delta^{(1)}_A$ and $\Delta^{(5)}_{ABCDE}$. For convenience we put:

$$\xi_A=\Delta^{(1)}_A,\qquad \zeta^A=\varepsilon^{ABCDEF}\Delta^{(5)}_{BCDEF}, \tag{36}$$

where ε^{ABCDEF} is the fully antisymmetric 6-th rank tensor. We note that

$$\xi_A \zeta^A = 0. \tag{37}$$

Now we use a decomposition (5) of the matrix η with the following ξ, z and δ :

$$\xi = \begin{pmatrix} 1 & \xi_{12} \cdots & \xi_{16} \\ 0 & I^4 & \begin{matrix}\xi_{26} \\ \vdots \\ \xi_{56}\end{matrix} \\ 0 & 0 & 1 \end{pmatrix}, \quad z = \begin{pmatrix} 1 & 0 & 0 \\ \begin{matrix} z_{21} \\ \vdots \\ z_{51} \end{matrix} & I^4 & 0 \\ z_{61} & z_{62} \cdots & 1 \end{pmatrix}, \quad \delta = \begin{pmatrix} d_1 & & \\ & D^4 & \\ & & d_6 \end{pmatrix}. \tag{38}$$

Here D^4 is of 4-th order with nonvanishing determinant and I^4 is the unity matrix of 4-th order. It is easy to show that

$$\Delta^{(1)}_A = d_6 z_{6A}, \qquad \Delta^{(5)}_{ABCDE} = D \begin{vmatrix} z_{2A} \cdots z_{2E} \\ \vdots \quad\quad \vdots \\ z_{6A} \cdots z_{6E} \end{vmatrix}, \tag{39}$$

where $D = \det D^4$. Therefore similarly to (8), the homogeneous functions of degree λ, μ and λ', μ' in ξ_A and ζ^A, respectively, have the form:

$$f(\eta) = f(\xi_A, \zeta^A) = d_6^{\lambda} \bar{d}_6^{\mu} (d_6 D)^{\lambda'} (\overline{d_6 D})^{\mu'} f'(z). \tag{40}$$

They transform also according to (12).

The invariant measure for the scalar product (9) is now:

$$d\sigma(\eta) = |D|^{10} |d_6|^{20} \prod_{A=2}^{6} \left(\frac{i}{2}\right) dz_{A1}\, d\bar{z}_{A1} \prod_{B=2}^{5} \left(\frac{i}{2}\right) dz_{6B}\, d\bar{z}_{6B}. \tag{41}$$

The unitarity of the representation follows from the invariance of $d\sigma(\eta)$. From these formulae we obtain for the constants λ, μ and λ', μ' :

$$\lambda = \frac{i\rho}{2} - \frac{5}{2} + \nu, \qquad \lambda' = \frac{i\rho'}{2} - \frac{5}{2} + \nu',$$
$$\mu = \frac{i\rho}{2} - \frac{5}{2} - \nu; \qquad \mu' = \frac{i\rho'}{2} - \frac{5}{2} - \nu', \tag{42}$$

where ν and ν' are integers or half-integers and ρ, ρ' are real numbers. The splitting of this $SL(6,C)$ representation into irreducible representations of the subgroup $SU(6)$ can be obtained as follows: first we consider the homogeneous functions of the form

$$\left(\xi_E \bar{\xi}^E\right)^{\frac{i\rho}{2}-\frac{5}{2}-\tau}\left(\zeta^F \bar{\zeta}_F\right)^{\frac{i\rho'}{2}-\frac{5}{2}-\tau'} X^{B_1\ldots B_{\tau-\nu};\,C_1\ldots C_{\tau'+\nu'}}_{A_1\ldots A_{\tau+\nu};\,D_1\ldots D_{\tau'-\nu'}}\,, \tag{43}$$

where

$$X^{B_1\ldots B_{\tau-\nu};\,C_1\ldots C_{\tau'+\nu'}}_{A_1\ldots A_{\tau+\nu};\,D_1\ldots D_{\tau'+\nu'}} = \xi_{A_1}\ldots\xi_{A_{\tau+\nu}}\bar{\xi}^{B_1}\ldots\bar{\xi}^{B_{\tau-\nu}}\zeta^{C_1}\ldots\zeta^{C_{\tau'+\nu'}}\bar{\zeta}_{D_1}\ldots\bar{\zeta}_{D_{\tau'-\nu'}}\,.$$

These spinors are symmetrical in each set of indices $\{A_1\ldots A_{\tau+\nu}\}, \{B_1\ldots B_{\tau-\nu}\}, \{C_1\ldots C_{\tau'+\nu'}\}, \{D_1\ldots D_{\tau'-\nu'}\}$. Because of (37) the traces in each pair of indices A_i and C_j (or B_i and D_j) vanish. In order to get i.r's of $SU(6)$ it is necessary to symmetrize in upper and lower indices according to all possible Young tableaux and then substract traces in all pairs of indices in a symmetrical way.

For definiteness let us take the multiplet with $\nu=\nu'=0$. It contains the following $SU(6)$ multiplets:

a singlet

$$\Phi_{(1)} = \left(\xi_A \bar{\xi}^A\right)^{\frac{i\rho}{2}-\frac{5}{2}}\left(\eta^A \bar{\eta}_A\right)^{\frac{i\rho'}{2}-\frac{5}{2}}\,, \tag{44}$$

two 35-plets

$$\begin{aligned}
\Phi_{(35)_1}{}^{B}_{A} &= \left(\xi_A \bar{\xi}^A\right)^{\frac{i\rho}{2}-\frac{7}{2}}\left(\eta^A \bar{\eta}_A\right)^{\frac{i\rho'}{2}-\frac{5}{2}}\left[\xi_A\bar{\xi}^B - \tfrac{1}{6}\delta^B_A\left(\xi_R\bar{\xi}^R\right)\right],\\
\Phi_{(35)_2}{}^{B}_{A} &= \left(\xi_A \bar{\xi}^A\right)^{\frac{i\rho}{2}-\frac{5}{2}}\left(\eta^A \bar{\eta}_A\right)^{\frac{i\rho'}{2}-\frac{7}{2}}\left[\eta^B\bar{\eta}_A - \tfrac{1}{6}\delta^B_A\left(\eta^S\bar{\eta}_S\right)\right],
\end{aligned} \tag{45}$$

a 189-plet, a $280+\overline{280}$-plet, three 405-plets e.t.c. Instead of the 35-plet in (45) it is convenient to take the symmetric and antisymmetric combinations

$$\begin{aligned}
\Phi^{(+)}_{(35)} &= \Phi_{(35)_1} + \Phi_{(35)_2}\,,\\
\Phi^{(-)}_{(35)} &= \Phi_{(35)_1} - \Phi_{(35)_2}\,.
\end{aligned} \tag{46}$$

It is possible to show that the noncompact generators act on the basis vectors (44) and (46) in the following way:

$$N^{\pm}\Phi_{(1)}=i\frac{\rho-\rho'}{2}(n^{\pm})^R_S\,\Phi^{(+)\,S}_{(35)\,R}+\left(i\frac{\rho+\rho'}{2}-5\right)(n^{\pm})^R_S\,\Phi^{(-)\,S}_{(35)\,R}\,, \tag{47}$$

$$\begin{aligned}N^{\pm}\Phi^{(+)\,B}_{(35)\,A}&=i\frac{\rho-\rho'}{2}\,\frac{2}{35}(n^{\pm})^B_A\,\Phi_{(1)}+i\frac{\rho-\rho'}{2}\Big\{-\frac{1}{48}\delta^B_A(n^{\pm})^R_S+\frac{1}{16}\delta^B_S(n^{\pm})^R_A+\\&+\frac{1}{16}\delta^R_A(n^{\pm})^B_S\Big\}\Phi^{(+)\,S}_{(35)\,R}+\Big\{\Big[\frac{1}{8}\Big(i\frac{\rho+\rho'}{2}-7\Big)-\frac{1}{6}\Big(i\frac{\rho+\rho'}{2}-\\&-5\Big)\Big]\delta^B_A(n^{\pm})^R_S+\Big[1+\frac{1}{8}\Big(i\frac{\rho+\rho'}{2}-7\Big)\Big]\Big[\delta^R_A(n^{\pm})^B_S\,(n^{\pm})^R_A\Big]\Big\}\Phi^{(-)\,S}_{(35)\,R}+\end{aligned} \tag{48}$$

+tensors with four indices,

$$\begin{aligned}N^{\pm}\Phi^{(-)\,B}_{(35)\,A}&=\Big[\frac{2}{3}+\frac{2}{35}\Big(i\frac{\rho+\rho'}{2}-5\Big)\Big](n^{\pm})^B_A\,\Phi_{(1)}+\Big\{\Big[\frac{1}{8}\Big(i\frac{\rho+\rho'}{2}-7\Big)-\\&-\frac{3}{16}\Big(i\frac{\rho+\rho'}{2}-5\Big)\Big]\delta^B_A(n^{\pm})^R_S+\Big[1+\frac{1}{8}\Big(i\frac{\rho+\rho'}{2}-7\Big)+\\&+\frac{1}{16}\Big(i\frac{\rho+\rho'}{2}-5\Big)\Big]\Big[\delta^R_A(n^{\pm})^B_S+\delta^B_S(n^{\pm})^R_A\Big]\Big\}\Phi^{(+)\,S}_{(35)\,R}+\\&+i\frac{\rho-\rho'}{2}\Big\{-\frac{1}{24}\delta^B_A(n^{\pm})^R_S+\frac{1}{8}\delta^B_S(n^{\pm})^R_A+\frac{1}{8}\delta^R_A(n^{\pm})^B_S\Big\}\Phi^{(-)\,S}_{(35)\,R}+\end{aligned} \tag{49}$$

+tensors with four indices.

Let us find now the conditions that must be satisfied in order to allow the introduction of the parity transformation within the given $SL(6,C)$ multiplet. Utilizing formulae (47-49), it is quite possible to show that the commutation relation (24) is compatible with the conditions:

$$P\Phi_{(1)} = \eta_1 \Phi_{(1)}, \quad P\Phi^{(\pm)}_{(35)} = \eta^{(\pm)}_{(35)} \Phi^{(\pm)}_{(35)}, \quad (\eta_1)^2 = \left(\eta^{(\pm)}_{35}\right)^2 = 1, \tag{50}$$

only if $\rho = \rho'$, and in this case:

$$\eta^{(+)}_{35} = \eta_1, \quad \eta^{(-)}_{35} = -\eta^{(+)}_{35}. \tag{51}$$

When studying processes in which particles with nonvanishing momenta are involed, it is necessary to pass from the $SL(6,C) \supset SU(6)$ type of reduction to the $SL(6,C) \supset SU(6)_p$ one. This can be carried out in the same way as in the case of the baryon multiplet. E.g. we have for the singlet:

$$\Phi_{(1)} = C_1 \left[\xi_A \left(-\frac{i\hat{p}}{\mu}\right)^A_{\dot{B}} \bar{\xi}^{\dot{B}}\right]^{\frac{i\rho}{2}-\frac{5}{2}} \left[\zeta^A \left(-\frac{i\hat{p}}{\mu}\right)^{\dot{B}}_A \bar{\zeta}_{\dot{B}}\right]^{\frac{i\rho}{2}-\frac{5}{2}}, \tag{52}$$

where C_1 is the normalization constant and μ is the meson mass. Let us describe the states of the 35-plet by the finite-dimensional $SL(6,C)$ spinors $\Phi^{(\pm)A}_{(35)B}$. Then we have:

$$\Phi^{(\pm)A}_{(35)B} = C_{35}\left[\xi_C\left(-\frac{i\hat{p}}{\mu}\right)^C_{\dot{D}}\bar{\xi}^{\dot{D}}\right]^{\frac{i\rho}{2}-\frac{3}{2}}\left[\zeta^E\left(-\frac{i\hat{p}}{\mu}\right)^{\dot{F}}_E\bar{\zeta}_{\dot{F}}\right]^{\frac{i\rho}{2}-\frac{3}{2}}\Big\{\left[\zeta^G\left(-\frac{i\hat{p}}{\mu}\right)^{\dot{H}}_G\bar{\zeta}_{\dot{H}}\right]\Big[\xi_B\left(-\frac{i\hat{p}}{\mu}\right)^A_{\dot{I}}\bar{\xi}^{\dot{I}} -$$

$$-\frac{1}{6}\delta^A_B\xi_I\left(-\frac{i\hat{p}}{\mu}\right)^I_{\dot{K}}\bar{\xi}^{\dot{K}}\Big] \pm \left[\xi_G\left(-\frac{i\hat{p}}{\mu}\right)^G_{\dot{H}}\bar{\xi}^{\dot{H}}\right]\Big[\zeta^A\left(-\frac{i\hat{p}}{\mu}\right)^{\dot{I}}_B\bar{\zeta}_{\dot{I}} - \frac{1}{6}\delta^A_B\zeta^I\left(-\frac{i\hat{p}}{\mu}\right)^{\dot{I}}_I\bar{\zeta}_{\dot{I}}\Big]\Big\}, \tag{53}$$

where C_{35} is the normalization constant. In what follows we shall put for the sake of simplicity $\rho = 0$.

§5. Meson - Baryon Vertex

Before studying the meson-baryon vertex we intend to find the solution of a more general problem:

let $f(\theta_A)$ be a homogeneous function from the Hilbert space of the unitary representation considered in §3. We want to construct a trilinear functional, linear with respect to $g(\xi_A, \zeta^A)$, bylinear in $f(\theta_A)$ and invariant under $SL(6,C)$.

Let us look for this functional in the integral form

$$I=\int \bar{f}(\theta)f(\theta')g(\xi,\zeta)K(\theta,\theta',\xi,\zeta)d\sigma(\theta)d\sigma(\theta')d\sigma(\xi,\zeta). \tag{54}$$

It is easy to prove that the kernel $K(\theta,\theta',\xi,\zeta)$ has to be an invariant function of its arguments.

Let $f(\theta)$ be a homogeneous function of degree $\frac{i\varsigma}{2}\pm\nu-3$, $g(\xi,\zeta)$ be a homogeneous function of degree $\frac{i\varsigma'}{2}\pm\nu'-\frac{5}{2}$ with respect to ξ, ξ' and of degree $\frac{i\varsigma''}{2}\pm\nu''-\frac{5}{2}$ with respect to ζ, ζ'. Then owing to the fact that the measures $d\sigma(\theta)$ and $d\sigma(\xi,\zeta)$ have the properties:

$$d\sigma(\lambda\theta)=\lambda^3\bar{\lambda}^3\, d\sigma(\theta),$$

$$d\sigma(\lambda\xi,\nu\zeta)=\lambda^{\frac{5}{2}}\bar{\lambda}^{\frac{5}{2}}\nu^{\frac{5}{2}}\bar{\nu}^{\frac{5}{2}}\, d\sigma(\xi,\zeta).$$

The integral (54) makes sense only if the kernel $K(\theta,\theta',\xi,\zeta)$ is a homogeneous function of its arguments too, namely

of degree $\frac{i\varsigma}{2}\pm\nu-3$	with respect to	θ	and $\bar{\theta}$,
of degree $-\frac{i\varsigma}{2}\mp\nu-3$	with respect to	θ'	and $\bar{\theta}'$,
of degree $-\frac{i\varsigma'}{2}\mp\nu'-\frac{5}{2}$	with respect to	ξ	and $\bar{\xi}$,
of degree $-\frac{i\varsigma''}{2}\mp\nu''-\frac{5}{2}$	with respect to	ζ	and $\bar{\zeta}$.

This kernel may contain explicitly the invariant variables

$$(\theta\zeta)=\theta_A\zeta^A, \qquad (\theta'\zeta)=\theta'_A\zeta^A, \tag{55}$$

but it can not be expressed by means of these variables only, for in such a case it would not depend on ξ. We note however that a possible invariant function of three variables is an integral of the form

$$J(\theta,\theta',\xi)=\int(\theta\alpha)^{M_1}(\theta'\alpha)^{M_1}(\xi\alpha)^{L_1}(\overline{\theta\alpha})^{M_2}(\overline{\theta'\alpha})^{M_2}(\overline{\xi\alpha})^{L_2}d\sigma(\alpha),$$

$$(\theta\alpha)=\theta_A\alpha^A, \quad (\theta'\alpha)=\theta'_A\alpha^A, \quad (\xi\alpha)=\xi_A\alpha^A. \tag{56}$$

This integral makes sense only if

$$N_i + M_i + L_i = -6 .$$

Let us write the kernel in the form of a linear combination of products of invariants (55) and integrals of the form (56). It can be proved that such a kernel exists only if $\rho' = \rho''$ and $\nu' = \nu''$. In this case it has the form

$$K(\theta,\theta',\xi,\zeta) = \sum_{\mu_1\mu_2} \int d\tau_1 d\tau_2 C(\tau_1,\tau_2,\mu_1,\mu_2) \overline{f(\theta)} f(\theta') g(\xi,\zeta) \times$$
$$\times (\theta\zeta)^{\frac{i\tau_1}{2}+\mu_1+R_1} \left(\overline{\theta\zeta}\right)^{\frac{i\tau_2}{2}+\mu_2+R_2} (\theta'\zeta)^{-\frac{i\tau_1}{2}-\mu_1+R_1} \left(\overline{\theta'\zeta}\right)^{-\frac{i\tau_2}{2}-\mu_2+R_2} \times \tag{57}$$
$$\times \int (\theta\alpha)^{N_1} \left(\overline{\theta\alpha}\right)^{N_2} (\theta'\alpha)^{M_1} \left(\overline{\theta'\alpha}\right)^{M_2} (\xi\alpha)^{L_1} (\xi\alpha)^{L_2} d\sigma(\alpha),$$

where

$$R_{1,2} = -\frac{i\rho'}{2} \mp \nu' - \frac{5}{2}, \qquad N_{1,2} = \frac{i\rho}{2} \pm \nu - 3 - \frac{i\tau_{1,2}}{2} - \mu_{1,2} - R_{1,2},$$

$$M_{1,2} = -\frac{i\rho}{2} \mp \nu - 3 + \frac{i\tau_{1,2}}{2} + \mu_{1,2} - R_{1,2}, \quad L_{1,2} = -\frac{i\rho'}{2} \mp \nu' - \frac{5}{2} . \tag{58}$$

Now we apply these results to the study of the structure of the meson-baryon vertex. We expand an arbitrary spinor from the Hilbert space of homogeneous functions characterizing the baryon multiplet in the form

$$\Psi_p(\theta_A) = \sum_{\tau=\nu}^{\infty} \Psi^{A_1 \ldots A_{\tau+\nu}}_{\dot{B}_1 \ldots \dot{B}_{\tau-\nu}}(p) \, \widetilde{\Phi}^{\dot{B}_1 \ldots \dot{B}_{\tau-\nu}}_{A_1 \ldots A_{\tau+\nu}}(\theta_A) . \tag{59}$$

Here the $\widetilde{\Phi}^{\dot{B}_1 \ldots \dot{B}_{\tau-\nu}}_{A_1 \ldots A_{\tau+\nu}}(\theta_A)$ are determined by formulae (33) and (34). On the other hand the spinors from the Hilbert space of homogeneous functions characterizing the mesons we write as

$$\Psi_q(\xi,\zeta) = \sum \left\{ \Psi(q) \widetilde{\Phi}_{(1)}(\xi,\zeta) + \Psi^{(+)}(q)^A_B \, \widetilde{\Phi}^{(+)\,B}_{(35)_A}(\xi,\zeta) + \Psi^{(-)}(q)^A_B \, \widetilde{\Phi}^{(-)\,B}_{(35)_A}(\xi,\zeta) + \cdots \right\}, \tag{60}$$

where $\widetilde{\Phi}_{(1)}(\xi,\zeta)$ and $\Phi^{(\pm)}_{(35)}(\xi,\zeta)$ are determined by (52) and (53). To obtain the vertex it is sufficient to insert expressions (59) and (60) into the formulae (57) for the invariant trilinear functional putting $\nu=3/2$, $\rho=\rho'=\nu'=0$. Extracting the part of the vertex corresponding to the interaction of the baryon 56-plet and the meson 35-plets, we get

$$\Gamma^{(\pm)}(p_1,p_2;q)=M^{(\pm)\dot{A}_1\dot{A}_2\dot{A}_3,C}_{B_1B_2B_3,D}\,\Psi^{(1)}(q)^D_C\,\overline{\Psi}_{\dot{A}_1\dot{A}_2\dot{A}_3}(p_2)\,\Psi^{B_1B_2B_3}(p_1), \tag{61}$$

where

$$\begin{aligned}M^{(\pm)\dot{A}_1\dot{A}_2\dot{A}_3,C}_{B_1B_2B_3,D}&=\sum_{\mu_1\mu_2}{}'\int d\tau_1 d\tau_2\, C(p_1^2,p_2^2q^2;\tau_1,\tau_2,\mu_1,\mu_2)\,\overline{\widetilde{\Phi}}^{\dot{A}_1\dot{A}_2\dot{A}_3}(\theta)\times\\&\times\widetilde{\Phi}_{B_1B_2B_3}(\theta')\,\widetilde{\Phi}^{(\pm)}_{(35)}(\xi,\zeta)^C_D\,(\theta\zeta)^{\frac{i\tau_1}{2}+\mu_1-\frac{5}{2}}\,\overline{(\theta\zeta)}^{\frac{i\tau_2}{2}+\mu_2-\frac{5}{2}}\,(\theta'\zeta)^{-\frac{i\tau_1}{2}-\mu_1-\frac{5}{2}}\times\\&\times\overline{(\theta'\zeta)}^{-\frac{i\tau_2}{2}-\mu_2-\frac{5}{2}}\int(\theta\alpha)^{1-\frac{i\tau_1}{2}-\mu_1}\,\overline{(\theta\alpha)}^{-2-\frac{i\tau_2}{2}-\mu_2}\,(\theta'\alpha)^{-2+\frac{i\tau_1}{2}+\mu_1}\,\overline{(\theta'\alpha)}^{1+\frac{i\tau_2}{2}+\mu_2}\times\\&\times(\xi\alpha)^{-\frac{5}{2}}\,\overline{(\xi\alpha)}^{-\frac{5}{2}}\,d\sigma(\alpha).\end{aligned} \tag{62}$$

In the expressions (33), (34) and (35) for $\widetilde{\Phi}_{ABC}$ and $\widetilde{\Phi}^{(\pm)}_{(35)}(\xi,\zeta)^C_D$ we must now put $m^2\to-p_i^2$ or $\mu^2\to-q^2$, respectively. For the other parts we have analogous expressions. These expressions show that the meson-baryon vertex in the theory of $SL(6,C)$ symmetry depends on an infinite number of scalar products $C(p_1^2,p_2^2,q^2;\tau_1,\tau_2,\mu_1,\mu_2)$. As far as now we are not able to prove whether the vertex depends essentially on all these functions or they form in (62) just a trivial combination. It is quite reasonable to expect that different functions $C(p_1^2,p_2^2,q^2;\tau_1,\tau_2,\mu_1,\mu_2)$ enter in the vertex (62) in different ways and that this vertex depends effectively on an infinite number of functions. This result of ours differs from that of Fronsdal/3/ and Rühl/5/. From the expression (62) one can derive different relations between physical formfactors.

The expressions of type (62) contain the complete information on the consequences of our symmetry scheme. A piece of this information can be derived without the evaluation of the integrals merely on the basis of transformation properties of the corresponding integrals under the auxiliary group $S'=SL(6,C)$ containing the Lorentz group (see §3). We remind that θ,θ',ξ and ζ transform according to finite dimensional spinor representations of the auxiliary group S'.

Let us now introduce certain spinors of this group: $P_{1\dot{B}}^{\;A}$, $P_{1B}^{\;\dot{A}}$, $P_{2\dot{B}}^{\;A}$, $P_{2B}^{\;\dot{A}}$, $Q_{\dot{B}}^{A}$, $Q_{B}^{\dot{A}}$ (we note that these spinors are not necessarily particles' momenta). Then we consider integrals of the type (62) in which momenta

$$P_{i\,B}^{\;\dot{A}}=p_{i\,\dot{\beta}}^{\;\dot{\alpha}}\,\delta_{\beta}^{\alpha},\quad Q_{B}^{\dot{A}}=q_{\dot{\beta}}^{\dot{\alpha}}\,\delta_{\beta}^{\alpha},\qquad i=1,2, \tag{63}$$

are replaced by arbitrary finite dimensional spinors of the group S'. The measures are invariant under S' therefore these integrals must have definite transformation properties under S' with the numbers of dotted and undotted indices given in the left hand side. On the other hand, these integrals are functions of the spinors $P_{i\,B}^{\;\dot{A}}$, $P_{i\,\dot{B}}^{\;A}$, $Q_{B}^{\dot{A}}$, $Q_{\dot{B}}^{A}$ and therefore are to be expressed in terms of the latter. Thus the integrals of the type (62) have to be linear combinations of the products of the type $P_{i\,B_1}^{\;\dot{A}_1}P_{j\,B_2}^{\;\dot{A}_2}P_{k\,B_3}^{\;\dot{A}_3}\delta_{D}^{C}$, $\delta_{B_1}^{\dot{A}_1}\delta_{B_2}^{\dot{A}_2}\delta_{B_3}^{\dot{A}_3}P_{i\,D}^{\;\dot{R}}P_{j\,\dot{R}}^{\;C}$, . . . , the coefficients being invariant functions. Replacing in the obtained expressions the arbitrary spinors $P_{i\,B}^{\;\dot{A}}$, $P_{i\,\dot{B}}^{\;A}$, $Q_{B}^{\dot{A}}$, $Q_{\dot{B}}^{A}$ by the momenta of particles according to (63) we get the general expression for vertices of the type (62). Vertices of this type were already proposed in the "old" theory of the intrinsically broken $SL(6,C)$ symmetry[6, 14,22,23]. The situation is the same for scattering amplitudes.

We have shown thus that in the developed framework of a theory with infinite multiplets it is possible to derive all experimental consequences resulting from the "old" non-unitary theory of intrinsically broken $SL(6,C)$ symmetry with finite multiplets.

In contrary to the situation there, in our approach the symmetry and unitarity of the S-matrix[8] are compatible.

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