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GAUGE FIELD VACUUM STRUCTURE IN GEOMETRICAL ASPECT

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1 Introduction

The main idea of this investigation is that a space-time geometry is created by the forces acting in space. Some prominent scientists, as Lobachevsky, Riemann, Einstein, Weyl and others were of such opinion ([1] - [5]). If corresponding dynamical geometry is local one, it must be given by the differential equations. These equations show how usual flat geometry, which is not connected with interactions (i.e. "rigid" in Weyl's terminology), propagates from point to point. In this case all notions used by modern theoretical physics have to obtain the local forms. In particular, it concerns the representations of finite Lie groups of symmetry and definitions of inertial motion and vacuum. In this talk the vacuum idea is analyzed, when it arises in classical gauge field theory in its geometrical form. It is shown that Einstein equations specify the vacuum structure of all gauge fields. The usual and hyperbolical instantons play an important role in definition of this relativistic vacuum structure, as their energy-momentum tensors are zero in spite of the fact that corresponding nonzero gauge fields are present. Therefore the instantons can be considered nongravitating matter.

2 Geometrical gauge field theory equations

In order that to construct the consistent relativistic quantum theory of interacting fields it is necessary to have the equation system describing any combinations of physical fields on classical level. In the gauge field theory such equation system is ([6]):

$$\nabla_\mu F^{\mu\nu}_a = j^\nu_a; \tag{1}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa(T^{(gf)}_{\mu\nu} + T^{(p)}_{\mu\nu}) \tag{2}$$

$a = 1, 2, ...r; \quad \mu, \nu = 1, 2, ..4.$

Here $F^a_{\mu\nu}$ - gauge field strength tensor, $R_{\mu\nu}$ - Ricci curvature tensor, R - scalar curvature of Riemannian space-time V_4 with metrics $g_{\mu\nu}$, $T^{(gf)}_{\mu\nu}$ - gauge field energy-momentum tensor, $T^{(p)}_{\mu\nu}$ - energy-momentum tensor of particles, κ - gravitational constant, j^ν_a - gauge field sources, ∇_μ - covariant derivative in fiber bundle space-time over 4D Riemannian space-time of GR, and fiber is gauge finite Lie group G_r. Latin indexes number parameters of internal symmetry group G_r. Greek indexes concern space-time V_4.

Any physical field describing interactions between particles can be regarded as a gauge field. Different Lie groups correspond to different kinds of forces. If several gauge fields are simultaneously present, then they can be jointly described by various ways.

Firstly, two different interactions may be united into a single interaction of more broad type. Then the gauge symmetry group should be extended in such a way that the symmetry group of each initial interaction became the subgroup of new extended symmetry group. Symplest variant of such unification is Weinberg-Salam model ([7]). In this model electromagnetic and weak interactions are unified into electroweak one by substitution of two individual gauge groups for more broad single gauge group $SU(2) \times SU(1)$ for electroweak interaction. This procedure does not change the equation system (1) - (2). But it is possible vacuum reconstruction.

Secondly, the equation system (1) - (2) may be extended by replacement of single equation (1) by several similar equations for each of interactions being combined. Then sum of energy-momentum tensors of all gauge fields and their sources will appear on the right side of equation (2). The solutions of such combined equation system will describe both the gauge fields themselves and these fields influence on geometry of space-time where they exist. If the different gauge fields will interact between themselves, modification of equation system (1) - (2) will become possible.

The important property of equation system (1) - (2) is its form independence on a gauge field treatment ([8], [9]). Symmetrical tensor of two rank $g_{\mu\nu}$ has the same rights both as Riemannian space-time V_4 metrics and as a tensor field in Minkowski space-time. In its geometrical form the gauge field vector-potentials A^a_μ become connection coefficients of fiber bundle space over Riemannian V_4. Here we shall consider equation system (1) - (2) the equations of unified geometrical theory of gauge fields.

3 Global and local geometry. Relativistic vacuum

In classical mechanics space-time geometry is globally given according with *long-range action hypothesis*. 3D space and disconnected with space 1D time are proposed infinite and plane. Space and time are absolute, i.e. are not connected with matter motion. Vacuum is present in the most part of Universe. In Newton mechanics vacuum is defined as emptiness, i.e. as any matter absence. In this sense it is universal for all matter kinds. Such vacuum is as absolute as world geometry is (i.e. not connected with matter motion) and global (i.e. its properties are the same in all points of Universe).

Relativistic physics is based on *short-range action hypothesis.* It states that all kinds of forces propagate in space from point to point. But if the world geometry is created by the forces acting in space, it must be also formed locally near one point and after that propagate to other points of space.

Einstein equations describe a construction process of world geometry step by step, when invariant square form $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ is given to start with. Although we can always choose the local geometry as flat, the space-time as a whole turns out to be curved. Its properties are described by Riemannian geometry without torsion.

Einstein equations connect Universe geometry with matter motion in it. Therefore in GR Riemannian space-time is not absolute. The form of Einstein equation solutions depends on energy-momentum tensors of different matter kinds being on the right side of these equations. Therefore in GR space-time in not universal. By electromagnetic field traces on the V_4 metrics this field can be reproduced to within dual rotations ([5]).

In GR the vacuum is described by the solutions of Einstein equations in emptiness, i.e. in the absence of any energy-momentum tensors on the right side of these equations. Usually emptiness means absence of any matter. But in the relativistic quantum field theory absence of particles (i.e. absence of the field quanta) does not mean nullification of corresponding field. It can be showed that the last would contradict to uncertainty principle. The fields in vacuum state are considered performing null oscillations ([10]). The vacuum solutions of Einstein equations describe Universe geometry both in absence of any nongravitational fields and in presence of them when their energy-momentum tensors are equal to zero. This situation is realized by instanton configurations of the fields ([11]). In a sense instanton configurations correspond to the matter without gravity.

In the quantum field theory instantons are classical trajectories connecting vacuums among themselves. They are being used for description of tunnel processes between vacuums ([12], [13], [14]).

In the gauge field theory instantons are usually called self- and antiself-dual solutions of Euclidean version of field equations, i.e. the solutions of self-duality equations

$$F^a_{\mu\nu} = \pm {}^*F^a_{\mu\nu}; \quad * \quad \text{means dual conjugation.}$$

These solutions turn gauge field action integral into a constant. On the other hand these solutions nullify gauge field energy-momentum tensor $T^{(gf)}_{\mu\nu}$. In ordinary opinion in the space-time with pseudoeuclidean metrics self-duality equations have no solutions. But the gauge field vector-potentials A^a_μ nullifying the field strength tensor $F^a_{\mu\nu}$, (i.e. so-called "pure gauges"), trivially satisfy self-duality equations and nullify the gauge field energy-momentum tensors in Minkowski space-time. Therefore they can be called trivial hyperbolical instantons, i.e. trivial instantons in space of pseudoeuclidean metrics.

When gravity is regarded as a gauge field in pseudoriemannian space-time V_4 then nontrivial hyperbolical instantons arise. They are the solutions of double self-duality equations of Riemannian curvature tensor $R_{\mu\nu\tau\lambda}$ of V_4 ([15]):

$$R_{\mu\nu\tau\lambda} = \pm^* R^*_{\mu\nu\tau\lambda}. \tag{3}$$

These equations appear in the gauge gravity theory when gravitational field is considered the gauge field associated with local Lorentz group of space-time symmetry ($SO(3,1)$-gauge gravity). The solutions of double self-duality equations nullify energy-momentum tensor of $SO(3,1)$-gauge gravity field $T^{(gg)}_{\mu\nu}$ and turn corresponding action integral containing Riemannian curvature tensor square into a constant. Contracted equations of self-duality (3) lead to the vacuum Einstein equations ([11]).

Thus, relativistic vacuum in GR is universal for all kinds of matter and their interactions. It is specified by the vacuum Einstein equations

$$R_{\mu\nu} = 0. \tag{4}$$

It is easy to see that in unified geometrical gauge field theory relativistic vacuum has to be determined in the same way as in GR. The equation (1) solutions corresponding to zero right side of equations (2) will be vacuum solutions in each gauge field theory and at the same time the vacuum solutions in GR.

Thus, the short-range action hypothesis works both in GR and in unified geometrical gauge field theory. Therefore in both cases geometry is local and connected with motion of matter. But vacuum in these theories is not connected with motion of matter and is absolute. It is defined by vacuum Einstein equations solutions. This is relativistic universal vacuum, because its appearance conditions are the same for all kinds of matter and their interactions. These conditions are nullification of right side of Einstein equations. Such relativistic vacuum is not global. Its properties are changing from point to point and specified by differential equations. In analogy with geometry in GR it can be named a dynamical vacuum. Between solutions of vacuum Einstein equations there are static and flat solutions describing flat space-time and globally given vacuum. Hence, vacuum Einstein equations ensure possibility of passage to nonrelativism and realization of corresponding principle.

4 Gravity as a gauge field

When we investigate physical processes in such region of space-time that its size is much more than our laboratory size, it is necessary to use Riemannian geometry. It arises just the situation when Universe properties are investigated. But the same situation can arise in elementary particle physics when the elementary particles are being used both as reference system and as observation means. Therefore Einstein equations appearence in elementary particle physics is quite natural.

Gauge fields arose as new mathematical (and later as physical) objects when global internal symmetry were localized. In other words the short-range action hypothesis being applied to the properties of elementary particles symmetry led to necessity to introduce into consideration the gauge fields as images of forces realizing interactions between these particles. If these forces are found to be of short range (as weak and nuclear forces), the elementary particles interact mainly with their nearest neighbours. The interaction propagates from one to another. Electromagnetic field is long-range acting. But it propagates also from point to point in correspondence with the short-range action hypothesis. Exception to the rules is static Coulomb potential. Infinite radius of this field action arises because of zero mass of photon as electromagnetic field quantum.

May gravity be considered a gauge field and where it comes from?

In GR the gravitational field propagates from point to point and, hence, the short-range action hypothesis is realized. Gravitational field is of infinite range of action. Therefore gravity forms Universe structure together with electromagnetic field.

Uniform approach to all interactions means that gravity is considered a gauge field. But it was be found that Einsteinian gravity takes peculiar place among fundamental interactions. Mathematical method (just compensation procedure) proposed by Weyl in 1929 ([16]) permits us to input electromagnetic field into the free electron theory. In 1954 Yang and Mills ([17]) applied this method to introduction of weak interactions. In 1956 Utiyama ([18]) proposed that all fields being in nature can be introduced into free particle theory by the compensation procedure when corresponding global symmetry becomes a local one. But his attempt of Einstein theory reproduction in this way has failed. Any way of successful GR regeneration by compensation procedure is not now exist. Therefore the opinion that quantum gravity must be noneinsteinian one is widely known ([19]). This opinion is not quite correct.

Really it is necessary to refuse the compensation procedure as formal and essentially restricted for application. Instead of that we have to do like Maxwell and carry attention center from particles and charges into space between them. When Maxwell has created electrodynamics electron was not yet being discovered and electric current nature was unknown. But this did not prevent him from correct formulating the electromagnetic field equations. The sources of any nature he denoted by letter j on the right side of the field equations, and elucidation of their physical sense was extracted as independent problem for next generations. Electron had experimentally been discovered only 30 years hence electrodynamics creation. Thus, in the field theory the field equations are primary, and sources nature is secondary.

We shall begin at the known fact that elementary particles physics language is Lie groups theory. All elementary particles are classified by the representations of finite Lie groups G_r, which transformations are specified by finite number of parameters independent on point. Global specification of transformations parameters simultaneously in all points of Universe contradicts the short range action hypothesis. Therefore in the relativistic physics of elementary particles the parameters of symmetry groups can not be globally given and must be the functions of V_4 point. But then the finite Lie groups

G_r become infinite Lie groups $G_{\infty r}$. In this case the symmetry transformations instead of number parameters are specified by r functions of point and their derivatives to k-th order.

When global symmetry becomes local one and finite Lie groups G_r turn into infinite Lie group $G_{\infty r}$ two fundamentally different kinds of infinite Lie groups representations arise. The representations, which transformations are independent on the parametric function derivatives, we shall use for classification of elementary particles as it is in the case of global symmetry. The representations, which transformations depend on these derivatives, we shall name gauge fields ([6]). If the parametric functions derivatives turn into zero the situation becomes nonrelativistic and the fields turn into the particles. Then fields can be as classified by finite Lie group G_r representations as particles are usually.

By Lagrange variational formalism for infinite Lie groups $G_{\infty r}$ being formulated in 1967 by N.P.Konopleva ([9]) it is possible to get any nongravitational gauge field equations in the form of equations (1). Here nongravitational fields are called those, which are disconnected with local space-time symmetries.

In order that to obtain the Einstein equations for gravity as a gauge field, above-cited Lagrange formalism for infinite Lie groups must be applied to a symmetric tensor of rank two $g_{\mu\nu}$ and its transformations under any continuous coordinate transformations of the form

$$x^{\mu'} = f^{\mu}(x^{\nu}), \tag{5}$$

where $f^{\mu}(x^{\nu})$ - arbitrary continuous functions of x^{ν}. Then the transformations (5) must be regarded as belonging to the local 4-parametric translation group $G_{\infty 4}$ ([9]).

In this case Einsteinian Lagrangian directly arises as the gauge field $g_{\mu\nu}$ Lagrangian in the form of scalar curvature $L = R$. By variation of this Lagrangian with respect to $g_{\mu\nu}$ we shall get vacuum Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0, \qquad \text{or} \qquad R_{\mu\nu} = 0. \tag{6}$$

These equations and Lagrangian appears independently on that is $g_{\mu\nu}$ metrics of V_4 or not. The $g_{\mu\nu}$ derivatives combinations will be the same both when $g_{\mu\nu}$ is the tensor field in flat V_4 and when we regard it as metrics of Riemannian space-time ([9]).

For obtaining of equation system (1) - (2) it is necessary to consider the system of two interacting fields: tensor fields $g_{\mu\nu}$ and some nongravitational gauge field described by vector-potential A^a_μ. Gauge field sources (particles) can also be introduced into the theory by corresponding field variables. The local symmetry of the theory permits to specify Lagrangian form of interacting fields and particles. There is no choice making principle in a globally symmetric theory in flat space-time.

The variational formalism for infinite Lie groups makes possible not only reconstruction of GR in terms of the gauge field theory, but also extension of our ability of gravity description. Real gravity manifests not only as curvature of test bodies trajectories, but as tidal forces which are not described by GR. In order to get corresponding equations let us discuss in detail what is a result of space-time symmetry localization of flat world?

It is well known that Minkowski space-time is symmetric with respect to the global Poincaré group P_{10} of transformations including 4D invariant subgroup of translations and Lorentz subgroup of rotations $SO(3,1)$. In contrast to localization of internal symmetry when space-time symmetry becomes a local one the conception of the world geometry becomes fundamentally different. Riemannian geometry in its usual form does not permit to construct the global invariants, as radius, energy, momentum, spin and others which are being used for description of physical processes in flat V_4.

But it is possible, following E. Cartan ([20]), to substitute Riemannian space-time V_4 for aggregate of flat Minkowski spaces tangent the Riemannian space-time in each its point. Then the global symmetry group of Minkowski space will fall into two different in essence local symmetry groups: 1) the local Lorentz group $SO(3,1)$ or as equivalent $G_{\infty 6}$, and 2) the local translations group $G_{\infty 4}$, which is the group of general relativistic coordinate transformations (5). First of them acts in tangent space as group rotating the local system of four mutually orthogonal vectors h_i^μ (reference system). Each of the tangent spaces is spanned with the reference system h_i^μ. The local traslations group corresponds to displacements of reference system origin from one point of Riemannian space V_4 to other point.

Thus, after localization of global space-time symmetry the local groups of space-time rotations and translations generated by it act in different spaces. In this case the local Lorentz group acting in the tangent space can be regarded as analog of internal symmetry group. But the local translation group can not be considered in the same way, because the coordinate transformations (5) concern the external world geometry.

Ricci connection coefficients $\Delta_\mu(ik)$ arise as the gauge field associated with the local Lorentz group $SO(3,1)$ (or $G_{\infty 6}$). They permit to make a parallel displacement of local system of four mutually orthogonal vectors h_i^μ in Riemannian V_4. Ricci coefficients are defined as:

$$\Delta_\mu(ik) = h_i^\tau h_\nu^k \left\{ {}_{\mu}{}^{\nu}{}_{\tau} \right\} + h_\tau^k \partial_\mu h_i^\tau . \tag{7}$$

It is easy to see that in addition to usual Christoffel symbols $\left\{ {}_{\mu}{}^{\nu}{}_{\tau} \right\}$ these connection coefficients include supplementary terms $h_\tau^k \partial_\mu h_i^\tau$. Such terms permit to remain orthogonality of vectors h_i^μ under parallel displacement in Riemannian space.

The gauge field $\Delta_\mu(ik)$ is gravity field although its Lagrangian structure is similar to that of Maxwell electrodynamics and contains the square of Riemannian curvature tensor as $L \sim R_{\mu\nu}(ik) R^{\mu\nu}(ik)$. Here $R_{\mu\nu}(ik)$ is usual Riemannian curvature tensor expressed in terms of vectors h_i^μ components. Therefore the parallel displacement does not result in rescaling. The lengths of segments and volumes keep their sizes under translations. This is just the point of the fundamental difference between Weyl geometrical approach ([4]) and the gauge field theory in geometrical interpretation ([6]).

In accordance with two kinds of local symmetries arising from localization of global symmetry of Minkowski space-time we have two gauge gravitational fields. Firstly, it is Einstein gravity described by tensor $g_{\mu\nu}$ and associated with infinite group of local translations $G_{\infty 4}$. Secondly, it arose a new gravitational field associated with the local Lorentz group $SO(3,1)$ being infinite group $G_{\infty 6}$. This field is described by Ricci connection coefficients and is called $SO(3,1)$-gauge gravity.

If these gravitational fields are separately considered and each of them theory is independently formulated we shall have system of two equations for noninteracting fields:

$$R^{\mu\nu}(ik)_{;\mu} = 0 \tag{8}$$

$$R_{\mu\nu} = 0 \,. \tag{9}$$

If we shall formulate the theory of two interacting gravitational fields a new equations system will arise.

In the geometrical gauge field theory gravitation as a gauge field is described by equation system including the equations of two kinds and being similar to the equation system (1) - (2). This equation system describes both relativistic gravitational vacuum, and real tidal gravitational forces. It has the form ([21]):

$$R^{\mu\nu}(ik)_{;\mu} = 0 \tag{10}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T^{(gg)}_{\mu\nu}, \tag{11}$$

where by analogy with electrodynamics the expression

$$T^{(gg)}_{\mu\nu} = R_{\mu\tau}(ik) R_{\nu}{}^{\tau}(ik) - \frac{1}{4} g_{\mu\nu} R_{\alpha\tau}(ik) R_{\alpha}{}^{\tau}(ik) \tag{12}$$

is named the energy-momentum tensor of $SO(3,1)$-gauge gravity field.

The equations system (10) - (11) as compared with the equations system (8) - (9) has only difference in one point. Appearance of interaction between two gravitational fields results in appearance of energy-momentum tensor on the right side of Einstein equations and vacuum reconstruction. The same situation takes place when we consider some nongravitational gauge field A^a_μ and tensor gauge field $g_{\mu\nu}$. When the gauge fields A^a_μ and $g_{\mu\nu}$ regard as separate noninteracting fields on the right side of equations (2) gauge field energy-momentum tensor is absent. When interaction of A^a_μ and $g_{\mu\nu}$ arise (i.e. space-time geometry becomes dynamical one) the vacuum turns immediately into the relativistic dynamical vacuum and on the right side of Einstein equations appears the gauge field energy-momentum tensor.

Having taken covariant divergence of equations (11) we shall get the equations (10). Thus, Einstein equations (11) for the tensor field $g_{\mu\nu}$ are followed by the equations (10). It could be said that being contained in quasimaxwellian equations (10) information is really already contained in Einsteinian equations (11), and hence in the metrics $g_{\mu\nu}$. However in contrast to Weyl we say that here the metric field contains not electromagnetic field, but additional information of gravitational field. This is information of tidal forces, which are indeed able to vary lengths and volumes.

The connection of Einstein equations (11) with equations (10) arose from localization of flat space-time translation group, i.e. because of general relativity principle. It is universal for all gauge fields.

Indeed, if on the right side of equations (2) only one gauge field energy-momentum tensor will be remained and covariant divergence of these equations will be taken, then without sources equations (1) will arise from equations (2). It appears because localization of tranlations group results in reduction of independent equations number in the system (1) - (2) and decrease of it by four (dimension of space-time V_4). Thus the connection between equations (1) and (2) arise from localization of translations. As far back as 1915 D.Hilbert paid attention to this connection, when he wrote a similar equations system for gravitational and electromagnetic fields ([22]). From this he concluded that four equations of the system (1) - (2) are unnecessary. Therefore the elecromagnetic field can be regarded as a certain gravity manifestation. Weyl as Hilbert's disciple was of such opinion also, but he realized above idea by other mathematical way.

Many different fields in addition to gravitational and electromagnetic fields are known at present. Writing equation system for all fields simultaneously (regarding them as gauge fields) we shall see that only four equations of this system will as before arise from others. It is evident that gravity alone is not ample for description of the whole interactions diversity.

In classical field theory in flat space-time equations of particles motion are independent on field equations and must be given in addition to them ([23]). But in Riemannian space-time V_4, i.e. in the presence of gravity, the equations of particles motion can be obtained from the field equations and Einstein equations. It is a result of general relativity principle and is known in GR.

If on right side of Einstein equations only energy-momentum tensor of noninteracting particles (dust) is remained and covariant divergence of these equations is taken, the geodesic lines equations appear as particles motion equations. Hence in GR hypothesis of test bodies geodesic motion is not independent hypothesis but follows from gravitational field equations.

The same situation take place in the classical gauge field theory. Without gravity in flat V_4 any connection between gauge field equations and particle motion equations is absent. These equations are independent. But in Riemannian V_4 (or in gravity presence) such connection appears. When we consider a system of gauge fields with their sources assumed charged particles in Riemannian V_4, the particles motion equations can be obtained as consequence of equation system (1) - (2) with the sources in the form of particles noninteracting among themselves. It is found that the structure of motion equations of all gauge charged particles will be in that case similar to Lorentz equation structure, but taking into account gravitational field $g_{\mu\nu}$. It is known that Lorentz equations describe motion of electron in external electromagnetic field. Our abovementioned generalized Lorentz equations describe the motion of electron as motion of a test body in external electromagnetic and gravitational fields. Similarly all gauge charged particles in accordance with equations (1) - (2) will behave as test bodies in corresponding external gauge and gravitational fields ([6]). Let us remember that the gravitational field $g_{\mu\nu}$ bears the responsibility for vacuum reconstruction.

In each infinitesimal segment of its trajectory test body are by definition moving free, i.e. by inertia. Therefore Einstein equations permit us to state a local concept of inertial motion. Experimental observation of test bodies motion give us a chance to see geometry of external world.

5 Fiber bundle space and Weyl geometry

In modern geometry a set of local reference systems associated with each point of Riemannian V_4 is described in terms of fiber bundle space geometry ([24]).

Fiber bundle space geometrically unites the spaces of two kinds. One of them is chosen as a base of fiber bundle space, and other one is declared a fiber. A copy of fiber is associated with each point of a base. Under displacements from one base point furnished with coordinate x^μ to other one with coordinate x^ν the fiber associated with initial point is carried to the next point. But in the point x^ν some copy of fiber has already been present before displacement. For substitution the fiber being previously in point x^ν for the fiber carried over from point x^μ it is necessary to tranform the original fiber in point x^ν. This trahsformation turns the fiber into itself because all fibers associated with all points of base are identical to each other and its transformations are automorphysms.

In the gauge field theory 4D pseudoriemannian space-time of GR is chosen as a base of fiber bundle space, and a finite Lie group G_r is chosen as a fiber of this space. Lie group G_r describes internal symmetry of the gauge field theory. Thus, for electrodynamics this is group $U(1)$, for weak interactions - $SU(2)$, for strong interactions (QCD) - $SU(3)$. In this case Lie group G_r is regarded geometrically as a manifold. As E.Cartan showed, all semisimple finite Lie groups under geometrical consideration are symmetric Riemannian spaces. Then tensor $g_{ab} = f_{a\ m}^{\ n} f_{b\ n}^{\ m}$ appears as a metrics on Lie group G_r.

For gravity, as said above, tangent space in given point of V_4 becomes a fiber, where Lorentz group $SO(3,1)$ acts. In all tangent spaces in each points of V_4 Lorentz group $SO(3,1)$ acts uniformly but parameters $\omega_k{}^n$ of corresponding rotations $\delta e_k = \omega_k{}^n e_n$, where e_k - arbitrary vector of the tangent space, can depend on the point. Therefore Lorentz group in Riemannian V_4 is said to be acting as a local group.

When the results of experiments fulfiled in different points of space-time are under discussion it is useful to apply the fiber bundle space geometry. Indeed, any experimental measurements are by their nature local. But for objectivity and science meaning of their results the experiments must be reproduced. They have to be repeated and compared with each other. If V_4 geometry is local one, comparison procedure requires to specify a process of parallel displacement so as to avoid a distortion of information obtained by the experiments when observation point is changed. But Riemannian V_4 has not any global symmetry in general case. Therefore the displacements in real space-time lead almost inevitably to distortions. At the same time description of experiment results requires an existence of globally symmetric space of parameters, because we have need of the invariants carrying the information of physical objects properties. The structure of fiber bundle space permit us to unite these two seeming incompatible demands: the global symmetry of parametric space (by fiber introduction) and the local space-time geometry in the base.

A tangent space exists only in infinitesimal neibourhood of each base point, but it is possessed of global symmetries of a flat space. All dynamical constants can be as usual defined in it. Segment lengths must be measured in tangent space as well.

How to do so that the local operations fulfiled in infinitesimal neibourhood of one point result in objective information remaining its valid under displacements to any other point of Riemannian space-time? Just for this the rules of displacement of different geometrical objects and of tangent space itself must be correctly specified.

Weyl geometry arose from an attempt to solve the question of measurement and displacement of segments in infinitesimally defined geometry.

In Riemannian space parallel displacement of vectors from point P to infinitesimally close to it point P' is defined by coefficients of affine connection (Christoffel symbols):

$$\left\{ {\lambda \atop \mu\ \nu} \right\} = \frac{1}{2} g^{\lambda\kappa} \left(\frac{\partial g_{\kappa\nu}}{\partial x^{\mu}} + \frac{\partial g_{\kappa\mu}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\kappa}} \right).$$

These coefficients give only displacement of direction. The displacement of segments must be given by other coefficients, namely by the coefficients of metric connection. According to Weyl ([4]) they have the form:

$$\Gamma_{\mu\ \nu}^{\ \lambda} = \left\{ {\lambda \atop \mu\ \nu} \right\} + \frac{1}{2} \left(\delta_{\mu}^{\lambda} \phi_{\nu} + \delta_{\nu}^{\lambda} \phi_{\mu} - g_{\mu\nu} g^{\lambda\kappa} \phi_{\kappa} \right), \qquad (13)$$

where $\phi_\mu = \frac{\partial \phi}{\partial x^\mu}$, $d\phi = \phi_\mu dx^\mu$ - infinitesimal multiplier independent on moving segment. This multiplier is a linear differential form. It specifies a difference of segment length dl on its parallel displacement from point P to infinitesimally close to it point P'. Under this

$$dl = -l d\phi.$$

Under arbitrary continuous coordinate transformations (5) both forms given on manifold (square form $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ and linear form $d\phi = \phi_\mu dx^\mu$) are invariant. Under arbitrary gauge transformations of scales the metrics $g_{\mu\nu}$ is substituted for $\lambda g_{\mu\nu}$, and the linear form coefficients ϕ_μ changes in correspondence with formula:

$$\phi_\mu{}' = \phi_\mu - \frac{1}{\lambda} \frac{\partial \lambda}{\partial x^\mu}. \qquad (14)$$

By Weyl these two fundamental forms characterize the metrics of manifold in some reference system ($\equiv$ coordinate system + gauge). He considered that the functions $g_{\mu\nu}$

and ϕ_μ have to be so brought into all values and relations, which express analytically metric respects, that would take place the invariance under 1) arbitrary continuous coordinate transformations (5) ("coordinate invariance"), and 2) substitutions $g_{\mu\nu} \rightarrow \lambda g_{\mu\nu}$, $\phi_\mu \rightarrow \phi_\mu - \frac{1}{\lambda}\frac{\partial\lambda}{\partial x^\mu}$. Here λ can be any positive function of coordinates ("gauge invariance"). Thus, Weyl extended Einsteinian general relativity principle by addition to it the requirement of the gauge invariance of theory.

Weyl multiplier λ in 4D interval definition in GR takes into account a possibility of rescaling, i.e. gauge variation of the scale measuring segment lengths. This gauge can be different in different points of Riemannian space. In Weyl space parallel displacement by the connection coefficients (13) is not preserving lengths and volums. In order to talk about equal segments or about the same segment in different points of metrical space *supplementary conditions* must be formulated. These conditions have to guarantee conservation of segment length under displacements in Weyl space. As Weyl showed, necessary and enough condition of it is nullification of "scale curvature" tensor $f_{\mu\nu}$ defined as

$$f_{\mu\nu} = \frac{\partial\phi_\nu}{\partial x^\mu} - \frac{\partial\phi_\mu}{\partial x^\nu}.$$

In this case $d\phi = d(lg\lambda)$, that permits us to choose λ so that $d\phi$ is equal to zero everywhere. So, a length comparison of different segments is possible if and only if the metrical space is Riemannian space, i.e. $f_{\mu\nu} = 0$. *This condition is the basis of any measurement procedure.* It was be later noted that in the case of central symmetry the aggregate of orbit radii in Weyl space along which the parallel displacement preserves volumes satisfies Bohr rules of hydrogen atom orbits quatization ([25]).

Extention of relativity principle leads to appearance of new fields. In Weyl theory of 1918 ([26]) a new field ϕ_μ appeared as a result of addition of new gauge invariance of gravitational theory. At that time there was known only one field besides gravity. It was electromagnetic field. The tensor of new field $f_{\mu\nu}$ satisfied, by definition, Maxwell equations. The gauge transformations of the field ϕ_μ (14) were by their structure similar to the gauge transformations of second type of electromagnetic vector-potential A^a_μ: $A'_\mu = A_\mu - \partial_\mu\alpha(x)$. The "scale curvature" tensor $f_{\mu\nu}$ was expressed in terms of ϕ_μ in just the same way as the electromagnetic field tensor $F_{\mu\nu}$ - in terms of A^a_μ. Therefore Weyl identified his new field with electromagnetic field and his theory - with unified geometrical theory of gravity and electromagnetism.

At present it became clear that all kinds of forces in nature considering gauge fields are described by vector-potentials A^a_μ, which are analogs of the electromagnetic vector-potential A_μ. Strength tensors of all gauge fields $F^a_{\mu\nu}$ and corresponding Lagrangians are by their structure similar to electromagnetic field strength tensor $F_{\mu\nu}$ and Maxwell Lagrangian $L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. All gauge field equations are similar to Maxwell equations, although they are nonlinear one. Physical fields differ from each other by the local gauge symmetry group. The parameters of these gauge groups, as ϕ in Weyl geometry, become the invariant linear forms. In geometrical interpretation the vector-potentials of all gauge fields become the connection coefficients of a fiber bundle space over V_4, but not space-time with generalized connections as Weyl proposed.

Weyl theory is the first simple example of a gauge field theory. But before Weyl ideas became to be extendable to other kinds of interactions his generalized connections had to be developed as long as generalized spaces, such as fiber bundle spaces, arose. In fiber bundle space all locally measured characteristics can be carried over Riemannian space without distortions. Separation of points displacement (i.e. displacement in Riemannian space of base) and displacement of geometrical objects defined in fiber over each point of the base permitted to solve the problems formulated by Weyl. In contrast to Weyl space in the fiber bundle space the parallel displacement in base preserves volume even if strength tensor of a gauge field $F^a_{\mu\nu}$ is nonzero. This tensor becomes the main geometrical characteristic - curvature tensor of fiber bundle space. Nonzero strength gauge field tensor means that the fiber bundle space is nontrivial.

6 Summary

In relativistic quantum field theory (i.e. in elementary particles theory) a ground state of field or system of fields is called a vacuum state. The vacuum state has zero energy and all other dynamical invariants. In this state there are no quanta of any field, i.e. any elementary particles are absent. In general, this vacuum can be both global and local depending on field energy distribution in space. In elementary particle physics space-time geometry is considered Minkowski geometry and not connected with matter motion.

In gauge field theory it happens that symmetry of the theory is spontaneously broken. Then there appear many local states of a stable equilibrium. They are called local vacuums. Quantum theory of perturbations are constructed in neibourhood of such local vacuums. The structure of the local vacuum set radically influences properties of quantum theory of corresponding gauge field. This terminology can not be regarded as successful, as it leads to misunderstanding and multiple attempts "to do energy from vacuum".

Here a new formulation of relativistic vacuum state is proposed. This approach follows from the gauge filds theory being invariant with respect to both an arbitrary continuous coordinate trasformations and local gauge groups. It is applicable to both local internal symmetries and local space-time symmetries forming a world geometry and directly concerning gravity. In accordance with the point of view stated here the relativistic vacuum has to satisfy following conditions: 1) local definition, 2) realizability of short-range action hypothesis, 3) being given by differential equations permitting to define the vacuum state step by step in any points of Universe.

It is shown that the local relativistic vacuum of any gauge fields and its sources is described by vacuum Einstein equations solutions. The gauge fields can take part in this consideration both individually and in aggregate. This vacuum is absolute and universal for all kinds of matter, but in contrast to Newtonian vacuum it is not globally given. The properties of the relativistic vacuum change from point to point. The vacuum solutions of Einstein equations describe the relativistic vacuum not only when fields and particles are absent, but also in the case when the fields form an instanton configurations with zero energy-momentum tensors.

We must make it a rule to add Einstein equations to any field and particle motion equations if the space-time geometry becomes a local one. This addition means that we give the equations specifying *the structure of new vacuum changing and propagating* from point to point, i.e. of the relativistic vacuum. Einstein equations permit to correct the field and particles equations due to the constraints between these equations arising from general relativity principle.

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