ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

00-211

E2-2000-211

D.V.Shirkov*

THE π^2 TERMS IN THE s-CHANNEL QCD OBSERVABLES

Submitted to «Physics Letters»

---

*E-mail: shirkovd@thsun1.jinr.ru

2000

1 Preamble

Usually, physical quantities in the time-like channel, like the cross-section ratio of the inclusive $e^+e^- \to$ hadron annihilation or the τ–decay process, are presented in the form of two- or three-term perturbation expansion

$$\frac{R(s)}{R_0} = 1 + r(s)\,; \quad r(s) = c_1\,\bar{\alpha}_s(s) + c_2\,\bar{\alpha}_s^2 + c_3\,\bar{\alpha}_s^3 + \ldots \tag{1}$$

(our coefficients $c_k = C_k\,\pi^{-k}$ are normalized differently from the commonly adopted, like in Refs.[1, 2, 3]) over powers of effective QCD coupling $\bar{\alpha}_s$ which is supposed *ad hoc* to be of the same form as in the Euclidean domain, e.g.,

$$\begin{aligned} \bar{\alpha}_s^{(3)}(s) \;&=\; \frac{1}{\beta_0 L} - \frac{b_1 \ln L}{\beta_0^2\, L^2} + \frac{1}{\beta_0^3 L^3}\left[b_1^2(\ln^2 L - \ln L - 1) + b_2\right]; \\ &+\; \frac{1}{\beta_0^4 L^4}\left[b_1^3\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3b_1 b_2 \ln L + \frac{b_3}{2}\right]. \end{aligned}$$

Here, $L = \ln(s/\Lambda^2)$ and for the beta-function we use normalization

$$\beta(\alpha) = -\beta_0\,\alpha^2 - \beta_1\,\alpha^3 - \beta_2\,\alpha^4 + \ldots = -\beta_0\,\alpha^2\left(1 + b_1\,\alpha + b_2\,\alpha^2 + \ldots\right),$$

that is also free of π powers. Numerically,

$$\beta_0(f) = \frac{33 - 2\,f}{12\pi}\,; \quad b_1(f) = \frac{153 - 19f}{2\pi(33 - 2f)}\,; \; b_1(4 \pm 1) = 0.490^{-0.089}_{+0.076}\,.$$

Coefficients $c_{k\geq 3} = d_k - \delta_k$ include "π^2 structures" δ_k proportional to lower c_k:

$$\delta_3 = \frac{(\pi\beta_0(f))^2\,c_1}{3}\,, \quad \delta_4 = (\pi\beta_0)^2\,(c_2 + \frac{5}{6}\,b_1\,c_1)\,; \; \pi^2\beta_0^2(4 \pm 1) = 4.340^{-.666}_{+.723}\,. \tag{2}$$

These structures δ_k arise[4, 5, 6, 7] in the course of analytic continuation from the Euclidean to Minkowskian region. Coefficients d_k should be treated as a genuine kth–order ones. Just they have to be calculated with the help of relevant Feynman diagrams.

To illustrate, consider the three–flavor case for τ–decay, $f = 4\,,5$ cases for $e^+e^- \to$ hadron annihilation and Z_0 decay (with $f = 5$) — see Table 1 in which we also give values for the π^2–terms.

Table 1

Process	f	c_1	$c_2 = d_2$	c_3	$d_3 = c_3 - \delta_3$	δ_3	δ_4
τ decay	3	$1/\pi$	.526	0.852	1.389	0.537	5.01
e^+e^-	4	.318	.155	-0.351	0.111	0.462	2.451
e^+e^-	5	.318	.143	-0.413	-0.023	0.390	1.752
Z_0 decay	5	.318	.095	-0.483	-0.094	0.390	1.576

Here, all coefficients c_k, d_k and δ_k, due to normalization (1), are of an order of unity. One can see that, in the high energy region, contribution of δ_3 prevails in c_3.

2 Preliminary quantitative estimate

In practice, the π^2–terms often dominate in higher expansion coefficients. This effect is especially strong in the $f = 5$ region. Meanwhile, just in this region people often use the so-called NLLA approximation, that is the two-term representation

$$O(s) = C_1(\bar{\alpha}_s/\pi) + C_2(\bar{\alpha}_s/\pi)^2 \tag{3}$$

for an observable $O(s)$ when next, the three-loop, coefficient C_3 is not known. This is the case, e.g., with event–shape[8] analysis.

On the basis of the numerical estimates of Table 1, in such a case, we recommend to use the three-term expression

$$O_3^{\Delta}(s) = d_1\left\{\bar{\alpha}_s - \frac{\pi^2\beta_0^2}{3}\bar{\alpha}_s^3\right\} + d_2\bar{\alpha}_s^2 = c_1\,\bar{\alpha}_s + c_2\bar{\alpha}_s^2 - \underline{\delta_3\,\bar{\alpha}_s^3} \tag{4}$$

i.e., to take into account the known predominant π^2 part of the next coefficient c_3. As it follows from the comparison of the last expression with the previous, two–term one, the $\bar{\alpha}_s$ numerical value extracted from eq.(4), for the same measured value O_{obs}, will differ by a positive quantity (e.g., in the $f = 5$ region with $\bar{\alpha}_s \simeq 0.12 \div 0.15$)

$$(\Delta\bar{\alpha}_s)_3 = \left.\frac{\pi\delta_3\,\bar{\alpha}_s^3}{1 + 2\pi d_2\bar{\alpha}_s}\right|_{20\div100\mathrm{GeV}}^{f=5} = \frac{1.225\,\bar{\alpha}_s^3}{1 + 0.90\,\bar{\alpha}_s} \simeq 0.002 \div 0.003$$

that turns to be numerically important.

Moreover, in the $f=4$ region, where the three-loop approximation is commonly used in the data analysis, the π^2 term δ_4 of the next order turns out also to be essential. Hence, we propose to use the four-term expression

$$O_4^{\Delta}(s)=d_1\,\bar{\alpha}_s+d_2\,\bar{\alpha}_s^2+c_3\,\bar{\alpha}_s^3-\underline{\delta_4\,\bar{\alpha}_s^4}\,;\quad c_3=d_3-\delta_3 \tag{5}$$

(instead of the three-term one (1)) that is equivalent to

$$O_4^{\Delta}(s)=d_1\left\{\bar{\alpha}_s-\frac{\pi^2\beta_0^2}{3}\bar{\alpha}_s^3-b_1\frac{5}{6}\pi^2\beta_0^2\,\bar{\alpha}_s^4\right\}+d_2\left\{\bar{\alpha}_s^2-\pi^2\beta_0^2\,\bar{\alpha}_s^4\right\}+d_3\,\bar{\alpha}_s^3 \tag{6}$$

with δ_3 and δ_4 defined[4, 7] in eq.(2).

The three– and two–term structures in curly brackets are related to specific expansion functions $\tilde{\alpha}$ and $\mathfrak{A}$ defined below (10) and entering into the non-power expansion (11).

To estimate roughly the numerical effect of using this last modified expression (5), we take the case of e^+e^- inclusive annihilation. For $\sqrt{s}\simeq 3\div 5$ GeV with $\bar{\alpha}_s\simeq 0.28\div 0.22$ one has

$$(\Delta\bar{\alpha}_s)_4=\frac{\pi\delta_4\,\bar{\alpha}_s^4}{1+2\pi d_2\bar{\alpha}_s}\Bigg|_{3\div 5\mathrm{GeV}}^{f=4}=\frac{1.07\,\bar{\alpha}_s^4}{1+0.974\,\bar{\alpha}_s}\simeq 0.005\div 0.002$$

— an important effect on the level of ca $1\div 2\%$.

Moreover, the $(\Delta\bar{\alpha}_s)_4$ correction turns out to be noticeable even in the lower part of the $f=5$ region! Indeed, at $\sqrt{s}\simeq 10\div 40$ GeV with $\bar{\alpha}_s\simeq 0.20\div 0.15$ we have

$$(\Delta\bar{\alpha}_s)_4\big|_{10\div 40\ \mathrm{GeV}}^{f=5}\simeq 0.71\,\bar{\alpha}_s^4\simeq(1.1\div 0.3)\cdot 10^{-3}\quad(\lesssim 0.5\%)\,.$$

3 Non-power expansion in the Minkowskian region

The so–called π^2 terms in the s–channel perturbative expansions for the invariant coupling and observables have a simple origin.

As it is well known, the usual invariant coupling originally defined [9] in terms of real constants z_i, counter-terms of finite Dyson renormalization transformation, can be expressed via a product of dressed symmetric vertex and propagator amplitudes taken at space-like values of their arguments.

$$\bar{\alpha}(Q^2,\alpha)=\alpha\Gamma^2(Q^2,\alpha)\prod_i d_i(Q^2,\alpha)\,.$$

Hence, by construction, it is a real function defined in the Euclidean region.

Transition to the time-like region, with logs branching $\ln Q^2\to\ln s-i\pi$ transforms all relevant amplitudes into complex functions $\Gamma(s,\alpha), d_i(s,\alpha)$. Here, the problem of appropriate defining of effective coupling in the time-like domain arises.

For this goal, we shall follow the idea devised in the early 80s by Radyushkin [4] and Krasnikov–Pivovarov [5]. There, an integral transformation $\mathbf{R}$ reverse to the dipole representation for the Adler function has been used.

We propose to treat this representation as an integral operation

$$R(s)\to D(z)=Q^2\int_0^\infty\frac{ds}{(s+z)^2}R(s)\equiv\mathbf{D}\left\{R(s)\right\} \tag{7}$$

transforming a function $R(s)$ of a real positive (time-like) argument into a function $D(z)$ given in the cut complex plane with analytic properties equivalent to those following from the Källen–Lehmann integral representation. In particular, the function $D(Q^2)$ is real on the positive (space-like) real axis at $z=Q^2+i0\,;Q^2\geq 0$.

The reverse operation is expressible in the form of a contour integral

$$R(s)=\frac{i}{2\pi}\int_{s-i\varepsilon}^{s+i\varepsilon}\frac{dz}{z}D_{\mathrm{pt}}(-z)\equiv\mathbf{R}\left[D(Q^2)\right]\,.$$

With the help of the latter, one can define[11, 12] an effective invariant time-like coupling $\tilde{\alpha}(s)=\mathbf{R}\left[\bar{\alpha}_s(Q^2)\right]$. Omitting some technical details, we give a few resulting[4, 5, 12] expressions.

E.g., starting with one–loop $\bar{\alpha}_s^{(1)}=[\beta_0\ln(Q^2/\Lambda^2)]^{-1}$ one has $\mathbf{R}\left[\bar{\alpha}_s^{(1)}\right]$

$$\tilde{\alpha}^{(1)}(s)=\frac{1}{\beta_0}\left[\frac{1}{2}-\frac{1}{\pi}\arctan\frac{L}{\pi}\right]_{L>0}=\frac{1}{\beta_0\pi}\arctan\frac{\pi}{L}\,;\quad L=\ln\frac{s}{\Lambda^2}\,. \tag{8}$$

At the same time, to $\left(\bar{\alpha}_s^{(1)}(Q^2)\right)^2$ and $\left(\bar{\alpha}_s^{(1)}(Q^2)\right)^3$ there correspond

$$\mathfrak{A}_2^{(1)}(s)\equiv\mathbf{R}\left[\left(\bar{\alpha}_s^{(1)}\right)^2\right]=\frac{1}{\beta_0^2\left[L^2+\pi^2\right]}\quad\text{and}\quad\mathfrak{A}_3^{(1)}(s)=\frac{L}{\beta_0^3\left[L^2+\pi^2\right]^2}\,.$$

In the two–loop case, for a "popular" expression

$$\beta_0\bar{\alpha}_{s,pop}^{(2)}(Q^2)=\frac{1}{l}-b_1(f)\frac{\ln l}{l^2}\,;\quad l=\ln\frac{Q^2}{\Lambda^2}$$

one obtains[4] the two-loop "pop" effective s-channel coupling

$$\tilde{\alpha}^{(2)}_{pop}(s) = \left(1 + \frac{b_1 L}{L^2+\pi^2}\right)\tilde{\alpha}^{(1)}(s) - \frac{b_1}{\beta_0}\frac{\ln\left[\sqrt{L^2+\pi^2}\right]+1}{L^2+\pi^2}. \qquad (9)$$

Both the expressions (8) and (9) are monotonically decreasing with a finite IR $\tilde{\alpha}(0) = 1/\beta_0(f=3) \simeq 1.4$ value. Meanwhile, higher functions go to the zero $\mathfrak{A}_k(0) = 0$ at the IR limit.

In the case $L \gg \pi$, it is possible to expand $\tilde{\alpha}$ and $\mathfrak{A}_k$ in powers of π^2/L^2. Then functions $\tilde{\alpha}$ and $\mathfrak{A}_2$ can be presented as expansions in powers of common $\bar{\alpha}_s \simeq 1/L$. They correspond to curly brackets in (6).

In [4, 5], as a starting point for observables in the Euclidean, i.e., space-like domain $Q^2 > 0$, the perturbation series

$$D_{\rm pt}(Q^2) = 1 + \sum_{k\geq 1} d_k\,\bar{\alpha}_s^k(Q^2)$$

has been assumed. It contains powers of usual, RG summed, invariant coupling $\bar{\alpha}_s(Q^2)$ that obeys unphysical singularities in the infrared (IR) region around $Q^2 \simeq \Lambda_3^2$.

By using the $\mathbf{R}$ transformation, we obtain in the Minkowskian region the "transformed" expansion over a non-power set of functions

$$R_\pi(s) \equiv \mathbf{R}\left[D_{\rm pt}(Q^2)\right] = 1 + \sum_{k\geq 1} d_k \mathfrak{A}_k(s)\,; \qquad \mathfrak{A}_k(s) = \mathbf{R}\left[\bar{\alpha}_s^k(Q^2)\right] \qquad (10)$$

free of the mentioned singularities. Properties of these functions have been analyzed in detail in our previous paper[13] — see also Ref. [14]. For a more detailed numerical information on the functions $\tilde{\alpha}$, $\mathfrak{A}_2$ and $\mathfrak{A}_3$ see Ref.[15].

Here, we give condensed information that will be enough for a few illustrations.

Table 2

Three-loop APT results for $\Lambda^{(5)}_{\overline{\rm MS}} = 290$ GeV; $\bar{\alpha}_s(M_z^2) = 0.125$

$\sqrt{s}$/GeV	5	10	15	20	30	50	60	90	150
$\bar{\alpha}_s(s)$	.235	.195	.177	.165	.153	.137	.133	.125	.115
$\tilde{\alpha}(s)$	.221	.186	.170	.160	.148	.136	.132	.123	.114
$10\mathfrak{A}_2$	.456	.330	.275	.246	.214	.180	.169	.149	.129
$100\mathfrak{A}_3$	.871	.555	.436	.357	.299	.232	.213	.177	.143

Both in the Figure 1 and in Table 2, we give 3-loop solutions for $\bar{\alpha}_s$ as well as for the modified, so-called *global* (for detail, see paper [13]) functions $\tilde{\alpha} = \mathfrak{A}_1$, $\mathfrak{A}_2$ and $\mathfrak{A}_3$ calculated within the $\overline{\rm MS}$ scheme for the cases $\Lambda_{(5)} = 215\,{\rm GeV}$, $\bar{\alpha}_s(M_Z^2) = 0.118$ and $\Lambda_{(5)} = 290\,{\rm GeV}$, $\bar{\alpha}_s(M_Z^2) = 0.125$.

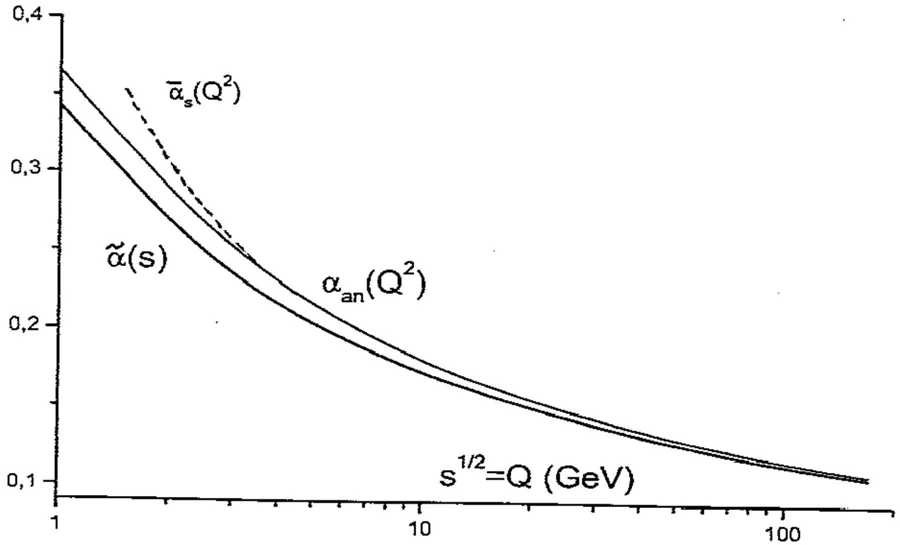

Figure 1: *Effective global Minkowskian, $\tilde{\alpha}$, and Euclidean, $\alpha_{\rm an}$ expansion functions, as compared with the standard one $\bar{\alpha}_s$ (at $\Lambda_{(5)} = 350\,{\rm MeV}$ and $\bar{\alpha}_s(M_Z^2) = 0.118$).*

We have chosen these two cases as limiting ones as far as in many practical cases real figures lie between these limits.

In the first figure we give three curves $\bar{\alpha}_s$, $\tilde{\alpha}$ and $\alpha_{\rm an}$ related to the same physical case for $\Lambda_3 = 350\,{\rm MeV}$ and $\bar{\alpha}_s(M_Z^2) = 0.118$. The curves $\tilde{\alpha}$ and $\alpha_{\rm an}$ on the figure go a bit slanting than usual, the $\bar{\alpha}_s$, dotted curve. This is quite natural, as they both are regular in the vicinity of the Λ singularity.

Meanwhile, only two first, $\tilde{\alpha}$ and $\alpha_{\rm an}$ have direct physical meaning (compare with conclusion of [13]). Just their values have to be determined from any given experiment. Nevertheless, in the four- and five-flavour regions one can still refer to $\bar{\alpha}_s$ and $\bar{\alpha}_s(M_Z^2)$ as to traditional theoretical objects.

Now, instead of (1), with due account to (10), we have

$$r(s)=\frac{\tilde{\alpha}(s)}{\pi}+d_2\,\mathfrak{A}_2(s)+d_3\,\mathfrak{A}_3(s) \tag{11}$$

with beautifully decreasing coefficients d_k. Just this nonpower expansion, strictly speaking, should be used instead of its approximations, eqs.(4) and (6), for data analysis in the time-like region.

At the same time, in the Euclidean, we have also non-power expansion

$$d(Q^2)=\frac{\alpha_{\rm an}(Q^2)}{\pi}+d_2\,\mathcal{A}_2(Q^2)+d_3\,\mathcal{A}_3(Q^2) \tag{12}$$

that can be related to (11) by transformation (7) in the framework of Invariant Analytic Approach (refs.[16, 17]).

These non-power expansions, free of unphysical singularities, jointly form a correlated system. The latter has been studied in detail in Refs.[13] and [18]. We call it Analytic Perturbation Theory (APT).

4 Numerical illustrations

To illustrate, let us start with a few cases in the $f=5$ region.

To begin with, consider the Υ decay. According to the Particle Data Group (PDG) overview (see their Fig.9.1 on page 88 of Ref.[1]), this is (with $\bar{\alpha}_s(M_\Upsilon)\simeq 0.170$ and $\bar{\alpha}_s(M_Z^2)=0.114$) one of the most "annoying" points of their summary of $\bar{\alpha}_s(M_Z^2)$ values. It is also singled out theoretically. The expression for the ratio of decay widths starts with the cubic term

$$R(\Upsilon)=R_0\,\bar{\alpha}_s^3(M_\Upsilon)(1+e_1\,\bar{\alpha}_s)\quad\text{with}\quad e_1\simeq 1\,. \tag{13}$$

Due to this, the π^2 correction[1] is rather big here

$$\mathfrak{A}_3\simeq\bar{\alpha}_s^3\left(1-2(\pi\beta_0)^2\bar{\alpha}_s^2\right)\,. \tag{14}$$

[1]First proposal of taking into account this effect in the Υ decay was discussed[5] more than a quarter of century ago. Nevertheless, in current practice it is neglected.

Accordingly,

$$\Delta\bar{\alpha}_s(M_\Upsilon)=\frac{2}{3}\,(\pi\beta_0)^2\,\bar{\alpha}_s^3(M_\Upsilon)\simeq 0.0123\,,$$

that corresponds to

$$\Delta\bar{\alpha}_s(M_Z)=0.006\quad\text{with}\quad\bar{\alpha}_s(M_Z)=0.120\,. \tag{15}$$

Now, let us turn to a few cases analyzed by the three-term expansion formula (1). For the first example, take e^+e^- hadron annihilation at $\sqrt{s}=$ 42 GeV and 11 GeV.

A common form (see, e.g., Eq.(15) in Ref.[2]) of theoretical presenting of the QCD correction in our normalization looks like

$$r_{e^+e^-}(s)=0.318\bar{\alpha}_s(s)+0.143\,\bar{\alpha}_s^2-0.413\,\bar{\alpha}_s^3\,. \tag{16}$$

Starting with $r_{e^+e^-}(42)\simeq 0.0476$, one has $\bar{\alpha}_s(42)=0.144$. Along with our new philosophy, one should use instead

$$r_{e^+e^-}(s)=0.318\,\tilde{\alpha}(s)+0.143\,\mathfrak{A}_2(s)-0.023\,\mathfrak{A}_3(s) \tag{17}$$

that yields $\tilde{\alpha}(42)=0.142$ with $\bar{\alpha}_s(42)=0.145$ and $\bar{\alpha}_s(M_Z^2)=0.127$ to be compared with $\bar{\alpha}_s(M_Z^2)=0.126$ under a usual analysis.

Quite analogously, for $r_{e^+e^-}(11)\simeq 0.0661$; $\bar{\alpha}_s(11)=0.200$, we obtain $\tilde{\alpha}(10)=0.190$ that corresponds to $\bar{\alpha}_s(M_Z^2)=0.129$ instead of 0.130.

For the next example, we take the Z_0 inclusive decay. Experimental ratio $R_Z=\Gamma(Z_0\to hadrons)/\Gamma(Z_0\to leptons)=20.783\pm.029$ is usually presented as follows: $R_Z=R_0\,(1+r_Z(M_Z^2))$ with $R_0=19.93$. A common form (see, e.g., Eq.(15) in Ref.[2]) of presenting of the QCD correction in our normalization looks like

$$r_Z(M_Z^2)=0.3326\bar{\alpha}_s+0.0952\,\bar{\alpha}_s^2-0.483\,\bar{\alpha}_s^3\,.$$

To $[r_Z]_{obs}=0.04184$ there corresponds $\bar{\alpha}_s(M_Z^2)=0.1241$ with $\Lambda^{(5)}_{\overline{\rm MS}}=$ 292 MeV. In the APT case,from

$$r_Z(M_Z^2)=0.3326\,\tilde{\alpha}(M_Z^2)+0.0952\,\mathfrak{A}_2(M_Z^2)-0.094\,\mathfrak{A}_3(M_Z^2) \tag{18}$$

we obtain $\tilde{\alpha}(M_Z^2)=0.122$ and $\bar{\alpha}_s(M_Z^2)=0.124$ that relates to $\Lambda^{(5)}=$ 290 MeV. Note that here the three-term approximation of (6) gives the same relation between the $\bar{\alpha}_s(M_Z^2)$ and $\tilde{\alpha}(M_Z^2)$ values.

Nevertheless, in accordance with our preliminary estimate for the $(\Delta\bar{\alpha}_s)_4$ role, even the so-called NNLO theory needs some π^2 correction in the $W = \sqrt{s} \lesssim 50$ GeV region.

Now, turn to the experiments in the HE Minkowskian (mainly with a shape analysis) that usually are confronted with two-term expression (3). As it has been shown above, the main theoretical error in the $f = 5$ region can be expressed in the form

$$(\Delta\bar{\alpha}_s(s)|^{f=5}_{20\div 100\mathrm{GeV}} \simeq 1.225\,\bar{\alpha}_s^3(s) \simeq 0.002 \div 0.003\,. \qquad (19)$$

An adequate expression for the shift of an equivalent $\bar{\alpha}_s(M_Z^2)$ value is

$$[\Delta\bar{\alpha}_s(M_Z^2)]_3 = 1.225\bar{\alpha}_s(s)\bar{\alpha}_s(M_Z^2)^2\,. \qquad (20)$$

Table 3

The APT revised[a] part ($f = 5$) of Bethke's[2] Table 6

Process	$\sqrt{s}$ GeV	loops No	$\bar{\alpha}_s$ (s) ref.[2]	$\bar{\alpha}_s(m_z^2)$ ref.[2]	$\bar{\alpha}_s$ (s) APT	$\bar{\alpha}_s(m_z^2)$ APT
Υ-decay [b]	9.5	2	.170	.114	.182	.120 (+6)
$e^+e^-[\sigma_{had}]$	10.5	**3**	.200	.130	.198	.129(**-1**)
$e^+e^-[j\ \&\ sh]$	22.0	2	.161	.124	.166	.127(+3)
$e^+e^-[j\ \&\ sh]$	35.0	2	.145	.123	.149	.126(+3)
$e^+e^-[\sigma_{had}]$	42.4	**3**	.144	.126	.145	.127(**+1**)
$e^+e^-[j\ \&sh]$	44.0	2	.139	.123	.142	.126(+3)
$e^+e^-[j\ \&sh]$	58	2	.132	.123	.135	.125(+2)
$Z_0 \to$ **had.**	91.2	**3**	.124	.124	.124	.124 (**0**)
$e^+e^-[j\ \&\ sh]$	91.2	2	.121	.121	.123	.123(+2)
$e^+e^-[j\ \&\ sh]$	133	2	.113	.120	.115	.122(+2)
$e^+e^-[j\ \&\ sh]$	161	2	.109	.118	.111	.120(+2)
$e^+e^-[j\ \&\ sh]$	172	2	.104	.114	.105	.116(+2)
$e^+e^-[j\ \&\ sh]$	183	2	.109	.121	.111	.123(+2)
$e^+e^-[j\ \&\ sh]$	189	2	.110	.123	.112	.125(+2)

Averaged $< \bar{\alpha}_s(M_Z^2) >_{f=5}$ values 0.121; 0.124;

[a] "j & sh" = jets and shapes; Figures in brackets in the last column give the difference $\Delta\bar{\alpha}_s(M_Z^2)$ between common and APT values.

[b] Taken from Ref.[1].

We give results of our approximate APT calculations, mainly by Eqs.(19) and (20), in the form of Table 3 and Figure 2. At the last column of the Table 3 in brackets we indicate difference between the APT and usual analysis. By bold figures the results of the three-loop analysis are singled out.

Let us note that our average over events from Table 6 of Bethke's review [2] nicely correlates with recent data of the same author (see Summary of Ref.[19]). The best χ^2 fit yields $\bar{\alpha}_s(M_Z^2)_{[2]} = 0.1214$ and $\bar{\alpha}_s(M_Z^2)_{APT} = 0.1235$. This gives minimum $\chi^2_{[2]} = 0.197$ and $\chi^2_{APT} = 0.144$ with impressive ratio ($\simeq 0.73$) illustrating the effectiveness of the APT procedure.

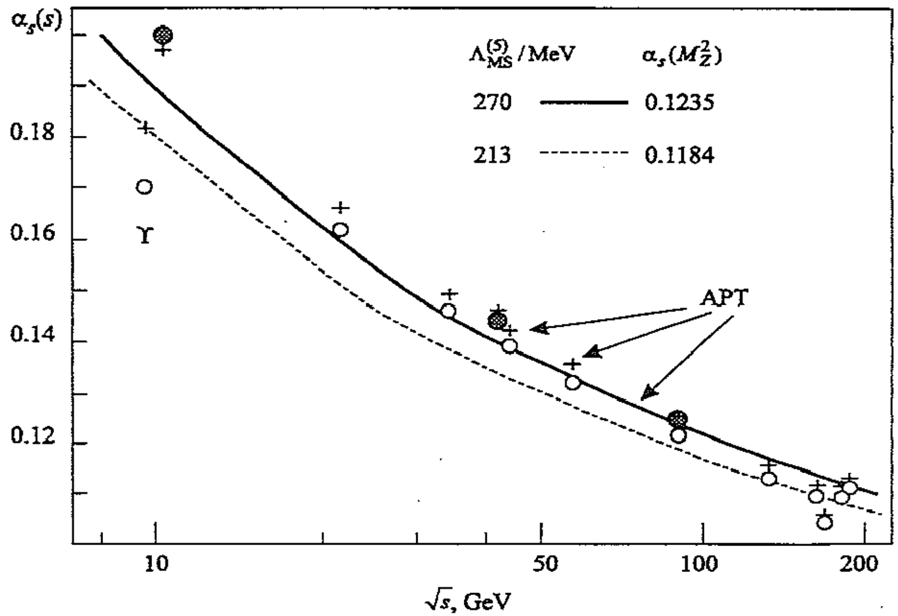

Figure 2: The new APT analysis for $\bar{\alpha}_s$ in the five-flavour time-like region. Crosses (+) differ from circles (∘, •) by π^2 correction (19). Solid APT curve relates to $\Lambda^{(5)}_{\overline{MS}} = 270$ MeV and $\bar{\alpha}_s(M_Z^2) = 0.124$. To compare, we give also the standard (dot-and-dash curve) $\bar{\alpha}_s$ (at $\Lambda^{(5)} = 213$ MeV and $\bar{\alpha}_s(M_Z^2) = 0.118$) taken from Fig.10 of paper [2].

On the Fig.2 by open circles and bullets (∘, •) we give two- and three-loops data mainly from Fig.10 of paper [2]. The only exclusion is the Υ

decay taken from the Table 6 of the same paper. By crosses we marked the new "APT values" calculated approximately mainly with help of Eq.(19).

For clearness of the π^2 effect, we skipped the error bars. They are the same as in the Bethke's figure and we used them for calculating χ^2.

5 Conclusion

We have established a few qualitative effects:

1. Effective positive shift $\Delta\bar{\alpha}_s = +0.002$ in the upper half ($\geq 50\,\mathrm{GeV}$) of the $f = 5$ region for all time-like events that have been analyzed up to now in the NLO mode.

2. Effective shift $\Delta\bar{\alpha}_s \simeq +0.003$ in the lower half ($10 \div 50\,\mathrm{GeV}$) of the $f = 5$ region for all time-like events that have been analyzed in the NLO modes.

3. The new value

$$\bar{\alpha}_s(M_Z^2) = 0.124 \tag{21}$$

by averaging over the $f = 5$ region.

These results are based on a plausible hypothesis on the "π^2 – terms" prevalence in expansion coefficients for observable in the Minkowskian domain. The hypothesis has some preliminary support but needs to be checked in a more detail.

Nevertheless, our result (21) being taken as granted, rises two physical questions:

– The issue of self-consistency of QCD invariant coupling behavior between the "medium ($f = 3, 4$)" and "high ($f = 5, 6$)" regions.

– The new "enlarged value" (21) can influence various physical speculations in the several hundred GeV region.

Acknowledgements

The author is indebted to D.Yu. Bardin, N.V. Krasnikov, B.A. Magradze, S.V. Mikhailov, A.V. Radyushkin, I.L. Solovtsov and O.P. Solovtsova for useful discussions and comments. This work was partially supported by grants of the Russian Foundation for Basic Research (RFBR projects Nos 99-01-00091 and 00-15-96691), by INTAS grant No 96-0842 and by INTAS-CERN grant No 2000-377.

References

[1] D.E.Groom *et al.*, *European Phys. J.* **C 15** (2000) 1.

[2] S. Bethke, "Determination of the QCD coupling α_s", *J. Phys.* **G 26** R27; Preprint MPI-PhE/2000-07, April 2000; hep-ex/0004021.

[3] D.Yu. Bardin, G. Passarino, The Standard model in the making, Clarendon Press, Oxford, 1999.

[4] A. Radyushkin, Dubna JINR preprint E2-82-159 (1982); see also *JINR Rapid Comm.* No.4[78]-96 (1996) pp 9-15 and hep-ph/9907228.

[5] N.V. Krasnikov, A.A. Pivovarov, *Phys. Lett.* **116 B** (1982) 168–170.

[6] J.D. Bjorken, "Two topics in QCD", Preprint SLAC-PUB-5103 (Dec 1989); in Proc. Cargese Summer Institute, eds. M. Levy et al., Nato Adv. Inst., Serie B, vol. 223, Plenum, N.Y., 1990.

[7] A.L. Kataev and V.V. Starshenko, *Mod. Phys. Lett. A* **19** (1995) 235-250.

[8] Delphi Collaboration, " Consistent measurements of $\bar{\alpha}_s$ from ...", CERN-EP/99-133 (Sept 1999).

[9] N.N. Bogoliubov and D.V. Shirkov, *Doklady AN SSSR*, **103** (1955) 203-206 (in Russian); also *Nuovo Cim.* **3** (1956) 845-637; *Sow. Phys. JETP* **3** (1956) 57–64 and Chapter "Renormalization group" in [10].

[10] N.N. Bogoliubov and D.V. Shirkov, Introduction to the theory of Quantized Fields, Wiley & Intersc. N.Y., 1959 and 1980.

[11] H.F. Jones and I.L. Solovtsov, *Phys. Let.* **B 349** (1995) 519-525.

[12] K.A. Milton and I.L. Solovtsov, *Phys. Rev.* **D 55** (1997) 5295-5298.

[13] D.V. Shirkov, "Toward the correlated analysis of observables in perturbative QCD", JINR preprint E2-2000-46; hep-ph/0003242.

[14] D.V. Shirkov, "Analytic Perturbation Theory for QCD observables", JINR preprint E2-2000-298; hep-ph/0012283

[15] B. Magradze, "The QCD coupling up to third order in standard and analytic perturbation theories", Preprint JINR, E2-2000-222; hep-ph/0010070;

[16] D.V. Shirkov and I.L. Solovtsov, *Phys.Rev.Lett.* **79** (1997) 1209-12; hep-ph/9704333.

[17] I.L. Solovtsov and D.V. Shirkov, *Theor. Math. Phys.* **120** (1999) 1210-1243; hep-ph/9909305.

[18] D.V. Shirkov, "Analytic Perturbation Theory for QCD observables", JINR preprint E2-2000-296; hep-ph/0012283.

[19] S. Bethke, "Standard Model Physics at LEP", hep-ex/0001023 and "QCD at LEP", (Talk at LEP Fest on Oct 11, 2000).

Received by Publishing Department
on September 8, 2000.