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**L.G.Zastavenko**

**SIMPLIFIED RENORMALIZATION  
THEORY (THE  $g\varphi^4$  MODEL).  
RENORMALIZATION IDENTITIES**

**1979**

Заставенко Л.Г.

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Упрощенное изложение теории перенормировок на примере модели  $g\phi^4$ . Перенормировочные тождества

Излагается упрощенный вариант теории перенормировок /на примере модели  $g\phi^4$ /. Упрощение достигнуто за счет 1/ рассмотрения рядов теории возмущений в терминах полного пропагатора, а не голого пропагатора как это делается обычно; 2/ применения R-операции Боголюбова-Парасюка к таким рядам; это применение дает перенормировочные тождества, которые и являются ключом к теории перенормировок. Например, для четырех-хвостки ( $\kappa = 12g$ )

$$G_4(p, g, \ell; G) = -2\kappa + 6\kappa^2 \text{ (diagram)} - 6\kappa^3 \text{ (diagram)} - 24\kappa^3 \text{ (diagram)} + \dots = \sum_{n,k} x^{nk} Q_{4nk} G_{4nk}(p, \ell; G)$$

/здесь каждой внутренней линии соответствует полный пропагатор  $G$ ,  $p$  - импульсы,  $g$  - константа связи,  $Q_{4nk}$  - диаграмма,  $Q_{4nk}$  - численный множитель при ней/ перенормировочное тождество имеет вид

$$G_4(p, g, \ell; G) = \sum_{n,k} [-G_4(\xi, g, \ell; G)/2]^{nk} Q_{4nk} R(\xi) G_{4nk}(p, \ell; G).$$

Здесь  $\xi$  - совокупность четырех произвольных 4-импульсов,  $R(\xi)$  - R-операция Боголюбова-Парасюка.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований, Дубна 1979

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Zastavenko L.G.

Simplified Renormalization Theory (the  $g\phi^4$  Model). Renormalization Identities

The article contains simplified version of the renormalization theory. It is based on the use of the renormalization identities of the form (19), (19a), (22). We start from the integral equation (10) for the full propagator; the higher-order Green functions are represented through the full propagator.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1979

### Sect.1. Introduction

The difficulty of divergences in quantum field theory gives rise to the renormalization theory, which has been developed by Dyson, Bogolubov and other authors. It is necessary to note that the present-day renormalization theory as given, e.g., in the book by Bogolubov and Shirkov /1/ remains rather complicated.

1.1. This article contains a simplified formulation of the renormalization theory. We begin with the equation (10) for the full propagator and formulae like (15), which define higher-order Green functions in terms of full propagator. We apply the Bogolubov-Parasjuk R-operation to the series in the right-hand-side of these equations. (In the book by Bogolubov and Shirkov the R-operation is applied to an analogous series in terms of the bare propagator). The R-operation allows us to get the renormalization identities (19), (19a), (22), which are the key to all the renormalization procedure.

We suppose the proof of the renormalization identities to be contained in the material of the book by Bogolubov and Shirkov /1/, so we only check these identities up to the terms of order  $g^4$  (see Appendix).



1.2. It is convenient to introduce the physical mass into eq. (10), (10a).

1.3. Later on we transform eq. (10a) with the help of eqs. (26)-(30) into eq. (32) for the renormalized propagator (30). Analogously, eq. (15) is transformed into eq. (33) for the renormalized (4)-point Green function.

1.4. We accept without proof that the renormalized Green functions contain no divergences <sup>[2]</sup>.

### Sect. 2. Some Useful Formulae

2.1. We restrict our consideration to the model  $g\varphi^4$  in four-dimensional Euclidean space-time. This model is defined by the Lagrangian

$$\mathcal{L} = -\frac{1}{2} \sum_{\alpha=1}^4 \left( \frac{\partial \tilde{\varphi}}{\partial x_{\alpha}} \right)^2 - \frac{1}{2} M^2 \tilde{\varphi}^2 - (2\pi)^4 g \tilde{\varphi}^4. \quad (1)$$

We use the Feynman formulation of quantum field theory. The Green functions  $\Gamma_n(p_1, \dots, p_n)$  are defined through the equation

$$\begin{aligned} \Gamma_n(p_1, \dots, p_n) \delta(p_1 + \dots + p_n) &= \\ &= \hat{C} \int (\prod_i \varphi(p_i)) e^{-S} \delta\varphi / \int e^{-S} \delta\varphi. \end{aligned} \quad (2)$$

Here we have included the operator  $\hat{C}$  in order to take only the connected part of the expression on the right of  $\hat{C}$  : e.g.,

$$\begin{aligned} \int (\prod_i \varphi(p_i)) e^{-S} \delta\varphi / \int e^{-S} \delta\varphi &= \delta(p_1 + \dots + p_n) \Gamma_1(p_1, \dots, p_n) \\ &+ \delta(p_1 + p_2) \delta(p_3 + p_4) \Gamma_2(p_1, p_2) \Gamma_2(p_3, p_4) + \dots \end{aligned} \quad (3)$$

$$\hat{C} \int (\prod_i \varphi(p_i)) e^{-S} \delta\varphi / \int e^{-S} \delta\varphi = \delta(p_1 + \dots + p_n) \Gamma_n(p_1, \dots, p_n) \quad (4)$$

The Action  $S$  in eq. (2) is the integral

$$S = \int \mathcal{L} d^4x, \quad (5)$$

where we have to introduce a cut-off  $\ell$  :

$$\begin{aligned} S &= \frac{1}{2} \int (p^2 + M^2) \varphi(p) \varphi(-p) dp \\ &\quad |p| < \ell \\ &+ g \int (\prod_i \varphi(p_i) d p_i) \delta(\sum_i p_i) \\ &\quad |p_i| < \ell \end{aligned} \quad (6)$$

here

$$\varphi(p) = (2\pi)^{-2} \int e^{i p x} \tilde{\varphi}(x) dx. \quad (7)$$

2.2. Using the equation

$$\int (\prod_i \varphi(p_i) d p_i) A(p_1, \dots, p_n) e^{-S_0} \delta\varphi / \int e^{-S_0} \delta\varphi \quad (8)$$

$$= \int \prod_i \frac{d p_i}{p_i^2 + M^2} [A(p_1 - p_1, p_2 - p_2, \dots, p_n - p_n) + A(p_1, p_2 - p_2, \dots, p_n - p_n) + \dots],$$

$$S_0 = \frac{1}{2} \int (p^2 + M^2) \varphi(p) \varphi(-p) dp, \quad (8a)$$

(... denotes  $(2n-1)!!$  -2 terms) one gets the Green function expansion in powers of  $g$ . Each term of this decomposition corres-

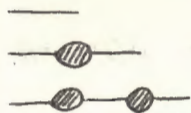


Fig. 1

ponds to some diagram. Summing up of diagrams, shown in fig. 1, gives the integral equation for the determination of propagator

$$\Gamma_2(p, -p) \quad . \text{ We denote}$$

$$\Gamma_2(p, -p) = G(p) \quad (9)$$

then

$$G(p)^{-1} = p^2 + M^2 + x \text{ (loop)} - \frac{2}{3} x^2 \text{ (loop)} + 2x^3 \text{ (loop)} - 2x^4 \text{ (loop)} - 4x^4 \text{ (loop)} - 4x^4 \text{ (loop)} + \dots$$

$$= p^2 + M^2 + \sum_{n, k} x^n Q_{2nk} G_{2nk} \quad (10)$$

Here

$$x = 12g \quad (11)$$

$Q_{2nk}$  are numerical factors,  $G_{2nk}$  diagrams. Each internal line of  $G_{2nk}$  corresponds to factor  $\int G(k) dk$ , each vertex (except of one) corresponds to factor  $\delta(\sum k_i)$ . It is convenient to introduce the physical mass  $m$  of the particle into eq. (10) and transform this eq. into

$$G(p, m, g, \ell)^{-1} = p^2 + m^2 + (1 - \hat{N}_m) \sum_{n, k} x^n Q_{2nk} G_{2nk}(p, m, \ell; g). \quad (10a)$$

Here the operator  $\hat{N}_m$  is defined by the formula<sup>1)</sup>

$$\hat{N}_m f(p^2) = f(-m^2). \quad (10b)$$

2.3. It is convenient to introduce instead of functions  $\Gamma_n$  (2) new functions  $G_n$  through the equation

$$\Gamma_n(p_1, \dots, p_n) = G_n(p_1, \dots, p_n) \prod_i G(p_i) \quad (14)$$

Analogously to eq. (10) one gets

$$G_4(p_1, p_2, p_3, p_4, m, g, \ell) = -2x + 6x^2 \text{ (loop)} - 6x^3 \text{ (loop)} - 24x^3 \text{ (loop)} + 6x^4 \text{ (loop)} + 12x^4 \text{ (loop)} + 24x^4 \text{ (loop)} + 96x^4 \text{ (loop)} + 24x^4 \text{ (loop)} + 24x^4 \text{ (loop)} + 16x^4 \text{ (loop)} + O(x^5)$$

$$= \sum_{n, k} x^n Q_{4nk} G_{4nk}(p, m, \ell; g) \quad (15)$$

<sup>1)</sup> One can write down eq. (10) in the form

$$G(p)^{-1} = p^2 + M^2 - 2 \frac{\delta F}{\delta G(p)}, \quad (12)$$

where

$$F = -\frac{x}{4} \text{ (loop)} + \frac{x^2}{12} \text{ (loop)} - \frac{x^2}{6} \text{ (loop)} + \frac{x^4}{8} \text{ (loop)} + \frac{x^4}{2} \text{ (loop)} - O(x^5). \quad (13)$$

Note also that one can get r.h.s. of eq. (15) by cutting off one vertex of  $F$ .



here  $Q_{4n\kappa}$  are numerical factors,  $G_{4n\kappa}$  diagrams. The diagrams in eq. (15) are invariant under all permutations of momenta  $p_1, p_2, p_3, p_4$  ( $p_1 + p_2 + p_3 + p_4 = 0$ ), e.g.:

$$\text{X} = \frac{1}{3} [\phi(p_1 + p_2) + \phi(p_1 + p_3) + \phi(p_1 + p_4)], \quad (16)$$

$$\text{D} = \frac{1}{6} \int d^4q d^4k_1 d^4k_2 G(k_1 + q + p_1) G(k_2 - q + p_2) \phi(q) \quad (17)$$

$$\begin{aligned} |p_1 + k_1 + q| < \ell \\ |p_2 + k_2 - q| < \ell \end{aligned}$$

$$\begin{aligned} &+ \frac{1}{6} [(p_1, p_2) \rightarrow (p_1, p_3)] + \frac{1}{6} [(p_1, p_2) \rightarrow (p_1, p_4)] \\ &+ \frac{1}{6} [(p_1, p_2) \rightarrow (p_2, p_3)] + \frac{1}{6} [(p_1, p_2) \rightarrow (p_2, p_4)] \\ &+ \frac{1}{6} [(p_1, p_2) \rightarrow (p_3, p_4)], \end{aligned}$$

$$\begin{aligned} \phi(q) &= \int d^4k_1 d^4k_2 G(k_1) G(k_2) \delta(k_1 + k_2 - q) \\ &|k_1| < \ell, \\ &|k_2| < \ell. \end{aligned} \quad (18)$$

### Sect. 3. Renormalization Identities

3.1. There exist the following identities

$$\begin{aligned} G_4(p, m, g, \ell) &= \sum_{n, \kappa} x^n Q_{4n\kappa} G_{4n\kappa}(p, m, \ell; G) \\ &= \sum_{n, \kappa} [-G_4(\xi, m, g, \ell/2)]^n Q_{4n\kappa} R(\xi) G_{4n\kappa}(p, m, \ell; G), \end{aligned} \quad (19)$$

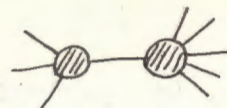


Fig. 2

$$\begin{aligned} \tilde{G}_{2i}(p, m, g, \ell) &= \sum_{n, \kappa} x^n Q_{2in\kappa} \tilde{G}_{2in\kappa}(p, m, \ell; G) \\ &= \sum_{n, \kappa} [-G_4(\xi, m, g, \ell/2)]^n Q_{2in\kappa} R(\xi) \tilde{G}_{2in\kappa}(p, m, \ell; G) \end{aligned} \quad (19a)$$

Here  $i \geq 2$ ,  $\xi$  is a set of four vectors  $\xi_1, \xi_2, \xi_3, \xi_4$ ,  $R(\xi)$  the Bogolubov - Parasjuk R-operation<sup>2)</sup>. The quantity  $\tilde{G}_{2i}$  in eq. (19a) is the strongly-connected part of  $G_{2i}$ , the diagrams  $\tilde{G}_{2in\kappa}$  are strongly-connected.

3.1.1. We stress eq. (19) to be valid only for values of  $Q_{2in\kappa}$  given in the eq. like (15).

3.2. In the case  $i=2$  eq. (19a) is correct only within a diagram  $\mathcal{D}$ . So we have  $x = 12g$  (21)

<sup>2)</sup> If the diagram  $\tilde{G}_{2in\kappa}$  is strongly connected (i.e., it does not have the form, shown in fig. 2) and does not contain self-energy parts then

$$R(\xi) \tilde{G}_{2in\kappa} \equiv \prod_j (1 - M_j) \tilde{G}_{2in\kappa}, \quad (20)$$

where the product is taken over all four-leg subdiagrams, which are contained in  $\tilde{G}_{2in\kappa}$  and  $M_j$  is the operator contracting the corresponding subdiagram into the point (see the book by Bogolubov and Shirkov, § 26.10)

$$\begin{aligned}
& (1 - \hat{N}_m) \sum_{n=2}^{\infty} \sum_K X^n Q_{2nK} G_{2nK}(p, m, l, \xi; \epsilon) = \\
& = (1 - \hat{N}_m) \sum_{n=2}^{\infty} \sum_K [-G_4(\xi, m, g, l)/2]^n Q_{2nK} \\
& R(\xi) G_{2nK}(p, m, l, \xi; \epsilon),
\end{aligned} \tag{22}$$

where the diagram  $\mathcal{Q}$  is cancelled out. Note that

$$R(\xi) \ominus = \ominus. \tag{23}$$

#### Sect. 4. Renormalization of the Propagator

4.1. Introduce the notation

$$\begin{aligned}
(1 - \hat{N}_m) R(\xi) G_{2nK}(p, m, l, \xi; \epsilon) & \equiv \\
& \equiv G_{2nK}(p, m, l, \xi; \epsilon).
\end{aligned} \tag{24}$$

Equation (10b) implies

$$G_{2nK}(p, m, l, \xi; \epsilon) \Big|_{p^2 = -m^2} = 0. \tag{25}$$

So one has

$$\begin{aligned}
G_{2nK}(p, m, l, \xi; \epsilon) & = (p^2 + m^2) K_{2nK}(m, l, \xi; \epsilon) \\
& + G'_{2nK}(p, m, l, \xi; \epsilon),
\end{aligned} \tag{26}$$

where<sup>3)</sup>

$$G'_{2nK}(p, m, l, \xi; \epsilon) = O((p^2 + m^2)^2) \tag{27}$$

at  $p^2 + m^2 \rightarrow 0$

4.2. Taking account of equations (10a), (22), (26) gives

$$\begin{aligned}
G(p, m, g, l)^{-1} & = (p^2 + m^2) Z(m, g, l, \xi; \epsilon) \\
& + \sum_{n=2}^{\infty} \sum_K [-G_4(\xi, m, g, l)/2]^n Q_{2nK} G'_{2nK}(p, m, l, \xi; \epsilon)
\end{aligned} \tag{28}$$

here

$$\begin{aligned}
Z(m, g, l, \xi; \epsilon) & = 1 + \sum_{n=2}^{\infty} \sum_K [-G_4(\xi, m, g, l)/2]^n Q_{2nK} \\
& K_{2nK}(m, l, \xi; \epsilon).
\end{aligned} \tag{29}$$

The function  $G'_{2nK}(p, m, l, \xi; \epsilon)$  contains  $(2n-1)$  internal lines with  $G$ ; So, introducing new functions  $G^r, \lambda$ ,

$$G^r(p, m, \lambda, \xi; l) \equiv G(p, m, g, l) Z(m, g, l, \xi; \epsilon), \tag{30}$$

$$\lambda = \lambda(m, g, l, \xi; \epsilon) \equiv -G_4(\xi, m, g, l) / (2Z^2(m, g, l, \xi; \epsilon)) \tag{31}$$

one can transform eq. (28) into the equation

<sup>3)</sup> It follows from the definition of the R-operation, that the function  $G_{2i}(p_i, m, l, \xi; \epsilon)$  does not depend on arguments  $(\xi_i, p_j)$ .



$$G^r(p, m, \lambda, \xi; l)^{-1} = p^2 + m^2 + \sum_{n=2}^{\infty} \sum_{\kappa} \lambda^n Q_{2n\kappa} G'_{2n\kappa}(p, m, l, \xi; G^r) \quad (32)$$

which is known not to give rise to infinities, (in the limit  $l \rightarrow \infty$ ) if the quantity  $\lambda$  is considered to be finite <sup>121</sup>.

The functions  $G^r$  and  $\lambda$  are, resp., the renormalized propagator and renormalized coupling constant;  $G^r$  has a finite limit as  $l \rightarrow \infty$  if  $\lambda$  is supposed to have a finite value.

### Sect. 5. Renormalization of the Function $G_{2i}$ .

The function  $R(\xi)G_{2i\kappa}$  in eq. (19a) contains  $2n-i$  internal lines with  $\xi$ , so, similarly to eq. (32), one gets the equation

$$\tilde{G}_{2i}^r(p, m, \lambda, \xi; l) \equiv \tilde{G}_{2i}(p, m, g, l) / Z^i(m, g, l, \xi; G)$$

$$= \sum_{n, \kappa} \lambda^n Q_{2i n \kappa} R(\xi) \tilde{G}_{2i n \kappa}(p, m, l; G^r) \quad (33)$$

defining the renormalized strongly-connected part of the function  $G_{2i}$  (14). The function  $\tilde{G}_{2i}^r$  has a finite limit as  $l \rightarrow \infty$ .

5.1. In order to renormalize the weakly connected part of the function  $G_{2i}$ , it is sufficient to know how the constituting it strongly connected parts and propagator are renormalized.

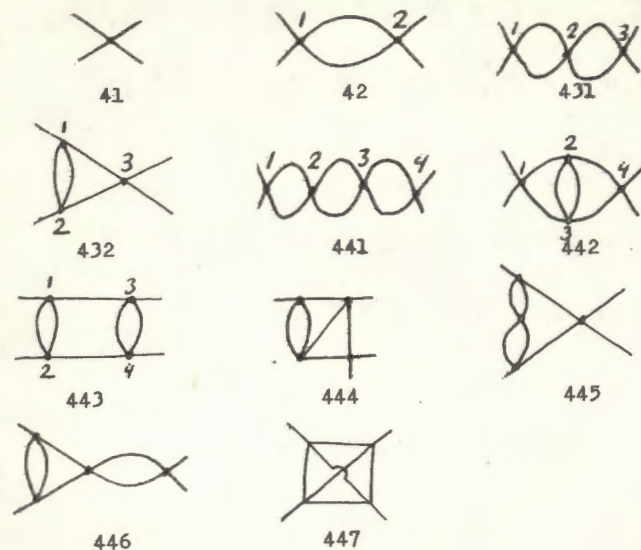


Fig. 3

### Appendix A

In this Appendix we check the identity (19) up to the order  $O(x^4)$ . The necessary diagrams are listed in eq. (15) and fig. 3; the numeration of the diagrams and vertices is shown in fig. 3.

We have

$$R(\xi)G_{41} = G_{41} = R_1; \quad (A.1)$$

$$R(\xi)G_{42} = (1 - M_{12})G_{42} = R_2,$$

$$G_{42} = \bigcirc R_1 + R_2; \quad \bigcirc = \begin{matrix} \xi_1 & & \xi_3 \\ & \diagdown & / \\ & \xi_2 & \\ & / & \diagdown \\ \xi_4 & & \xi_4 \end{matrix} \quad (A.2)$$

$$\begin{aligned}
R(\mathbb{E})G_{431} &= (1 - M_{123})(1 - M_{12})(1 - M_{23})G_{431} = \\
&= (1 - M_{123} - M_{12} - M_{23} + M_{123}(M_{12} + M_{23}) \\
&\quad + M_{12}M_{23} - M_{123}M_{12}M_{23})G_{431} = \\
&= G_{431} - \infty X - 2\circ X X + 2\circ^2 X \\
&\quad + \circ^2 X - \circ^2 X
\end{aligned}$$

(A.3a)

so that

$$G_{431} = \infty R_1 + 2\circ R_2 + R_{31}$$

(A.3)

$$\begin{aligned}
R(\mathbb{E})G_{432} &= (1 - M_{123})(1 - M_{12})G_{432} = \\
&= (1 - M_{123} - M_{12} + M_{123}M_{12})G_{432} = \\
&= G_{432} - \triangleright X - \circ X X + \circ^2 X
\end{aligned}$$

(A.4.a)

and

$$G_{432} = \triangleright R_1 + \circ R_2 + R_{32}$$

(A.4)

later on we consider the diagram  $G_{442}$ ; it contains four four-leg subdiagrams (23) (123) (234) (1234), so that

$$\begin{aligned}
R(\mathbb{E})G_{442} &= (1 - M_{1234})(1 - M_{123})(1 - M_{234})(1 - M_{23})G_{442} \\
&= (1 - M_{1234})(1 - M_{123} - M_{234})(1 - M_{23})G_{442} \\
&\quad (\text{for } (1 - M_{1234})M_{123}M_{234} = 0)
\end{aligned}$$

$$\begin{aligned}
&= \{1 - M_{1234} - M_{123} - M_{234} - M_{23} \\
&\quad + M_{1234}(M_{123} + M_{234} + M_{23}) + (M_{123} + M_{234})M_{23} \\
&\quad - M_{1234}(M_{123} + M_{234})M_{23}\}G_{442} = \\
&= G_{442} - \circ X - 2\triangleright X X - \circ X X \\
&\quad + [2\triangleright \circ + \circ \infty]X + 2\circ^2 X X \\
&\quad - 2\circ^3 X;
\end{aligned}$$

here

$$X X = G_{42}$$

$$X X X = G_{431}$$

(A.5.b)

Equations (A.5.a) (A.5.b), (A.3), (A.2) imply

$$G_{442} = \circ R_1 + 2\triangleright R_2 + \circ R_{31} + R_{42} \quad (\text{A.5})$$

All the formulae of that kind we have collected in the Table.

The result is:

$$\begin{aligned}
G_4 &= \alpha_1 R_1 + \alpha_2 R_2 + \alpha_{31} R_{31} + \alpha_{32} R_{32} \\
&\quad + \alpha_{41} R_{41} + \dots + \alpha_{47} R_{47} + O(X^5),
\end{aligned}$$

(A.6)

where

$$\begin{aligned}
\alpha_1 &= -2X + 6X^2 \circ - 6X^3 \infty - 24X^3 \triangleright \\
&\quad + 6X^4 \infty \infty + 12X^4 \circ \infty + 24X^4 \square \\
&\quad + 96X^4 \square + 24X^4 \triangleright \circ + 24X^4 \circ \triangleright \\
&\quad + 16X^4 \square + O(X^5) = G_4(\mathbb{E}, m, g, \ell),
\end{aligned}$$

(A.7)



Table

$G_{4n\alpha}$	$R_1$	$R_2$	$R_{31}$	$R_{32}$	$R_{41}$	$R_{42}$	$R_{43}$	$R_{44}$	$R_{45}$	$R_{46}$	$R_{47}$	$Q_{4n\alpha}$
$G_{41}$	1											-2
$G_{42}$	$\bigcirc$	1										6
$G_{431}$	$\infty$	$2\bigcirc$	1									-6
$G_{432}$	$\triangle$	$\bigcirc$		1								-24
$G_{441}$	$\infty\infty$	$2\infty + \bigcirc^2$	30		1							6
$G_{442}$	$\bigcirc\bigcirc$	$2\triangle$	$\bigcirc$			1						12
$G_{443}$	$\text{cylinder}$	$\bigcirc^2$		$2\bigcirc$			1					24
$G_{444}$	$\square$	$\triangle$		$\bigcirc$				1				96
$G_{445}$	$\triangle$	$\infty$		$2\bigcirc$					1			24
$G_{446}$	$\infty$	$\bigcirc^2 + \triangle$	$\bigcirc$	$\bigcirc$						1		24
$G_{447}$	$\square$										1	16

$$\alpha_2 = 6X^2 - 36X^3\bigcirc + 36X^4\infty + 144X^4\triangle + 54X^4\bigcirc^2 + O(X^5) = 6[-G_4(\xi, m, g, \ell)/2]^2, \quad (\text{A.8})$$

$$\alpha_{31} = -6X^3 + 54X^4\bigcirc + O(X^5) = -6[-G_4(\xi, m, g, \ell)/2]^3, \quad (\text{A.9})$$

$$\alpha_{32} = -24X^3 + 216X^4\bigcirc + O(X^5) = -24[-G_4(\xi, m, g, \ell)/2]^3. \quad (\text{A.10})$$

So, we have checked eq. (19) up to the terms  $O(X^4)$ .

#### Appendix B

In this Appendix we check the identity (22) up to the terms  $O(X^4)$ . The necessary diagrams are listed in eq. (10) and fig. 4; the numeration of the diagrams is shown in fig. 4. Similarly to the Table one gets the formulae

$$G_{22}|_{-m^2}^{p^c} \equiv G_{22}(p, m, g, \ell) - G_{22}(im, m, g, \ell) = R_2', \quad (\text{B.1})$$

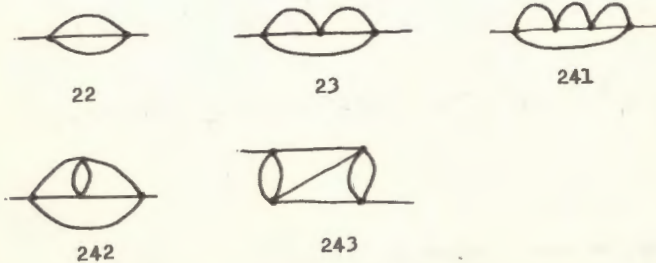


Fig.4

$$G_{23} \Big|_{-m^2}^{\rho^2} = 2 \circ R_2' + R_3', \quad (\text{B.2})$$

$$G_{241} \Big|_{-m^2}^{\rho^2} = 2[\text{fish} + \text{bubble}^2] R_2' + 2 \circ R_3' + R_{41}', \quad (\text{B.3})$$

$$G_{242} \Big|_{-m^2}^{\rho^2} = 2 \triangle R_2' + \circ R_3' + R_{42}', \quad (\text{B.4})$$

$$G_{243} \Big|_{-m^2}^{\rho^2} = (2 \triangle + \circ) R_2' + 2 \circ R_3' + R_{43}'. \quad (\text{B.5})$$

Taking the coefficients  $Q$  from eq. (10) and using eqs. (B.1) - (B.5) one obtains

$$(1 - \hat{N}_m) \sum_{n\kappa} x^n Q_{2n\kappa} G_{2n\kappa} = \beta_2 R_2' + \beta_3 R_3' + \beta_{41} R_{41}' + \beta_{42} R_{42}' + \beta_{43} R_{43}' + \dots \quad (\text{B.6})$$

where

$$\beta_2 = -\frac{2}{3}x^2 + 4x^3 \circ - 4x^4 \text{fish} - 16x^4 \triangle - 6x^4 \circ^2 + O(x^5) = -\frac{2}{3}[-G_4(\frac{k}{2}, m, g, \ell)/2]^2, \quad (\text{B.7})$$

$$\beta_3 = 2x^3 - 18x^4 \circ + O(x^5) = 2[-G_4(\frac{k}{2}, m, g, \ell)/2]^3. \quad (\text{B.8})$$

In this way, we have checked eq. (22).

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