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# GALILEO-INVARIANT THEORY OF LOW ENERGY PION-NUCLEUS SCATTERING. Part II

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## INTRODUCTION

Two substantially different Galileo-invariant models are constructed in Section 3*. The first is based on the assumption that the matrix $f(\tilde{E})$ depends only on the kinetic energy of the pion and the target nucleon. The second model utilizes a (A+1)-body generalization of the auxiliary scattering matrices which enter Faddeev equations [1] for a three-body system. Kinematical regions where the two models should lead to different predictions are determined. The concept of effective nucleon momenta is useful also in studying the pion-nucleus inelastic processes, particularly the pion-induced knock-out reaction.

Since the work of Lenz [2] it has been known that the exact first-order optical potential describes a non-local propagation of the resonating pion-nucleon system through the nucleus. On the other hand, if the static approximation is postulated, the pion-nucleon system decays at the point of its creation. If the leading m/M corrections are retained in the optical potential via the aforementioned factorization procedure, the potential mediates a nonlocal propagation of the pion-nucleon system of a fairly general type. We arrive at this conclusion in Section 4. Disregarding the fact that we deal with the pion-nucleus scattering in Sections 2-4, all the results obtained are fully applicable to the scattering of an arbitrary particle on a composite system if the projectile mass is considerably less than that of the target particles.

## 3. GALILEO-INVARIANT OPTICAL MODELS

There are two obvious possibilities of restoring the Galilean invariance of the optical model, or, generally speaking, of models based on the impulse approximation. The definition of the auxiliary matrix $t(\tilde{E})$ is to be changed in such a way that the Green function $d(\tilde{E})$, Eq. (2.8), may contain either the two-body quantities or the (A+1)-body ones, but not a mixture of both. In subsections 3.1 and 3.2 the correspondingly modified IA schemes are developed and the properties of the resulting optical potentials are discussed.

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*For Sections 1 and 2 see R.Mach, JINR, E2-12932, Dubna, 1979.

### 3.1. Two-body model

In the early applications of the impulse approximation, the projectile-scatterer energy was sometimes identified with the two-body kinetic energy rather than with the total reaction energy $E$. In this case the prescritpion for IA can be written as

$$\tau(E) \simeq \int d^3p\, d^3p'\, d^3k_1\, d^3k'_1 |\vec{k}'_1 \vec{p}'\rangle\langle \vec{p}' \vec{k}'_1 | t(\tilde{E}) | \vec{k}_1 \vec{p}\rangle\langle \vec{p} \vec{k}_1 | \qquad (3.1)$$

and

$$\tilde{E} \equiv E_2 = \frac{1}{2}\left(\frac{p'^2 + p^2}{2m} + \frac{k_1'^2 + k_1^2}{2M}\right). \qquad (3.2)$$

Here, $t(E_2)$ is again the pion-free nucleon scattering matrix (2.7). In the optical model, however, the energy $E_2$ ceases to be a fixed parameter, being related to the reaction energy indirectly, via the equation of motion (2.1). Energy $E_2$ has been chosen as the mean sum of the kinetic energies in the initial and final states. This choice meets the detailed balance condition. Unfortunately, we did not succeed in expressing the relation between the matrices $\tau(E)$ and $t(\tilde{E})$ in a closed form similar to (2.9a). However, the iterative expansion of the matrix $\tau(E)$ in terms of $t(\tilde{E})$ can be easily obtained.

Recently, Wilkin[3] suggested a configuration space version of the two-body model. On the analogy of electrodynamics he argued that the physically plausible choice of the energy $\tilde{E}$ is the local pion kinetic energy

$$\tilde{E} = E - V(\vec{r}), \qquad (3.3)$$

where $V(\vec{r})$ is an average potential felt by the pion inside the nucleus at the point $\vec{r}$. The two-body choice (3.3) or its momentum space analogue (3.2) should be reasonable one in the absence of strong nucleon-nucleon correlations. Otherwise, the mean nuclear field is not a well-defined concept. It is very difficult, however, to make some quantitative estimates.

Having chosen the energy $\tilde{E} \equiv E_2$, we can repeat the derivation of the optical potential. The equation (2.17b) is obtained again with the stipulation that the energy $z$ is to be replaced by the manifestly Galileo-invariant expression

$$z_2 = \frac{1}{2\mu}\left(\frac{q_{f,0}^2 + q_{i,0}^2}{2} + \frac{\mu}{M}\vec{v}_1 \cdot (\vec{q}_{f,0} + \vec{q}_{i,0}) + \left(\frac{\mu}{M}\right)^2 v_1^2\right). \qquad (3.4)$$



If the factorization approximation is applied ($\vec{v}_1 \equiv 0$), we end up with the optical potential (2.19), where $z_0$ is to be replaced by

$$z_{2,0} = \frac{1}{2m}\left(\frac{Q'^2 + Q^2}{2} - \frac{A-1}{A}\frac{\mu}{M}\vec{Q}' \cdot \vec{Q} + O\left(\frac{m^2}{M^2}\right)\right). \qquad (3.5)$$

According to the previous discussion, the error of the resulting optical potential is of the order of $(m/M)^2$.

### 3.2. (A+1)-body model

An attempt has been made by Revai[4] to approximate the matrix $\tau(E)$ by means of a more complex operator than a two-body one. According to his suggestion, a special three-body problem should be solved, where the role of the third particle is played by the remaining (A−1) nucleons forming the nuclear core. The resulting pion-nucleon scattering matrix, which contains also some binding corrections, is then used as a starting point in IA. In actual calculations[5], however, the energy $\tilde{E}$ is calculated from the pion-nucleon-core kinematics rather than from the solution of the three-body equations. The model being Galileo-noninvariant is suitable probably for heavier nuclei, where the concept of the nuclear core has a sound meaning.

We suggest a Galileo-invariant (A+1)-body generalization of IA, which consists of replacing the matrix $\tau(E)$ by an auxiliary matrix $T_{\pi A}$ defined as

$$T_{\pi A} = v + v g(E) T_{\pi A}(E), \qquad (3.6)$$

where

$$g(E) = \left(E - h_0 - \sum_{i=1}^{A} h_i - \langle 0 | U_A | 0\rangle + i\epsilon\right)^{-1}. \qquad (3.7)$$

In contrast with the exact Green function $G(E)$, Eq. (2.2), only the ground state matrix element of the nuclear potential $U_A$ is retained in (3.7). The relation between matrices $\tau(E)$ and $T_{\pi A}(E)$ is analogous to (2.9a). Energy $E$ is scaled here as

$$E = E_{Ac} + \frac{P^2}{2(m + AM)} - \epsilon_B. \qquad (3.8)$$

where $E_{Ac}$ is the energy of the relative pion-nucleus motion and $\epsilon_B > 0$ the nuclear binding energy.

If the element $<0|U_A|0>$ were dropped in (3.7), we would have the same auxiliary matrix, which is commonly used in solving the Faddeev equations (in the case of $A=2$ or 3 nuclei). Nevertheless, we prefer to keep the matrix element in (3.7), otherwise the matrix $T_{\pi A}(E)$ would be evaluated at energies considerably remote from the reaction energy. This fact causes the well-known inadequacy of the leading term in the iterative expansion of the Faddeev equations, which usually strongly violates the unitarity condition. The defect is compensated, of course, by higher order iterative terms; however, it could lead to serious difficulties in the approximation scheme developed here.

Prior to the derivation of the optical potential, we recall the obvious identity

$$<\vec{p}', \vec{k}'_1, \dots, \vec{k}'_A | T_{\pi A}(E) | \vec{k}_A, \dots, \vec{k}_1, \vec{p}> =$$

$$= (2\pi)^{3A} \delta^3 (\vec{p}' + \vec{k}'_1 - \vec{p} - \vec{k}_1) \tag{3.9}$$

$$\times \delta^3 (\vec{k}'_2 - \vec{k}_2) \dots \delta^3 (\vec{k}'_A - \vec{k}_A) <\vec{q}_f | t_r (z_{A+1}) | \vec{q}_i>,$$

where

$$z_{A+1} = E - \frac{(\vec{p}+\vec{k}_1)^2}{2(m+M)} - \frac{1}{2M} \sum_{i=2}^{A} k_i^2 - <0|U_A|0>. \tag{3.10}$$

Here, $t(z_{A+1})$ is the pion-free nucleon scattering matrix as defined by (2.7). Besides the substitution (2.16), we made use of others

$$\vec{k}_j = \frac{\vec{\kappa}}{A} + \frac{1}{2A}(\vec{Q} - \vec{Q}') - \vec{v}_j$$

$$\vec{k}'_j = \frac{\vec{\kappa}'}{A} - \frac{1}{2A}(\vec{Q} - \vec{Q}') - \vec{v}'_j, \tag{3.11}$$

$j = 2, 3, \dots, A$ for momenta of "spectator" nucleons in evaluating the matrix elements $<\vec{p}' 0 | T_{\pi A}(E) | 0 \vec{p}>$. The resulting optical potential is

$$<\vec{Q}' 0 | U_r(E_{Ac}) | 0 \vec{Q}> = \frac{A}{(2\pi)^{3A-3}} \int <0 | \vec{\xi}'_1, \vec{\xi}'_2, \dots, \vec{\xi}'_{A-1}>$$



$$\times \exp[i \frac{A-1}{A} (\vec{Q}' - \vec{Q}) \cdot (\vec{\xi}'_{A-1} + \vec{\xi}_{A-1})] \exp[i \sum_{j=1}^{A-1} \vec{\eta}_j \cdot (\vec{\xi}'_j - \vec{\xi}_j)]$$

$$\times <\vec{q}_f | t_r (z_{A+1}) | \vec{q}_i> <\vec{\xi}_{A-1}, \dots, \vec{\xi}_2, \vec{\xi}_1 | 0> \tag{3.12}$$

$$\times \prod_{j=1}^{A-1} d^3 \xi'_j \prod_{j=1}^{A-1} d^3 \xi_j \prod_{j=1}^{A-1} d^3 \eta_j .$$

The momenta

$$\vec{\eta}_1 = \frac{1}{2} (\vec{v}_A - \vec{v}_{A-1}) \qquad \vec{\eta}_{A-1} = \frac{A-1}{A-2} \left( \frac{\vec{v}_A + \vec{v}_{A-1} + \dots + \vec{v}_2}{A-1} - \vec{v}_1 \right) \equiv -\vec{v}_1$$

$$\vdots \tag{3.13}$$

$$\vec{\eta}_{A-2} = \frac{A-2}{A-3} \left( \frac{\vec{v}_A + \vec{v}_{A-1} + \dots + \vec{v}_3}{A-2} - \vec{v}_2 \right) \qquad \vec{\eta}_A = \vec{v}_A + \dots + \vec{v}_1 \equiv 0.$$

are canonically conjugated with the Jacobi coordinates $\vec{\xi}_j$.

The optical potential (3.12) is Galileo-invariant since

$$z_{A+1} = E_{Ac} - \frac{1}{2m} \frac{\mu}{\mathfrak{M}} \{ \frac{A-1}{A} \frac{(\vec{Q}' + \vec{Q})^2}{4} - \vec{\eta}_{A-1} \cdot (\vec{Q}' + \vec{Q}) - \tag{3.14}$$

$$- \frac{\mathfrak{M}}{M} \eta_{A-1}^2 \} - C(\eta_1, \eta_2, \dots, \eta_{A-1})$$

and

$$C(\eta_1, \eta_2, \dots, \eta_{A-1}) = \frac{1}{2M} \sum_{j=1}^{A-1} \frac{j+1}{j} \eta_j^2 + <0|U_A|0> + \epsilon_B . \tag{3.15}$$

The expression (3.15) represents the "binding" correction and the terms in curly brackets in (3.14) can be interpreted as the corrections due to the Fermi motion on the target nucleon. Nevertheless, the separation between the binding effects and the Fermi motion can be made only approximately.

Taking into account the identity

$$<0| C(\hat{\eta}_1, \hat{\eta}_2, \dots, \hat{\eta}_{A-1}) |0> = 0, \tag{3.16}$$

we neglect the term (3.15) in Eq. (3.14). The assumption is made here that the omitted term yields only a small contribution to the energy $z_{A+1}$ for typical momenta of nucleons bound in the nucleus. It is important to note that the leading "binding" correction of the order of $C(\eta_1,\eta_2,\ldots,\eta_{A-1})/E_{Ac}$ vanishes in (3.12) in the limit of low momentum transfer $(\vec{Q}'-\vec{Q})\to 0$ as a consequence of (3.16). Therefore, the attempts [5,6] to simulate the binding corrections by a constant shift in the energy $z$ are hardly justified.

Neglecting the "binding" effects in (3.14), we are left with the optical potential (2.17b), where the energy $z$ is now, of course, somewhat modified. When the momentum $\vec{\eta}_{A-1}$ is further neglected in (3.14), we arrive at the factorized optical potential (2.19) the energy dependence of which is given by

$$z_{A+1,0} = E_{Ac} - \frac{1}{8M}\frac{A-1}{A}\frac{\mu}{\mathfrak{M}}(\vec{Q}'+\vec{Q})^2. \qquad (3.17)$$

The error of the last approximation is again of the order of $(m/M)^2$.

### 3.3. Comparison of the two models

Two different IA schemes were developed in the previous subsections yielding two Galileo-invariant optical potentials. When the "binding" effects are neglected and the technique of the optimal effective nucleon momenta is used, the two optical potentials have the same factorized form (2.19), the validity of which is guaranteed up to the $m/M$ terms. Then the only difference between the two- and (A+1)-body model consists in the energy dependence of the optical potential, which is given by $z_{2,0}$, Eq. (3.5), and $z_{A+1,0}$, Eq. (3.17), respectively. Since the last energies depend on dynamical variables $\vec{Q}'$ and $\vec{Q}$, the corresponding optical potentials are substantially nonlocal. This is true especially for the two-body model since the energy $z_{2,0}$ is built up solely from the momenta $\vec{Q}'$ and $\vec{Q}$.

Some points of comparison can be drawn from the inspection of the energies $z_{2,0}$, $z_{A+1,0}$ and $z_0$, Eq. (2.21). In the expressions for $z_0$ and $z_{A+1,0}$, the energy of the pion-nucleon subsystem is multiplied by $((A-1)/A)^2$ and $(A-1)/A$, respectively. Therefore, we can expect that the predictions of the (A+1)-body and the Galileo-noninvariant optical models will differ appreciably only in the case of the lightest nuclei. It is interesting to note that our (A+1)-body model



as well as the three-body one suggested in ref. [5] implies the evaluation of the pion-nucleon scattering matrix at somewhat lower energies than the standard Galilean noninvariant optical model.

On the energy shell, $Q'=Q=\sqrt{2\mathfrak{M}E_{Ac}}$, the energies $z_{2,0}$ and $z_{A+1,0}$ increases with the increasing scattering angle $\theta$. The angular dependence of an analogous type was earlier discussed in the case of the Galileo-noninvariant optical model [7]. Assuming, for example, the pion elastic scattering by $^4He$ at $E_\pi$ = 20 MeV in the laboratory system, the energies $z_{2,0}$ and $z_{A+1,0}$ reach the values from the intervals 17.4-21.2 and 17.4-19.3 MeV, respectively. For higher energies $E_\pi$, the range of energies $z_{A+1,0}$ and $z_{2,0}$ becomes larger; then however, the relativistic kinematics is to be used. For the reactions going predominantly via the single scattering mechanism, the predictions of the two Galilean invariant optical models may differ substantially only in the region of large scattering angles, since $z_{2,0}(\theta=0°)=z_{A+1,0}(\theta=0°)$ and $z_{2,0}(\theta=180°)-z_{2,0}(\theta=0°)=2(z_{A+1,0}(\theta=180°)-z_{A+1,0}(\theta=0°))$.

A different situation occurs when the multiple scattering terms contribute significantly and, especially, when also the off-shell effects are important. Actually, off-energy-shell values of the optical potential are required in solving the dynamical equation (2.1), the energy $z_2$ being bound in the interval zero-plus infinity. On the other hand, the values reached by $z_{A+1}$ extend from $E_{Ac}$ to minus infinity. Therefore, the pion-nucleus scattering matrix can reflect quite different aspects of the underlying pion-nucleon amplitude in the two optical models studied. The pion-nucleus elastic scattering will provide a severe test of the optical models considered in the $\Delta_{33}$ resonance region as well as in the small energy ($E_\pi \leq$ 60 MeV) interval since the multiple scattering and the off-shell effects seem to play an important role here.

From the technical point of view, the two-body optical model exhibits some attractive features in comparison with the (A+1)-body one. Because the energy $z_2$ is always positive, there is no need to construct models for pion-nucleon amplitude in the nonphysical negative energy region. Moreover, the optical potential is never too far from the energy shell in eq. (2.1) since the energy $z_2$ lies between the energies corresponding to the initial and final state momenta $\vec{Q}$ and $\vec{Q}'$. This is not the case of the energy $z_{A+1,0}$. Therefore, there are only minor input uncertainties in the two-body optical model.

The inherent difference between the two optical models becomes apparent when the characteristics of $\pi$-mesoatoms are evaluated. In this case, the energies $z_{A+1,0} < 0$ and $z_{2,0} \geq 0$ represent two nonoverlapping intervals. Neglecting the remote poles and cuts, the pion-nucleon amplitude is a real quantity for $z_{A+1,0} < 0$. Therefore, the optical potential of the (A+1) -body model is real, too, and the model describes the $\pi$-mesoatom as a stable system. Because of $z_{2,0} \geq 0$, the corresponding optical potential is complex and provides some phenomenological model for the true pion absorption. Using this model, the $\pi$-mesoatomic level shifts and widths are estimated and the possible physical origin of the imaginary part of the optical potential is discussed in Section 5.

## 4. PROPAGATION OF THE PION-NUCLEON SUBSYSTEM

The meaning of the particular IA scheme chosen in constructing the optical model becomes more transparent if the optical potential is transformed into the configuration space. Closely following the considerations of Lenz[2], we assumed here that the pion-nucleon amplitude is dominated by the $\Delta_{33}$ resonance of a Breit-Wigner form. The model is simple enough to allow for analytic solutions in the case of the configuration space optical potential and provides an insight into the propagation of the pion-nucleon subsystem in the nuclear medium.

The pion-nucleon scattering matrix reads

$$\langle \vec{q}_f | t(z) | \vec{q}_i \rangle = \frac{2\pi}{\mu} f_0 \frac{\Gamma/2}{z - R + i\Gamma/2} \frac{\vec{q}_f \cdot \vec{q}_i}{p_{2c}^2} . \qquad (4.1)$$

where R = 297 MeV, $\Gamma/2$ = 55 MeV, $p_{2c} = \sqrt{2\mu z}$ and the normalization

$$f_0 = \frac{p_{2c}\,\bar{\sigma}}{4\pi} \qquad (4.2)$$

is determined via the optical theorem at the resonance energy. Further, $\bar{\sigma}$ is the spin-isospin averaged total pion-nucleon cross section.

For the scattering matrix (4.1), the optical potential (2.17b) reduces to



$$\langle \vec{Q}' 0 | U_r (E_{Ac}) | 0 \vec{Q} \rangle = \frac{t_0}{(2\pi)^3} \int e^{i \frac{A-1}{2A} (\vec{Q}' - \vec{Q}) \cdot (\vec{\xi}'_{A-1} + \vec{\xi}_{A-1})}$$

$$\times e^{i \vec{v}_1 \cdot (\vec{\xi}'_{A-1} - \vec{\xi}_{A-1})} \frac{\rho_{00} (\vec{\xi}'_{A-1}, \vec{\xi}_{A-1})}{z - R + i\Gamma/2} \frac{\vec{q}_f \cdot \vec{q}_i}{p_{2c}^2} \qquad (4.3)$$

$$\times d^3\xi'_{A-1} d^3\xi_{A-1} d^3 v_1$$

with

$$t_0 = \frac{A \bar{\sigma} \Gamma p_{Ac}}{4\pi} . \qquad (4.4)$$

Momenta $\vec{q}_f$ and $\vec{q}_i$ depend on $\vec{Q}'$, $\vec{Q}$ and $\vec{v}_1$, cf. (2.18a,b). Since we are not interested in the well-known nonlocality of the optical potential[8] caused by the term $(\vec{q}_f \cdot \vec{q}_i)$, we set approximately $(\vec{q}_f \cdot \vec{q}_i)/p_{2c}^2 = 1$. Effects associated with the finite range of the pion-nucleon interaction are also left aside since the problem of our main concern is the relationship between the energy dependence of the pion-nucleon amplitude and the modes of propagation of the pion-nucleon subsystem in nuclei.

In the standard Galileo-noninvariant model, the energy z is given by (2.18c) (we set $\vec{P} = 0$ in (2.18c)), and the optical potential in the configuration space is obtained

$$\langle \vec{r}' 0 | U_r (E_{Ac}) | 0 \vec{r} \rangle = -\frac{1}{2\pi} \left(\frac{A}{A-1}\right)^2 t_0 (m+M) \rho (\vec{r}', \vec{r}) \frac{e^{iK|\vec{r}' - \vec{r}|}}{|\vec{r}' - \vec{r}|} . \qquad (4.5)$$

The nonlocal potential (4.5) describes the intermediate propagation of the free resonating pion-nucleon system from the point $\vec{r}$ where the pion-nucleon collision takes place to $\vec{r}'$ where the system decays back into the pion-nucleus channel. The propagation is characterized by the complex momentum

$$K^2 = 2\left(\frac{A}{A-1}\right)^2 (m+M)(E_{Ac} - R + i\Gamma/2). \qquad (4.6)$$

If the m/M term is neglected in Eq. (2.21) for the energy z the optical potential reduces to

$$\langle \vec{r}' 0 | U_r (E_{Ac}) | 0 \vec{r} \rangle = t_0 \frac{\rho(r)\, \delta^3 (\vec{r}' - \vec{r})}{E_{Ac} - R + i\Gamma/2} . \qquad (4.7)$$

Therefore, the intermediate propagation of the pion-nucleon system is neglected in the static approximation.

We extend now the configuration space analysis of the optical potential to the Galileo-invariant models.

## 4.1. (A+1)-body model

The energy $z \equiv z_{A+1}$ is given by (3.14). In the extreme static approximation picture $z_{A+1} \to E_{Ac}$ and we arrive at the expression (4.7) for the optical potential the error of which is of the order of $m/M$.

The technique of effective nucleon momenta developed in the preceeding section makes it possible to incorporate the effects of the order of $m/M$ into the optical potential. To this end we use (3.17) for the energy $z_{A+1,0}$ and obtain the potential

$$<\vec{r}'0|U_r(E_{Ac})|0\vec{r}> = -\frac{1}{2\pi}\frac{A}{A-1}t_0\frac{\mathfrak{M}M}{\mu}\rho\left(\frac{\vec{r}'+\vec{r}}{2}\right) \times \frac{\exp(iK_{A+1}|\vec{r}'-\vec{r}|)}{|\vec{r}'-\vec{r}|}. \tag{4.8}$$

The $m/M$ terms obviously restore the main features of the nonlocal propagation of the pion-nucleon system in a nucleus. The propagation is characterized by the slightly modified momentum

$$K^2_{A+1} = 2\frac{A}{A-1}\frac{\mathfrak{M}M}{\mu}(E_{Ac} - R + i\Gamma/2) \tag{4.9}$$

and by the mass $\mathfrak{M}M/\mu = (m+M)/(1+m/AM)$. It can be concluded that even the factorized optical potential (2.19) describes the nonlocal propagation of the pion-nucleon system, but the density matrix is approximated by the usual nuclear density $\rho(\vec{r})$ taken midway between the points where the system is created and where it decays.

The appearance of the density matrix $\rho(\vec{r}', \vec{r})$ is connected with the terms of the order of $(m/M)^n$, $n \geq 2$. It is instructive to use the exact Eq. (3.14) for the energy $z_{A+1}$ (of course, neglecting $C(\eta_1, \eta_2, \ldots, \eta_{A-1})$) in constructing the configuration space optical potential. We obtain

$$<\vec{r}'0|U_r(E_{Ac})|0\vec{r}> = -\frac{1}{2\pi}\frac{A}{A-1}t_0\frac{\mathfrak{M}M}{\mu} \tag{4.10}$$



$$\times \int e^{i\vec{v}\cdot\vec{\eta}}\frac{e^{iK_{A+1}(v)|\vec{r}'-\vec{r}|}}{|\vec{r}'-\vec{r}|}\cdot\rho(\vec{r}'+\vec{\eta}/2, \vec{r}-\vec{\eta}/2)\frac{d^3v\, d^3\eta}{(2\pi)^3},$$

where

$$K^2_{A+1}(v) = 2\frac{A}{A-1}\frac{\mathfrak{M}M}{\mu}\left(E_{Ac} - R + \frac{A-1}{A}\frac{v^2}{2M} + \frac{i\Gamma}{2}\right). \tag{4.11}$$

As a result of the (A+1)-body nature of the model, the pion-nucleon system does not move through nucleus as a free particles with a fixed momentum. The presence of the "spectator" nucleons causes an additional nonlocality of the potential and leads to the effective downward shift in the resonance position, cf. (4.11).

## 4.2. Two-body model

Quite different aspects of the pion-nucleon off-shell dynamics are emphasized by the two-body model in comparison with the (A+1)-body or the noninvariant one. If the pion-nucleon scattering is dominated by an s-channel resonance (e.g., by $\Delta_{33}$), the resonance condition

$$z - R \simeq 0 \tag{4.12}$$

indicates in the case of the noninvariant model that the resonating quantity is the energy which belongs to the motion of the pion-nucleon system as a whole. The kinetic energy of the relative pion-nucleon motion is not directly influenced by the resonance. On the contrary, the last energy is the resonating quantity in the two-body model ( $z \equiv z_2$ , cf. (3.4)) whereas the motion of the pion-nucleon system is not affected by the resonance. This is the reason why the optical potential

$$<\vec{r}0|U_r|0\vec{r}> = \frac{t_0}{2\pi}\frac{\mu^2}{\mathfrak{M}}\int e^{i\frac{\mu}{\mathfrak{M}}\vec{Y}\cdot\vec{\eta}}\frac{e^{-iK_2(Y)|\vec{r}'-\vec{r}|}}{|\vec{r}'-\vec{r}|} \times\rho\left(\frac{\vec{r}'+\vec{r}}{2}+\vec{\eta}\right)d^3\eta\frac{d^3Y}{(2\pi)^3} \tag{4.13}$$

characterizes rather the propagation of the pion inside the pion-nucleon system than the motion of the system as a whole.

The potential (4.13) was obtained from (4.3), where $z \equiv z_{2,0}$, cf. (3.5). We have

$$\mathcal{K}_2^2(Y) = 2\mathfrak{M}(R - i\Gamma - \frac{Y^2}{8\mathfrak{M}}) \qquad (4.14)$$

and (4.13) is correct up to the terms of the order of $m/M$.

If the complete energy $z_2$ were used in (4.3), the density matrix $\rho(\vec{r}', \vec{r})$ would enter the expression for the optical potential instead of the nuclear density $\rho(\vec{r})$. Therefore, also in the two-body model, the appearance of the density matrix $\rho(\vec{r}', \vec{r})$ is connected with the terms of the order of $(m/M)^n$, $n \geq 2$. It is not surprising that even in the extreme static limit $m/M \to 0$ the optical potential remains nonlocal. The nonlocality simulates the dependence of the optical potential on the reaction energy $E_{Ac}$, which has been suppressed in the two-body model.

The pion-nucleon scattering matrix enters the optical potential at the energy which is indirectly related to the reaction energy $E_{Ac}$ (due to the equation of motion). Therefore, the two-body model is not expected to work very well in the vicinity of the s-channel resonances, where the pion-nucleus cross sections are rapidly varying functions of the reaction energy. Particularly in the region of the $\Delta_{33}$ resonance, the (A+1)-body model seems to be more appropriate. It will be shown in the next section that a natural area of applicability of the two-body model seems to be a region where the t-channel resonances play an important role in the pion-nucleon scattering. Such a situation occurs in the low energy interval ($E_{Ac} \leq$ 80-100 MeV) where the pion-nucleon s-waves are dominated by the $\rho$-and $\sigma$-meson exchange [9].

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