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дубна

E2 - 12949

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A STOCHASTIC DERIVATION OF THE SIVASHINSKY EQUATION FOR THE SELF-TURBULENT MOTION OF A FREE PARTICLE

Submitted to "Foundations of Physics"

1979

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E2 - 12949

Стохастическое получение уравнения Сивашинского для самотурбулентного движения свободной частицы

В рамках подхода Кершоу и гипотезы о стохастичности пространства получены релятивистские уравнения стохастической механики. В нашей модели существует еще совокупность уравнений гидродинамического типа для регулярной $\vec{v}(\vec{x},t)$ и стохастической $\vec{u}(\vec{x},t)$ скоростей частицы. Если учесть члены порядка ℓ^2, где ℓ элементарная длина, то эти уравнения дают уравнения Сивашинского для $\vec{v}(\vec{x},t)$ в предельном переходе, когда $\vec{u}(\vec{x},t) \Rightarrow 0$. А в пределе $\ell \to 0$ получается уравнение Ньютона.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1979

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E2 - 12949

A Stochastic Derivation of the Sivashinsky Equation for the Self-Turbulent Motion of a Free Particle

Within the framework of the Kershaw approach and of a hypothesis on the spatial stochasticity, relativistic equations of Lehr-Park, Guerra-Ruggiero and Vigier for the stochastic Nelson mechanics are obtained. There is another set of equations of the hydrodynamical type for a drift $\vec{v}(\vec{x},t)$ and stochastic $\vec{u}(\vec{x},t)$ velocities of a particle in our model. Taking into account quadratic terms in ℓ, the universal length, we obtain from these equations the Sivashinsky equations for the $\vec{v}(\vec{x},t)$ in the case $u \to 0$. In the limit $\ell \to 0$, these equations acquire the Newtonian form.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1979

1. INTRODUCTION

Recently, Sivashinsky[1] has noted a formal analogy between the equation of motion for a flame front and the Hamilton-Jacobi equation for the motion of a free particle. He has shown that if one introduces terms, which contain higher order derivaties with respect to x_i and describe a flame structure of the front, then a plane flame front is unstable to perturbations of a sufficiently long wavelenght[2]. As a result, the initial deterministic equation can generate a solution of a random-function type. An attempt was made to interpret the equation with higher order derivatives as an equation which describes a motion of a "quantized" particle.

However, within the Sivashinsky approach a selection of a unique destabilizing inherent field (self-generated field potential) cannot be solved and a clear physical basis of this selection is absent.

In this paper we wish to show that an equation of the Sivashinsky type for the self-turbulent motion of a free particle may be constructed in the stochastic theory which is based on the hypothesis of the spatial stochasticity[3]. At the same time we make an attempt to give a proper foundation for the mentioned selection of the potential of the self-generated field.

There are three approaches to the construction of the theory of the stochastic processes in physics. Two of them are connected with diffusion processes and with properties of electromagnetic vacuum as a source for the randomness present in the Nature. Basic ideas and problems of these two schools are reviewed in[4,5].

The third approach is based on the postulate that the random behaviour of a physical system is caused by the stochastic character of the physical space. A stochastic space, which can be used in theories of elementary particles was first considered in papers[6], (see also review[7]).

Mathematical spaces with a stochastic metric and a quantized domain were investigated by Frederick[8] and Roy[9], respectively. Paper[10] is devoted to the construction of the relativistic kinematics of massive and massless particles in the stochastic phase space.

Following an idea of Blokhintsev[7] we have investigated in the previous paper[3] the problem of motion of a particle in the stochastic space with a small stochastic component and we have obtained the equations of Nelson stochastic mechanics in both the nonrelativistic and relativistic cases.

2. NONRELATIVISTIC EQUATIONS OF MOTION

We consider motion of a single scalar particle, the coordinates of which in a stochastic space $R_3(\hat{x}_i)$ are defined by two terms

$$\hat{x}_i = x_i + b_i ,$$

x_i being the regular part of the coordinate and b_i, some small random vector with a distribution $\tau(b_i)$ obeying the condition

$$\int d\tau(b_i) = 1, \quad d\tau(b_i) \geq 0 .$$

Since in our model the actual points of the space are of a stochastic nature, neither these points can be used as a basis for a coordinate system, nor one can take a derivative with respect to them. However, the space of common experience (i.e., the laboratory frame) is nonstochastic on a large scale. It is only in the micro-world where the stochasticity manifests itself. One can then continue mathematically from the microworld to this large-scale nonstochastic space. This mathematical construction provides a nonstochastic space which the stochastic physical space can be referred to. This is the Frederick argument[8]. In our case the mathematical construction reduces to averaging with the distribution $\tau(b_i)$ at any point of the space $R_3(\hat{x}_i)$ at a given time.

Therefore the averaged quantity $\langle f(\hat{x}_i, t)\rangle$ on $R_3(\hat{x}_i)$ with $\tau(b_i)$ is called the physical value of $f(x_i, t)$. Assumption about smallness of the stochastic component in the space $R_3(\hat{x}_i)$ means that

$$F(x_i, t) = \langle f(x_i + b_i, t)\rangle = \int d^3b\, \tau(b_i) f(x_i + b_i, t) =$$

$$= \langle f(x_i, t) + b_j \frac{\partial}{\partial x_j} f(x_i, t) + \frac{1}{2} b_i b_j \frac{\partial^2}{\partial x_i \partial x_j} f(x_k, t) + \ldots \rangle \approx \tag{1}$$

$$\approx f(x_i , t) + \ell^2 \Delta f(x_i , t), \tag{1}$$

where ℓ is some universal length. We suppose that $\tau(b_i) = \tau(-b_i)$. In the first approximation in the parameter ℓ we have

$$<f(x_i , t)> \approx f(x_i , t).$$

Namely, our previous paper[3] has concerned with this approximation, but now we shall not neglect the second terms in (1). Then in the space-time of the large-scale a physical value $f(x_i, t)$ has the form

$$F(x_i , t) = f(x_i , t) + f_\ell (x_i , t). \tag{2}$$

WE shall now make an attempt to obtain a general form of dynamical equations for a scalar particle, in the case when the term of order ℓ^2 is present in expression for the velocity and the force due to the equality(2)

$$v_j^{\pm} \Rightarrow v_j^{\pm} + v_{j\ell}^{\pm} \qquad \text{and} \qquad f_j^{\pm} \Rightarrow f_j^{\pm} + f_{j\ell}^{\pm} .$$

We assume that the small values $v_{j\ell}^{\pm}$ and $f_{j\ell}^{\pm}$ in the Smoluchowski-type equations for the $v_j^{\pm}$ take part only in a symmetrical combination with respect to transformations $\Delta t \to -\Delta t$ and $\delta x_i \to -\delta x_i$, i.e., they are even functions of Δt and δx_i. Following Kershaw[11] we can construct equations for $v_j^{\pm}$ by the formulas[3]

$$v_j^{\pm} (x_i , t \pm \Delta t) + \sum_{\{\Delta t\}} v_{j\ell}^{\pm} (x_i , t \pm \Delta t) = \frac{1}{N^{\pm}} \int [v_j^{\pm} (x_i \mp \delta x_i^{\pm}, t) +$$

$$+ \sum_{\{\delta x_i^{\pm}\}} v_{j\ell}^{\pm} (x_i \mp \delta x_i^{\pm} , t) \pm \frac{\Delta t}{m} (f_j^{\pm} (x_i \mp \delta x_i^{\pm} , t) + \tag{3}$$

$$+ \sum_{\{\delta x_i^{\pm}\}} f_{j,\ell}^{\pm} (x_i \mp \delta x_i^{\pm}, t))]$$

$$\times \rho (x_i \mp \delta x_i^{\pm} , t) \Psi_{\pm} (x_i \mp \delta x_i^{\pm}, t; \delta x_i^{\pm} , \Delta t) d^3 (\delta x^{\pm}),$$

$$v_j^{\pm}(x_i, t \mp \Delta t) + \sum_{\{\Delta t\}} v_{j,\ell}^{\pm}(x_i, t \mp \Delta t) = \frac{1}{N^{\mp}} \int [v_j^{\pm}(x_i \pm \delta x_i^{\mp}, t) +$$

$$+ \sum_{\{\delta x_i^{\mp}\}} v_{j,\ell}^{\pm}(x_i \pm \delta x_i^{\mp}, t) \mp \frac{\Delta t}{m}(f'^{\pm}_j(x_i \pm \delta x_i^{\mp}, t) +$$

$$+ \sum_{\{\delta x_i^{\mp}\}} f'^{\pm}_{j,\ell}(x_i \pm \delta x_i^{\mp}, t))] \times$$

$$\times \rho(x_i \pm \delta x_i^{\mp}, t)\Psi_{\mp}(x_i \pm \delta x_i^{\mp}, t; \delta x_i^{\mp}, \Delta t) d^3(\delta x^{\mp}), \qquad (4)$$

where

$$N^{\pm} = \int \rho(x_i \mp \delta x_i^{\pm}, t)\Psi_{\pm}(x_i \mp \delta x_i^{\pm}, t; \delta x_i^{\pm}, \Delta t) d^3(\delta x^{\pm})$$

are normalization constants, the symbol $\sum_{\{\dots\}}$ means symmetrization in the variables $\{\dots\}$ and

$$\Psi_{\pm} = \frac{1}{(4\pi D_{\pm}\Delta t)^{3/2}} \exp\{-(\delta x_i^{\pm} - v_i^{\pm}(x_j, t)\Delta t)^2 / 4D_{\pm}\Delta t\}.$$

In our case we have

$$\sum_{\{y\}} g_{j,\ell}^{\pm}(x-y) = \sum_{\{y\}} g_{j,\ell}^{\pm}(x+y) \equiv \frac{1}{2}[g_{j,\ell}^{\pm}(x+y) + g_{j,\ell}^{\pm}(x-y)],$$

for any function $g_{j,\ell}(\dots)$.
Upper (lower) sign corresponds to v_j^{+} (v_j^{-}). We assume $D_+ = D_- = D$, expand $v_j^{\pm}$, ρ, $\Psi_{\pm}$, $f_j^{\pm}$, and $f'^{\pm}_j$ in Taylor series, integrate and retain only the terms of the first order in Δt, then we get

$$m\left(\frac{\partial v_j^{\pm}}{\partial t} + v_i^{\pm} \nabla_i v_j^{\pm}\right) = f_j^{\pm} \pm mD\left(2\frac{\nabla_i \rho}{\rho} \nabla_i v_j^{\pm} + \nabla_\ell^2 v_j^{\pm}\right),$$

$$m\left(\frac{\partial v_j^{\pm}}{\partial t} + v_i^{\mp} \nabla_i v_j^{\pm}\right) = f'^{\pm}_j \mp mD\left(2\frac{\nabla_i \rho}{\rho} \nabla_i v_j^{\pm} + \nabla_\ell^2 v_j^{\pm}\right), \qquad (5)$$

where $\nabla_\ell^2 = \nabla^2 + \ell^2 \nabla^4$.

We pass to the variables $v_j = \frac{1}{2}(v_j^+ + v_j^-)$, $u_j = \frac{1}{2}(v_j^+ - v_j^-) =$
$= D\nabla_j \ln\rho$ and sum (subtract) the equations in (5) in pairs, so we obtain the equations describing different processes

$$d_c v_j - \lambda d_s u_j = F^+_{j,\lambda}/m$$
$$d_c u_j + \lambda d_s v_j = F^-_{j,\lambda}/m \qquad (6)$$

$$d_c v_j - \lambda d_s^* v_j = F_{j,\lambda}/m$$
$$d_c u_j + \lambda d_s u_j = F'_{j,\lambda}/m \qquad (7)$$

where $d_c = \frac{\partial}{\partial t} + v_i \nabla_i$, $d_s = u_i \nabla_i + D(\nabla^2 + \ell^2 \nabla^4)$, $\lambda = \pm 1$,

$$F^+_{j,1} = \frac{1}{2}(f_j^+ + f_j^-),\ F^+_{j(-1)} = \frac{1}{2}(f'^+_j + f'^-_j),\ F^-_{j,1} = \frac{1}{2}(f'^+_j - f'^-_j),$$

$$F^-_{j(-1)} = \frac{1}{2}(f_j^+ - f_j^-),\ F_{j1} = \frac{1}{2}(f_j^+ + f'^-_j),\quad F_{j(-1)} = \frac{1}{2}(f'^+_j + f_j^-),$$

$$F'_{j,1} = \frac{1}{2}(f'^+_j - f_j^-),\ F'_{j(-1)} = \frac{1}{2}(f_j^+ - f'^-_j).$$

Notice that the left-hand sides of equations (6) possess a definite parity under the time reversal operation. Indeed, since

$$v_i \to -v_j,\quad u_i \to u_i,\quad d_c \to -d_c \qquad \text{and} \qquad d_s \to d_s,$$

by $t \to -t$, then it can be easily seen that the expression $d_c v_j - \lambda d_s u_j$ does not change, but $d_c u_j + \lambda d_s v_j$ $(\lambda = \pm 1)$ changes sign under $t \to -t$. Therefore, the right-hand side of the corresponding equations (6) describing a force must be chosen so that the separated equation will remain invariant under the time reversal. This requirement is fulfilled, if we assume $f_j^+ \to f_j^-$, $f'^+_j \to f'^-_j$ under the $t \to -t$. Then $F^+_{j,\lambda}$ does not change, but $F^-_{j,\lambda}$ changes sign and

$$F_{j,1} \to F_{j(-1)} \quad -F'_{j,1} \to F'_{j(-1)} \text{ by } t \to -t.$$

Therefore the four equations (7) indeed reduce to a pair of equations.

If the terms of order $D\ell^2$ in the expression for d_s are neglected, then, as one can expect, we obtain the same fundamental equations (6) of Nelson, Pena-Auerbach and Skagerstam, which we have obtained in [3].

The other set of equations (7) is analogous to the hydrodynamical equations for the "liquids" v_j and u_j, provided D is interpreted formally as a viscosity coefficient.

In the limit $u_i \Rightarrow 0$, i.e., $v_j^+ \equiv v_j^-$ we obtain from (6) and (7) the Newtonian equation

$$d_c v_j = F_j/m$$

and, the following equations

$$d_c v_j - \lambda D(\nabla^2 + \ell^2 \nabla^4) v_j = F_{j,\lambda}/m$$

for a particle, respectively.

The last equations in the case $F_{j,\lambda} = 0$ represent equations of the Sivashinsky type for a free particle. For example, assuming $\lambda = -1$, we get

$$d_c v_j = -D\nabla^2 v_j - D\ell^2 \nabla^4 v_j \, , \qquad (8)$$

which is invariant under the Galilean transformation.

3. RELATIVISTIC GENERALIZATION OF THE SCHEME

In paper [3] we have considered a method for an extension of our techniques to the relativistic case. A basic hypothesis was the following:

i) the stochasticity of the space $R_4(\hat{x}_\mu)$ appears in the Euclidean region of the variables $\hat{x}_\mu$ only;

ii) a shift of the coordinate $x_0 \to x_0 + i\tau$, is equivalent to the consideration of the physical quantities as functions of complex times $t + i\tau$ in the limit $\tau \to 0$.

In our case τ is the random variable which makes possible introducing of the hypothesis about a stochasticity of the Euclidean space $E_4(\hat{x}_i, \tau)$.

The importance of this shift in the time variable was noted in [12]. In particular, a connection between quantum mechanics and the Markov processes may be made more transparent (see [13]).

In the relativistic case the expression (1) acquires the following form

$$F(x_i,t) = \langle f(x_i + y_i, x_0 + iy_4)\rangle = \int d^4 y_E \, r(y_E) \cdot$$
$$f(x_i + y_i, x_0 + iy_4) \approx f(x_i, t) + \ell^2 \Box f(x_i,t), \tag{9}$$

where

$$\Box = -\frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial x_i^2} \qquad \text{and} \quad r(y_\mu^E) = r(-y_\mu^E),$$

Using the language of random fluctuations in the Euclidean space stochasticity means that the fluctuations appear in the Euclidean space $E_4(\hat{x}_i, r)$. Thus, in this case we have the following expression for the transition probability density

$$\Psi(y_E, \Delta s) = \frac{1}{(4\pi D \Delta s)^2} \exp\left\{-\frac{y_E^2}{4D\Delta s}\right\}, \tag{10}$$

where s is some invariant parameter (proper time).

According to our model it is possible to generalize the equations (3) and (4) in the following way

$$u_\pm^\mu(x_\nu, s+\epsilon\Delta s) + \sum_{\Delta s} u_\pm^{\mu,\ell}(x_\nu, s+\epsilon\Delta s) = \frac{1}{N^\mp}\int [u_\pm^\mu(x_i - \epsilon y_i, x_0 + iy_4, s) +$$
$$+ \sum_{\{\epsilon y_i, iy_4\}} u_\pm^{\mu,\ell}(x_i - \epsilon y_i, x_0 + iy_4, s) + \frac{\epsilon \Delta s}{m}(F_\pm^\mu(x_i - \epsilon y_i, x_0 + iy_4) +$$
$$+ \sum_{\{\epsilon y_i, iy_4\}} F_\pm^{\mu,\ell}(x_i - \epsilon y_i, x_0 + iy_4))]\rho(x_i - \epsilon y_i, x_0 + iy_4, s) \times$$
$$\times \Psi_\pm(x_i - \epsilon y_i, x_0 + iy_4, s; y, \Delta s) d^4 y_E, \tag{11}$$

$$u_\pm^\mu(x_\nu, s-\epsilon\Delta s) + \sum_{\{\Delta s\}} u_\pm^{\mu,\ell}(x_\nu, s-\epsilon\Delta s) = \frac{1}{N^\mp}\int [u_\pm^\mu(x_i + \epsilon y_i, x_0 + iy_4, s) +$$

$$+ \sum_{\{\epsilon y_i, iy_4\}} u_{\pm}^{\mu,\ell}(x_i+\epsilon y_i, x_0+iy_4, s) - \frac{\epsilon\Delta s}{m}(F_{\pm}^{\prime\mu}(x_i+\epsilon y_i, x_0+iy_4) +$$

$$+ \sum_{\{\epsilon y_i, iy_4\}} F_{\pm}^{\prime\mu,\ell}(x_i+\epsilon y_i, x_0+iy_4))]\rho(x_i+\epsilon y_i, x_0+iy_4, s) \times$$

$$\times \Psi_{\pm}(x_i+\epsilon y_i, x_0+iy_4, s; y, \Delta s)\, d^4y_E, \tag{12}$$

where

$$N^{\pm} = \int d^4y_E\, \Psi_{\pm}(x_i \mp y_i, x_0+iy_4, s; y, \Delta s)\rho(x_i \mp y_i, x_0+iy_4, s)$$

$$\epsilon = \begin{cases} +1 & \text{for } u_+^{\mu} \\ -1 & \text{for } u_-^{\mu}, \end{cases}$$

$F_{\pm}^{\mu}$ and $F_{\pm}^{\prime\mu}$ are some forces, and $\Psi_{\pm}$ can be chosen in the form

$$\Psi_{\pm} = \frac{1}{(4\pi D_{\pm}\Delta s)^2} \exp\{-\frac{(y - y_{\pm}^{\mu})^2}{4D_{\pm}\Delta s}\},\quad y_{\pm}^{\mu} = (\pm iu_{\pm}^{0}\Delta s, u_{\pm}^{i}\Delta s),$$

$$(D_- = D_+ = D)$$

$u_{\pm}^{\mu}$ are four-dimensional velocity vectors, and D is the diffusion coefficient. From (11) and (12) we obtain after some calculations

$$m(\frac{\partial u_{\pm}^{\mu}}{\partial s} + u_{\pm}^{\nu}\partial_{\nu}u_{\pm}^{\mu}) = F_{\pm}^{\mu} \pm mD(\frac{2u^{\nu}}{D}\partial_{\nu}u_{\pm}^{\mu} + \Box_{\ell}u_{\pm}^{\mu}),$$

$$m(\frac{\partial u_{\pm}^{\mu}}{\partial s} + u_{\mp}^{\nu}\partial_{\nu}u_{\pm}^{\mu}) = F_{\pm}^{\prime\mu} \mp mD(\frac{2u^{\nu}}{D}\partial_{\nu}u_{\pm}^{\mu} + \Box_{\ell}u_{\pm}^{\mu}), \tag{13}$$

where $\partial_{\mu} = (\frac{\partial}{\partial x_0}, \nabla_i)$, $u^{\mu} = \frac{1}{2}(u_+^{\mu} - u_-^{\mu}) = -D\partial^{\mu}\ln\rho$

and $\Box_\ell = \Box + \ell^2\Box^2$.

Then the relativistic equations for velocities v^μ and u^μ acquire the form

$$\begin{aligned} D_c v^\mu - \lambda D_s u^\mu &= \frac{1}{m}\phi_\lambda^{(+)\mu} \\ D_c u^\mu + \lambda D_s v^\mu &= \frac{1}{m}\phi_\lambda^{(-)\mu} \end{aligned} \qquad (14)$$

$$\begin{aligned} D_c v^\mu - \lambda D_s v^\mu &= \frac{1}{m}\phi_\lambda^{\mu} \\ D_c u^\mu + \lambda D_s u^\mu &= \frac{1}{m}\phi_\lambda^{\prime\mu} \end{aligned} \qquad (15)$$

here $D_c = \frac{\partial}{\partial s} + v^\nu \partial_\nu$, $D_s = u^\nu \partial_\nu + D(\Box + \ell^2\Box^2)$, $\lambda = \pm 1$.

The functions $\phi_\lambda^{(+)\mu},\dots,\phi_\lambda^{\prime\mu}$ are expressed through $F_+^\mu,\dots,F_-^{\prime\mu}$ in the same way as in the nonrelativistic case. $\phi_\lambda^{(+)\mu}(\phi_\lambda^{(-)\mu})$ does not change (changes) sign, but $\phi_{1}^{\mu} \to \phi_{-1}^{\mu}$ and $\phi_1^{\prime\mu} \to (-1)\phi_{-1}^{\prime\mu}$ under the "time" reserval $s \to -s$.

Sivashinsky-type equations (8) now have the form

$$D_c v^\mu = -D(\Box + \ell^2\Box^2)v^\mu \qquad (16)$$

invariant under the Lorentz transformation.

We see that if we neglect the term of order $D\ell^2$ in the definition of the operator D_s, then in the case $\lambda = 1$ we have from (14) the relativistic equations of Guerra-Ruggiero /14/ and Vigier /15/, which we have, however, obtained by a different method.

The application of the method /16/ which is based on the concept of a derivative with respect to the direction of some time-like vector is useful and interesting in the relativistic description of a particle motion. So, for example, by this method the equations (11) and (12) become essentially simple due to the elimination of the dependence upon $u_\pm^\mu$ in the expression for the $\Psi_\pm$. In this case $\Psi_\pm$ can be chosen in the following form

$$\Psi_\pm = (4\pi D\Delta s_\pm)^{-2} \exp\{-\frac{y^2}{4D\Delta s_\pm}\},$$

where the quantities s_+ and s_- may be taken instead of the proper time; they can be interpreted as some parameters such that the derivativies with respect to them are equal to $\partial/\partial s_\pm = u^\nu_\pm \partial_\nu$.

Then the equations (13) acquire the form

$$\frac{\partial u^\mu_+}{\partial s_\pm} = \pm 2u^\nu \partial_\nu u^\mu_+ \pm D\Box_\ell u^\mu_+ + \left\{ \begin{matrix} F^\mu_+ \\ F'^\mu_+ \end{matrix} \right\} m^{-1},$$

$$\frac{\partial u^\mu_-}{\partial s_\pm} = \pm 2u^\nu \partial_\nu u^\mu \pm D\Box_\ell u^\mu_- + \left\{ \begin{matrix} F'^\mu_- \\ F_-^\mu \end{matrix} \right\} m^{-1}.$$

Having in mind the definition of the derivative with respect to directions u^μ_+ and u^μ_-, we obatin from these equations the equations (14) and (15) with $D_c = v^\nu \partial_\nu$ and $D_s = u^\nu \partial_\nu + D\Box_\ell$. The equations of the type (14) with $\lambda = 1$ and $D_c = v^\nu \partial_\nu$, $D_s = u^\nu \partial_\nu + D\Box$ have been obtained first by Pena-Auerbach /17/ and Lehr-Park /18/. They are, of course, equivalent to the one with $D_c = \frac{\partial}{\partial s} + v^\nu \partial_\nu$ and $D_s = u^\nu \partial_\nu + D\Box$.

4. CONCLUSION

We see that starting with the hypothesis about the spatial stochasticity and the assumption $u_i \Rightarrow 0$, we obtain the Sivashinsky equation. He has shown that Galilei-invariant turbulence - producing potentials imply instability of the uniform rectilinear motion of a particle and yield random fluctuations of its trajectory. Despite the fact that the classical-trajectory concept is retained, the mechanics of the particle then admits quantum-type effects: an uncertainty relation, de Broglie-type waves and their interference, discrete energy levels, and zero-point fluctuations.

These zero-point fluctuations, as we have seen previously, are responsible for the space stochasticity on a small scale. The self-interaction potential of a particle (the right-hand side of equations (8) and (16)) which generates turbulences in the motion of a free particle is of a stochastic origin. In other words, stochasticity (disappearing in the limit $u_i \Rightarrow 0$) as a self-memory makes the motion for $v_i(x_j, t)$ unstable.

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Received by Publishing Department
on November 29 1979.

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Издательский отдел Объединенного института ядерных исследований.
Заказ 27412. Тираж 680. Уч.-изд. листов 0,73.
Редактор Э.В.Ивашкевич.
Набор Л.М.Кашехлебовой, Е.М.Граменицкой.
Макет В.А.Халякиной. Подписано к печати 17.12.79 г.