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# GALILEO- INVARIANT THEORY OF LOW ENERGY PION-NUCLEUS SCATTERING. Part I

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## 1. INTRODUCTION

The scattering of a particle by a system of bound scatterers can yield valuable information about the amplitude of the elementary scattering act and about the properties of the bound system as well. Since the pioneering works of Fermi[1] and Goldberger and Watson[2] the impulse approximation (IA) has been the basic ingredient of practically all microscopic models of elastic scattering (optical model, Glauber theory) and inelastic scattering (plane or distorted wave impulse approximation, coupled channel method) on a composite target. IA refers to procedure which makes it possible to reduce effectively the many-body problem of projectile-composed system scattering to the two-body one. Practically, one assumes that the projectile is scattered by a free target particle the impulse distribution of which is determined by the wave function of the bound system.

In spite of the fact that the calculations based in IA have been refined considerably in the last few years[3], there is a fundamental uncertainty concerning the choice of the energy dependence of the elementary scattering amplitude. In fact, the energy $\tilde{E}$ at which the projectile-free scatterer amplitude is calculated represents a free parameter of the model. This deficiency would not be so serious if IA were used only in such kinematical situations where the elementary amplitude depends weakly on the energy (as was required in the original derivation[1,2]). Nevertheless, for lack of other techniques, one often assumes IA to be valid also in the resonance region ( $\Delta_{33}$ resonance in the pion-nucleon system) or at energies which are considerably less than the mean kinetic energy of the bound particle ( $\pi$ , K-mesoatoms). In such a type of calculations, the elementary amplitude is usually evaluated at the reaction energy E of the projectile and the whole composite system.

A further defect of IA has been recently pointed out by Nagarajan et al.[4], namely that the standard choice of the energy $\tilde{E} = E$ leads to models which are not Galileo-invariant. We believe that the low energy scattering should be described in terms of Galileo-invariant expressions (if, however, the number or type of particles does not change in the reaction). Moreover, the requirement of the Galilean invariance could restrict somewhat the arbitrariness in choosing the energy $\tilde{E}$. Those are the reasons for a careful reexamination of IA.

Elastic scattering of a pion by nuclei is studied here in the framework of a nonrelativistic optical model. Only when numerical calculations are reported, some relativistic corrections are introduced. The Galileo-invariant optical model is derived from the elementary pion-nucleon scattering amplitude $f(\tilde{E})$ using a correspondingly modified IA. In Section 2, an explanation is given as to why the optical potential and other models based on the standard IA violate the Galilean invariance. Further, a relationship is revealed between the choice of the energy $\tilde{E}$ and the validity of the factorization approximation.

The factorization approximation is connected with an approximate treatment of the motion of the target nucleon (Fermi motion). If the dependence of the pion-nucleon amplitude on nucleon momenta is either completely neglected (static approximation) in constructing the optical potential or the pion-nucleon amplitude is evaluated at some fixed effective nucleon momenta, the optical potential is obtained in factorized form: the elementary amplitude by the nuclear form factor. Usually, one assumes the target nucleon to be at rest in laboratory system: however, more appropriate effective nucleon momenta have been also introduced[5-8].

The main result of Section 2 consists in the following statement: If the energy $\tilde{E}$ is chosen in such a way that the resulting optical model is Galileo-invariant, then there exists a unique combination of effective target nucleon momenta in the initial and final states, by means of which the optical potential can be expressed in factorized form while the error caused by the factorization procedure is of the order of $(m/M)^2 \approx 1/50$. An arbitrary factorization procedure used in connection with a Galileo-noninvariant potential leads to an error of the order of $m/M \sim 1/7$, $m$ and $M$ being the mass of pion and nucleon, respectively.



## 2. IMPULSE AND FACTORIZATION APPROXIMATIONS

In the framework of the nonrelativistic potential theory, the pion-nucleon scattering matrix $T(E)$ is given by the two-body equation

$$T(E) = U(E)(1 + G(E)PT(E)). \tag{2.1}$$

Here, $P = |0\rangle\langle 0|$ is the projection operator, which projects onto the nuclear ground state. The Green function

$$G(E) = (E - h_0 - H_A + i\epsilon)^{-1} \tag{2.2}$$

contains the pion kinetic energy operator $h_0$ and the nuclear Hamiltonian $H_A$. The nucleus consists of $A$ nucleons, and the Hamiltonian $H_A$ can be split into its kinetic and potential energy parts

$$H_A = \sum_{i=1}^{A} h_i + U_A . \tag{2.3}$$

The complex many-body character of the problem is hidden in the potential matrix $U(E)$, which fulfils the equation

$$U(E) = A\tau(E) + (A-1)\tau(E)G(E)U(E) - \\ - A\tau(E)G(E)PU(E). \tag{2.4}$$

The Watson formulation [2,4] of the optical model is used here.

In the following, the first order optical potential

$$U^{(1)}(E) = A\tau(E) \tag{2.5}$$

will be considered. Neglecting the remaining terms in (2.4), we assume roughly that virtual nuclear excitations do not yield an important contribution to the elastic scattering (coherent scattering approximation). The pion scattering matrix for a bound nucleon $\tau(E)$ is defined as[9]

$$\tau(E) = v + vG(E)\mathcal{A}\tau(E). \tag{2.6}$$

Here, $v$ is a pion-nucleon potential and $\mathcal{A}$ is the projection operator which projects on the antisymmetric nuclear states. The operator $\tau(E)$ is still a very complicated many-body quantity.

In order to reduce the scattering problem to a two-body one, we introduce the pion-free nucleon scattering matrix $t(\tilde{E})$ as

$$t(\tilde{E}) = v + v d(\tilde{E}) t(\tilde{E}), \qquad (2.7)$$

where

$$d(\tilde{E}) = (\tilde{E} - h_0 - h_1)^{-1}. \qquad (2.8)$$

The following relation holds between the matrices $\tau(E)$ and $t(\tilde{E})$

$$\tau(E) = t(\tilde{E}) + t(\tilde{E}) \ (G(E) - d(\tilde{E}))\, \tau(E). \qquad (2.9a)$$

Further, the impulse approximation is introduced by the relation

$$\tau(E) \simeq t(\tilde{E}). \qquad (2.9b)$$

The energy of the pion-nucleon subsystem is a dynamical variable in many-body theories (e.g., in the Faddeev theory /10/; therefore we lack any reliable guide for the choice of the energy $\tilde{E}$. The validity of IA and particularly the choice made for $\tilde{E}$ can be tested, in principle, via Eq. (2.9a). However, this tedious task has never been undertaken up to now and one usually postulates

$$\tilde{E} = E. \qquad (2.10)$$

Adopting for a moment the choice (2.10), we analyze in detail the first order optical potential

$$\langle \vec{p}\,' \vec{\kappa}\,' 0 | U^{(1)}(E) | 0 \vec{\kappa} \vec{p} \rangle = \frac{A}{(2\pi)^6} \int e^{i(\vec{r}_1\,' \cdot \vec{k}_1\,' - \vec{r}_1 \cdot \vec{k}_1)} \rho_{\vec{\kappa}\,', \vec{\kappa}}(\vec{r}_1\,', \vec{r}_1) \qquad (2.11)$$

$$\times \langle \vec{p}\,' \vec{k}_1\,' | t(E) | \vec{k}_1 \vec{p} \rangle \, d^3k_1' \, d^3k_1 \, d^3r_1 \, d^3r_1'$$

in arbitrary system. To accomplish this, we introduce the density matrix

$$\rho_{\vec{\kappa}\,'\vec{\kappa}}(\vec{r}_1\,', \vec{r}_1) = \int \langle \vec{\kappa}\,' 0 | \vec{r}_1\,' \vec{r}_2 \ldots \vec{r}_A \rangle \langle \vec{r}_A \ldots \vec{r}_2 \vec{r}_1 | 0 \vec{\kappa} \rangle \prod_{i=2}^{A} d^3 r_i \qquad (2.12)$$



for the target nucleon "1". The spin and isospin variables are suppressed in (2.11), assuming that the nuclear spin J and isospin T are both equal to zero. In the Table, the kinematical variables are introduced into the three systems of our interest. The pion-nucleus and the pion-nucleon centre-of-mass system will be referred to as Acm and 2cm, respectively. Familiar relations hold for the momenta defined in the Table

$$\vec{Q} = \mathfrak{M} \left(\frac{\vec{p}}{m} - \frac{\vec{\kappa}}{AM}\right), \quad \vec{q}_i = \mu \left(\frac{\vec{p}}{m} - \frac{\vec{k}_1}{M}\right), \text{etc.}, \qquad (2.13)$$

where

$$\mu = mM/(m+M) \qquad \mathfrak{M} = mAM/(m+AM), \qquad (2.14)$$

is the pion-nucleon and pion-nucleus reduced mass, respectively.

Table

Notation used for energies and momenta of the interacting particles

| | Arbitrary system | Acm | 2cm |
|---|---|---|---|
| Initial (final) pion momentum | $\vec{p}(\vec{p}\,')$ | $\vec{Q}(\vec{Q}\,')$ | $\vec{q}_i(\vec{q}_f)$ |
| Initial (final) nuclear momentum | $\vec{\kappa}(\vec{\kappa}\,')$ | $-\vec{Q}(-\vec{Q}\,')$ | |
| Initial (final) momentum of nucleon "1" | $\vec{k}_1(\vec{k}_1\,')$ | $\vec{k}(\vec{k}\,')$ | $-\vec{q}_i(-\vec{q}_f)$ |
| Reaction energy (momentum) | E | $E_{Ac}(p_{Ac})$ | $z(p_{2c})$ |

In order to express the optical potential in terms of Galileo-invariant variables, it is advantageous to introduce the Jacobi coordinates /5/

$$\vec{\xi}_1 = \vec{r}_A - \vec{r}_{A-1} \qquad \vec{\xi} = \frac{\vec{r}_A + \vec{r}_{A-1} + \ldots + \vec{r}_2}{A-1}$$

$$\vec{\xi}_2 = \frac{\vec{r}_A + \vec{r}_{A-1}}{2} - \vec{r}_{A-2} \qquad \vec{\xi}_{A-1} = \vec{\xi} - \vec{r}_1$$

$$\vdots \tag{2.15a}$$

$$\vec{\xi}_{A-2} = \frac{\vec{r}_A + \vec{r}_{A-1} + \dots + \vec{r}_3}{A-2} - \vec{r}_2 \qquad \vec{\xi}'_{A-1} = \vec{\xi} - \vec{r}'_1$$

into (2.11) and (2.12) and to define

$$<\vec{p}'\vec{k}'_1|t(E)|k_1 p> = (2\pi)^3\delta^3(\vec{p}'+\vec{k}'_1-\vec{p}-\vec{k}_1)<\vec{q}_f|t_r(z)|\vec{q}_i>. \tag{2.15b}$$

If the substitutions

$$\vec{k}_1 = \frac{\vec{\kappa}}{A} - \frac{A-1}{2A}(\vec{Q}-\vec{Q}') - \vec{v}_1 \equiv \vec{k}_{eff} - \vec{v}_1$$

$$k'_1 = \frac{\vec{\kappa}'}{A} + \frac{A-1}{2A}(\vec{Q}-\vec{Q}') - \vec{v}'_1 \equiv k'_{eff} - \vec{v}'_1 \tag{2.16}$$

are used in (2.11), we encounter two delta functions - $\delta^3(\vec{p}'+\vec{\kappa}'-\vec{p}-\vec{\kappa})\,\delta^3(\vec{v}'_1-\vec{v}_1)$. After integration over $d^3\vec{v}'_1$, the resulting expression for the optical potential is obtained

$$<\vec{p}'\vec{\kappa}'0|U^{(1)}(E)|0\vec{\kappa}\vec{p}> = (2\pi)^3\delta^3(\vec{p}'+\vec{\kappa}'-\vec{p}-\vec{\kappa})<\vec{Q}'0|U_r(E_{Ac})|0\vec{Q}> \tag{2.17a}$$

$$<\vec{Q}'0|U_r(E_{Ac})|0\vec{Q}> = \frac{A}{(2\pi)^3}\int e^{i\frac{A-1}{2A}(\vec{Q}'-\vec{Q})\cdot(\vec{\xi}'_{A-1}+\vec{\xi}_{A-1})}$$

$$\times\, e^{i\vec{v}_1\cdot(\vec{\xi}'_{A-1}-\vec{\xi}_{A-1})}\rho_{00}(\vec{\xi}'_{A-1},\vec{\xi}_{A-1})<\vec{q}_f|t_r(z)|\vec{q}_i>$$

$$\times\, d^3\xi'_{A-1}\,d^3\xi_{A-1}\,d^3v_1. \tag{2.17b}$$

The density matrix $\rho_{00}(\vec{\xi}'_{A-1},\vec{\xi}_{A-1})$ does not contain the motion of the nucleus as a whole. In obtaining Eq. (2.17b)



we used the invariance of the transferred momentum $\vec{p}'-\vec{p} = \vec{Q}'-\vec{Q}$ under the Galilean transformation.

It is instructive to write down the explicit expressions for the kinematical variables on which the pion-nucleon scattering matrix depends

$$\vec{q}_f = \vec{Q}' - \frac{A-1}{2A}\frac{\mu}{M}(\vec{Q}'+\vec{Q}) + \frac{\mu}{M}\vec{v}_1 = \vec{q}_{f,0} + \frac{\mu}{M}\vec{v}_1 \tag{2.18a}$$

$$\vec{q}_i = \vec{Q} - \frac{A-1}{2A}\frac{\mu}{M}(\vec{Q}'+\vec{Q}) + \frac{\mu}{M}\vec{v}_1 = q_{i,0} + \frac{\mu}{M}\vec{v}_1 \tag{2.18b}$$

$$z = E - \frac{(\vec{p}+\vec{k}_1)^2}{2(m+M)} = E_{Ac} + \frac{P^2}{2(m+AM)} - \frac{1}{2(m+M)}\left(\frac{\mathfrak{M}}{\mu}\frac{\vec{P}}{A} + \frac{A-1}{2A}(\vec{Q}'+\vec{Q}) - \vec{v}_1\right)^2. \tag{2.18c}$$

From (2.18c) we can see that the energy z depends on the momentum $\vec{P} = \vec{p} + \vec{\kappa}$ of the whole pion-nucleus system. The dependence of the optical potential on the momentum $\vec{P}$ cannot be removed by any substitution for the variable $\vec{v}_1$ since, except for the energy z, the other terms in (2.17b) contain the obviously Galileo-invariant quantities $\vec{Q}',\vec{Q}$ and $E_{Ac}$. It can be concluded from (2.18c) that the optical potential depends on the system in which it is evaluated and the Galilean invariance defect is more serious in the case of light nuclei.

The step of our derivation, in which the dependence on momentum P was introduced into the optical potential, can easily be traced out in (2.8) and (2.10). In fact, the Green function d(E) contains the (A+1)-body energy E and the two-body Hamiltonian $(h_0+h_1)$ as well. The behaviour of these quantities is apparently quite different with respect to the Galilean transformation. In the next Section we shall give two alternative definitions of the auxilliary matrix $t(\tilde{E})$, i.e., we shall construct different impulse approximation schemes, which maintain the Galilean invariance of the optical potential. Now we investigate in some detail the problems associated with the factorization approximation.

The optical potential (2.17b) becomes much simpler when the terms $\mu\vec{v}_1/M$ are neglected in the expression for $<q_f|t_r(z)|q_i>$. Then we have

$$<Q'0|U_r(E_{Ac})|0Q> = AF(\vec{Q}'-\vec{Q})<q_{f,0}|t_r(z_0)|q_{i,0}>, \tag{2.19}$$

where the momenta $q_{f,0}$ and $q_{i,0}$ are defined by (2.18a,b) and

$$F(\vec{Q}'-\vec{Q}) = \int e^{i\frac{A-1}{A}(\vec{Q}'-\vec{Q})\cdot\vec{\xi}_{A-1}} \rho_{00}(\vec{\xi}_{A-1}, \vec{\xi}_{A-1})\, d^3\xi_{A-1} \qquad (2.20)$$

is the nuclear form factor. For definiteness, we work in $Acm(\vec{P}=0)$, where

$$z_0 = E_{Ac} - \frac{1}{8}\left(\frac{A-1}{A}\right)^2 \frac{(\vec{Q}'+\vec{Q})^2}{m+M}. \qquad (2.21)$$

The meaning of the factorization procedure leading to (2.19) can be explained in the following way. The pion-nucleon scattering matrix is evaluated at fixed effective nucleon momenta $\vec{k}'_{eff}$ and $\vec{k}_{eff}$ rather than folded with the density matrix over all nucleon momenta (compatible with the momentum conservation in the elementary scattering act).

It should be noted that in the standard static approximation one neglects not only the terms containing the variable $\vec{v}_1$, but all the terms in (2.18) proportional to $m/M$. A more consistent approach relies on the assumption that the target nucleon is at rest in the laboratory system. This is equivalent to the following definition of the effective nucleon momenta:

$$\vec{k}_{eff,0} = \frac{\vec{\kappa}}{A}, \qquad \vec{k}'_{eff,0} = \frac{\vec{\kappa}}{A} + \frac{A-1}{A}(\vec{Q}'-\vec{Q}). \qquad (2.22)$$

While the early use [5-7] of the more sophisticated definition (2.16) has been motivated mainly by intuitive considerations, it will be shown here that the effective momenta (2.16) represent the optimal choice.

In order to estimate the error caused by the factorization approximation, we represent the pion-nucleon scattering matrix in (2.17b) as a power series

$$\begin{aligned} <\vec{q}_f|t_r(z)|\vec{q}_i> &= <\vec{Q}'|t_r(E_{Ac})|\vec{Q}> \\ &+ x\left[\left(\frac{\partial}{\partial\vec{q}_f} + \frac{\partial}{\partial\vec{q}_i}\right)<\vec{q}_f|t_r(E_{Ac})|\vec{q}_i>\right]_{x=0} \\ &\times\left(\vec{v}_1 - \frac{A-1}{2A}(\vec{Q}'+\vec{Q})\right) - \frac{x}{2p_{Ac}}\left[\frac{\partial}{\partial p_{2c}}<\vec{Q}'|t_r(z)|\vec{Q}>\right]_{x=0} \\ &\times\left[\left(\frac{A-1}{2A}\right)^2(\vec{Q}'+\vec{Q})^2 - \frac{A-1}{A}\vec{v}_1\cdot(\vec{Q}'+\vec{Q}) + v_1^2\right] + O(x^2). \end{aligned} \qquad (2.23)$$



Here, $x = m/M$ and $p_{2c} = \sqrt{2\mu z}$. Therefore, the terms linear in $x$ contain the variable $\vec{v}_1$ in two different combinations: $v_1^2$ and $(\vec{a}\cdot\vec{v}_1)$, where $\vec{a}$ is a constant vector. Earlier it has been shown [5] (see also [9]) that the identity

$$\int e^{i\frac{A-1}{2A}(\vec{Q}'-\vec{Q})\cdot(\vec{\xi}'_{A-1}+\vec{\xi}_{A-1})} e^{i\vec{v}_1\cdot(\vec{\xi}'_{A-1}-\vec{\xi}_{A-1})}(\vec{a}\cdot\vec{v}_1) \times \rho_{00}(\vec{\xi}'_{A-1}, \vec{\xi}_{A-1})\, d^3\xi'_{A-1}\, d^3\xi_{A-1}\, d^3v_1 = 0 \qquad (2.24)$$

holds for $J=0$ nuclei. Thus the $(\vec{a}\cdot\vec{v}_1)$ terms vanish when (2.23) is substituted into Eq. (2.17b). The proof of (2.24) is based on the substitution

$$\vec{v}_1 e^{i\vec{v}_1\cdot(\vec{\xi}'_{A-1}-\vec{\xi}_{A-1})} = \frac{i}{2}(\vec{\nabla}\xi_{A-1} - \vec{\nabla}\xi'_{A-1})\, e^{i\vec{v}_1\cdot(\vec{\xi}'_{A-1}-\vec{\xi}_{A-1})} \qquad (2.25)$$

Inserting (2.25) in the integral (2.24) leads to the vanishing Fourier component of the nuclear current. By virtue of the substitution (2.16) the coordinates $\vec{\xi}'_{A-1}$ and $\vec{\xi}_{A-1}$ enter the expression $\exp(i(A-1)(\vec{Q}'-\vec{Q})\cdot(\vec{\xi}'_{A-1}+\vec{\xi}_{A-1})/A)$ symmetrically. That is the reason why no $(\vec{Q}'-\vec{Q})F(\vec{Q}'-\vec{Q})$ terms arise when the per partes integration succeeding the substitution (2.25) is performed in (2.24). The effective nucleon momenta $\vec{k}'_{eff}$ and $\vec{k}_{eff}$ defined by (2.16) are unique in the following sense: Any other (linear) combination of pion and nucleon momenta necessarily cause an error of the order of $m/M$ in the optical potential since the $(\vec{a}\cdot\vec{v}_1)$ terms do not vanish. Particularly, it is the case of the effective momenta $\vec{k}'_{eff,0}$ and $\vec{k}_{eff,0}$ as defined by (2.22).

The $v_1^2$ term in (2.23) yields a nonvanishing contribution to the optical potential. Solely due to this term, the inaccuracy of the resulting optical potential (2.19) is of the order of $m/M$. The question arises as to whether the peculiar appearance of the $v_1^2$ term in the two-body energy $z$ has something to do with the Galilean noninvariance of the last quantity. Let us suppose for a moment that the energy $z$ depends only on the scalar combinations of the Galileo-invariant kinematical variables, such as $E_{Ac}$, $q^2_{i(f)}$, $\vec{q}_i\cdot\vec{q}_f$ and maybe also $q'^2_{i(f)}$, $\vec{q}'_i\cdot\vec{q}'_f$, etc., where

$$\vec{q}'_i = \left(\frac{\vec{p}}{m} - \frac{\vec{k}'_1}{M}\right) \qquad q'_f = \left(\frac{\vec{p}'}{m} - \frac{\vec{k}_1}{M}\right). \qquad (2.26)$$

Evidently, the $v_1^2$ term enters such combinations only together with the coefficient $(m/M)^2$, e.g.,

$$\vec{q}_f \cdot \vec{q}_i = \vec{q}_{f,0} \cdot \vec{q}_{i,0} + \frac{\mu}{M} \vec{v}_1 \cdot (\vec{q}_{f,0} + \vec{q}_{i,0}) + (\frac{\mu}{M})^2 v_1^2 . \qquad (2.27)$$

Therefore, no $v_1^2$ terms linear in $\mu/M$ arise in (2.23). We can conclude that the factorization approximation yields an optical potential correct up to $m/M$ terms, provided that the effective momenta $\vec{k}'_{eff}$ and $\vec{k}_{eff}$ are defined according to (2.16) and the Galileo-invariant version of the impulse approximation is used.

A detailed discussion of the possible Galileo-invariant forms of the energy $z$ is postponed until the next Section. We would like to emphasize once more that the quality of the factorization approximation is intimately connected with the particular IA scheme used in constructing the optical potential. Generally speaking, the factorization approximation based on the optimal effective momenta $\vec{k}'_{eff}$ and $\vec{k}_{eff}$ will give much better results in Galileo-invariant models than in the noninvariant ones.

The factorization approximation is not, of course, of primary importance since the integration over $d^3v_1$ and $d^3\xi'_{A-1}$ can be performed numerically in (2.17b). Such studies have been recently undertaken by several authors [12,13]. Practically, however, the time-consuming numerical integration makes it impossible to take into account other important corrections, such as corrections to the coherent scattering approximation or to the true pion absorption. According to our opinion, the surplus in accuracy achieved when the factorization approximation is abandoned hardly warrants the additional effort since the main difference between the exact and the approximate results comes from the dubious $v_1^2$ term in the Galileo-noninvariant expressions for the two-body energy $z$. The calculated results of [13] are worth mentioning in this respect. The differential cross sections for elastic $\pi - {}^4He$ scattering lie closer to the experimental data when calculated in the factorization approximation frame than the exact results (i.e., when the $d^3v_1$ and $d^3\xi'_{A-1}$ integrations are performed).

The concept of optimal effective nucleon momenta appeared to be a useful tool also in studying inelastic pion-nucleus processes. Recently, we have investigated the pion-induced knock-out reactions [14] on $^4He$ (e.g., the reaction $^4He(\pi^-,\pi^- p)^3H$ ) in the modified plane-wave Galileo-invariant impulse approximation. In order not to violate the Pauli



principle, we assumed the pion quasielastic scattering by a nucleon and by a three nucleon cluster as well. Also for the knock-out reaction we succeeded in finding effective nucleon momenta, which render the factorization approximation valid up the $m/M$ terms. In this case, the effective momenta differ, of course, from those defined by (2.16).

In concluding this Section - remark will be made concerning the $J \neq 0$ nuclei. Now, Eq. (2.24) is not valid any more and terms like

$$\frac{m}{AM} \langle 0 | \vec{J} | 0 \rangle F(\vec{Q}' - \vec{Q}) \qquad (2.28)$$

contribute to the optical potential. We have shown [15] that except for very special situations such terms can be neglected.

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