объединенный
институт
ядерных
исследований
дубна

E2 - 12919

V.N.Kapshay, N.B.Skachkov

EXACT SOLUTION OF THE RELATIVISTIC COULOMB PROBLEM FOR TWO-PARTICLE BOUND STATES IN THE QUASIPOTENTIAL APPROACH

1979

E2 - 12919

V.N.Kapshay, N.B.Skachkov

EXACT SOLUTION OF THE RELATIVISTIC COULOMB PROBLEM FOR TWO-PARTICLE BOUND STATES IN THE QUASIPOTENTIAL APPROACH

Submitted to ТМФ

Капшай В.Н., Скачков Н.Б. **E2 - 12919**

Точное решение релятивистской кулоновской проблемы для связанных состояний двух частиц в квазипотенциальном подходе

Целью работы является нахождение точных решений релятивистского квазипотенциального уравнения Кадышевского, которое представляет собой прямое релятивистское обобщение уравнения Шредингера в импульсном представлении. Выбранный в качестве квазипотенциала пропагатор обмена безмассовым мезоном также представлен в виде геометрического обобщения нерелятивистского потенциала Кулона. Показано, что соответствующим решением релятивистского двухчастичного уравнения является прямое геометрическое обобщение нерелятивистских решений в смысле замены эвклидовой геометрии пространства импульсов на геометрию Лобачевского.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1979

Kapshay V.N., Skachkov N.B. **E2 - 12919**

Exact Solution of the Relativistic Coulomb Problem for Two-Particle Bound States in the Quasipotential Approach

A composite system of two relativistic particles is studied on the basis of the Kadyshevsky quasipotential equation, in which the "Coulomb" potential is taken in the form of a propagator of the massless-scalar-particle exchange. The obtained exact solutions to this equation are shown to be a geometrical generalization of nonrelativistic Coulomb wave functions in the sense of change of the Euclidean geometry of momentum space to the Lobachevsky geometry.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1979

1. INTRODUCTION

The description of the two-particle relativistic system is one of the principal problems of the quantum field theory. To study this problem within the four-dimensional Feynman-Dayson formalism a completely covariant equation has been derived and named by the Bethe-Salpeter equation [1,2]. However there is no clear physical interpretation of wave function dependence on the relative time of two particles in the framework of the four-dimensional Bethe-Salpeter approach. A quasipotential approach suggested by Logunov and Tavkhelidze [3,4] does not contain this shortcoming.

A quasipotential approach of Kadyshevsky [5] is based on the Hamiltonian formulation of quantum field theory [6]. A distinguishing feature of such a formulation is that 4-momenta of all the particles even in intermediate states are on mass surfaces*

$$p_0^2 - \vec{p}^{\,2} = m^2 . \qquad (1.1)$$

As a result quasipotential equations for a relativistic scattering amplitude and wave function appear to be three-dimensional. In the case of spinless particles with equal masses ($m_1 = m_2 = m$) these equations are written in the form

$$A(\vec{p},\vec{q}) = -\frac{m}{4\pi} V(\vec{p},\vec{q};E_q) + \frac{1}{(2\pi)^3}\int V(\vec{p},\vec{k};E_q)\frac{m\,d\vec{k}}{\sqrt{\vec{k}^2+m^2}}\,\frac{A(\vec{k},\vec{q})}{2E_q-2E_k+i\epsilon}; \qquad (1.2)$$

*When it is not needed we use a unit system $h = c = 1$, $\frac{e^2}{4\pi} = \frac{1}{137}$, where e is the charge of an electron.

$$(2E_q - 2E_p)\Psi_q(\vec{p}) = \frac{1}{(2\pi)^3}\int V(\vec{p},\vec{k};E_q)\frac{m\,d\vec{k}}{\sqrt{\vec{k}^2+m^2}}\Psi_q(\vec{k}). \qquad (1.3)$$

Here all the momenta are given in the system of the inertial centre ($\vec{p}_1 = -\vec{p}_2 = \vec{p}$), $E_p = \sqrt{\vec{p}^{\,2} + m^2}$, $E_k = \sqrt{\vec{k}^{\,2} + m^2}$, $2E_q$ is the total energy of the system, and $V(\vec{p},\vec{k},E_q)$ is the quasipotential.

Condition (1.1) in the 4-momentum space gives a hyperboloid surface, the upper sheet of which represents a constant negative curve space - three-dimensional Lobachevsky space. Forms of equations (1.2) and (1.3) differ from corresponding nonrelativistic Lippman-Schwinger and Schrödinger ones only by relativistic sense of expressions for E_p, E_k and element of integration volume

$$d\Omega_{\vec{k}} = \frac{m\,d\vec{k}}{\sqrt{\vec{k}^{\,2}+m^2}}, \qquad (1.4)$$

which is an element of volume in the Lobachevsky space.

For bound state problem, according to procedure [4,5], a quasipotential is formed with the use of the relativistic scattering amplitude on the mass surface, as solution of (1.2) relative to unknown function $V(\vec{p},\vec{k};E_q)$. In this case the amplitude $A(\vec{p},\vec{q})$ is considered to be given, for instance, by diagrams of field theory. In ref. [9] there was suggested a modification of this approach, allowing to keep the locality of one-boson exchange amplitudes, chosen as a quasipotential in the Lobachevsky space beyond the energetic surface $E_p = E_q$.

In the present paper, equation (1.3) in the case of a quasipotential, corresponding to the exchange by a massless scalar particle, - the "relativistic Coulomb"

$$V(\vec{p},\vec{k};E\ ) = \frac{e^2}{(p-k)^2}, \qquad (1.5)$$

is being investigated.

The difference between (1.5) and quasipotentials, arising from the production in the pointed manner, is in the absence of a factor, containing a parameter dependence on a system energy $2E_q$. We have omitted this factor because its calculation, needed in practical applications does not cause any difficulty, and its absence makes a geometrical analogy of nonrelativistic and relativistic equations more evident.

As is seen from ref. [10] quasipotentials of one-boson exchange in the momentum representation can be represented in the "absolute form",that is,in the form of a direct geometrical generalization of nonrelativistic potentials. This allows us to represent equation (1.3) with quasipotential (1.5) in the form of an analogue of the Schrödinger equation with the Coulomb potential in the Lobachevsky momentum space, and to find solutions (1.3) also in the form of a geometrical generalization of known nonrelativistic wave functions.

In the next paragraph we shall give necessary information about the Lobachevsky geometry. In §3 equation (1.3) with quasipotential (1.5) is transformed to the "absolute form". In §4 exact relativistic wave functions - solutions of equation (1.3) are found and a relativistic condition of level quantization of two-particle system energy is derived.

2. THE LOBACHEVSKY GEOMETRY AND A RELATIVISTIC KINEMATICS

It is known that all the formulae of the relativistic kinematics, including a relativistic velocity addition law, can be obtained by the substitution of the Euclidean geometry of the three-dimensional velocity space for the Lobachevsky geometry[11-13]. Thus in a special relativity theory the relative velocity of two particles, moving at the velocities $\vec{v}_p$ and $\vec{v}_k$,

$$\vec{v}_{rel} = \frac{\vec{v}_p - \vec{v}_k - [1-(1-\frac{\vec{v}_k^2}{c^2})^{-1/2}]\frac{\vec{v}_k}{\vec{v}_k^2}(\vec{v}_p\vec{v}_k - \vec{v}_k^2)}{(1-\frac{\vec{v}_k^2}{c^2})(1-\frac{\vec{v}_p\vec{v}_k}{c^2})}, \qquad (2.1)$$

from the geometrical point of view represents the difference between two vectors: $\vec{v}_p$ and $\vec{v}_k$ in the Lobachevsky space[12-16].

In the relativistic theory, where the energy of the particle p_0 is expressed over the momentum p by formula (1.1), a three-dimensional velocity is determined by the relation $\vec{v}_p = \frac{\vec{p}}{p_0}$.

One may introduce various coordinate systems on surface (1.1). As coordinates in the Lobachevsky space we choose the Cartesian coordinates of vector $\vec{p}$ on the hyperplane $p_0 = 0$ onto which the hyperboloid is mapped while projecting from the point $(\infty, \vec{0})$. Thus, all the three-dimensional $\vec{p}$ - space with a metric:

$$ds^2 = d\vec{p}^{\,2} - \frac{(\vec{p}\,d\vec{p})^2}{m^2 + \vec{p}^{\,2}} \equiv g_{ij}(\vec{p})\,dp^i dp^j \tag{2.2}$$

will serve now as a model of the Lobachevsky momentum space. Volume element (1.4) can be obtained from this metric. In the nonrelativistic limit ($c \to \infty$) a hyperboloid curve tends to zero and the Lobachevsky space turns into an ordinary plane three-dimensional Euclidean momentum space. Here $d\Omega_p \to d\vec{p}$; $ds^2 \to d\vec{p}^{\,2}$. A group of the Lobachevsky space motions realized on hyperboloid (1.1) is the Lorentz group. Pure Lorentz transformations L_k ("boosts"), when $L_k(m,\vec{0}) = (k_0, \vec{k})$,

$$\overrightarrow{L_k^{-1} p} \equiv \vec{p}(-)\vec{k} \equiv \vec{\Delta}_{p,k} = \vec{p} - \frac{\vec{k}}{m}\left(p_0 - \frac{\vec{p}\cdot\vec{k}}{k_0 + m}\right), \tag{2.3}$$

$$(L_k^{-1} p)^\circ \equiv (p(-)k)^\circ \equiv \Delta^\circ_{p,k} = \frac{p_0 k_0 - \vec{p}\vec{k}}{m} \tag{2.4}$$

in the nonrelativistic limit turn into transformations of the translation in the plane Euclidean space: $\vec{p}(-)\vec{k} \to \vec{p} - \vec{k}$. In the spherical coordinates

$$E_p = p_0 = m\,\mathrm{ch}\,\chi_p, \ \vec{p} = \vec{n}_p\, m\,\mathrm{sh}\,\chi_p, \ \vec{n}_p = \frac{\vec{p}}{|\vec{p}|};$$
$$E_k = k_0 = m\,\mathrm{ch}\,\chi_k, \ \vec{k} = \vec{n}_k\, m\,\mathrm{sh}\,\chi_k, \ \vec{n}_k = \frac{\vec{k}}{|\vec{k}|}; \tag{2.5}$$

equality (2.4) takes on the form of a theorem on a cosine of a compound angle in the Lobachevsky trigonometry /12-16/

$$\mathrm{ch}\,\chi_{p,k} = \mathrm{ch}\,\chi_p\,\mathrm{ch}\,\chi_k - \vec{n}_p\cdot\vec{n}_k\,\mathrm{sh}\,\chi_p\,\mathrm{sh}\,\chi_k. \tag{2.6}$$

The hyperbolic angle χ in the relativistic kinematics is called rapidity /17/. A distance in the velocity space of relativistic particles, that is, in the Lobachevsky space, is known to be measured in terms of rapidity /17,18/. Expression

$$ds_p = m\,d\chi_p \tag{2.7}$$

as it follows from (2.2) serves as an analog of a radial Euclidean length element

$$ds_{p,eucl} = d|\vec{p}| \tag{2.8}$$

in the momentum Lobachevsky space.

The vector $\vec{\Delta}_{p,k} = \vec{p}(-)\vec{k}$ can be considered as a relativistic geometric generalization of the momentum transfer vector $\vec{p}-\vec{k}$ and is a difference between two vectors in the Lobachevsky momentum space. Really, using a relativistic particle three-dimensional velocity definition $\vec{v}_k = \frac{\vec{k}}{k^\circ}$, $\vec{v}_p = \frac{\vec{p}}{p^\circ}$, it is easy to show from (2.3) that

$$\vec{v}_{rel} = \frac{\vec{\Delta}_{p,k}}{\Delta^\circ_{p,k}} = \frac{\vec{p}(-)\vec{k}}{(p(-)k)^\circ},$$

where the sign (-) means a difference of vectors in the Lobachevsky space [7,8]. With the help of (2.3) and (2.4) it is easy to see that the quadrate of the four-dimensional vector of the momentum transfer is expressed by the vector $\vec{p}(-)\vec{k}$ in the following way [7,8]

$$t = (p-k)^2 = 2m^2 - 2m\sqrt{m^2 + (\vec{p}(-)\vec{k})^2}\,. \tag{2.9}$$

In the space of relativistic velocities a particle half-velocity notion, proposed in ref. [13], plays an important role. Values of analogous sense in the Lobachevsky momentum space: half-momentum of a particle

$$\pi_p = (\pi_p^\circ, \vec{\pi}_p) = m(\mathrm{ch}\,\frac{\chi_p}{2}, \vec{n}_p\,\mathrm{sh}\,\frac{\chi_p}{2}) \tag{2.10}$$

and half-transfer-momentum κ

$$\begin{aligned} \kappa^\circ_{p,k} &= m\,\mathrm{ch}\,\frac{\chi_\Delta}{2}, \quad \Delta^\circ_{p,k} = m\,\mathrm{ch}\,\chi_\Delta; \\ \vec{\kappa}^\circ_{p,k} &= m\,\vec{n}_\Delta\,\mathrm{sh}\,\frac{\chi_\Delta}{2}, \quad \vec{\Delta}_{p,k} = m\vec{n}_\Delta\,\mathrm{sh}\,\chi_\Delta . \end{aligned} \tag{2.11}$$

have been considered in [10]. Relation (2.9) in terms of half-transfer momentum vector takes an "absolute" form

$$t = (k-p)^2 = -4\vec{\kappa}^2_{p,k} \tag{2.12}$$

because in the non-relativistic limit (when $\vec{\pi}_p \to \vec{\pi}_{p,eucl} = \frac{\vec{p}}{2}$, and $\vec{\kappa}_{p,k} \to \vec{\kappa}_{p,k,eucl} = \frac{\vec{p}-\vec{k}}{2}$) it turns into a nonrelativistic relation

$$t = (k-p)^2 \to -(\vec{k}-\vec{p})^2 = -4\vec{\kappa}^{\,2}_{p,k,eucl} . \qquad (2.13)$$

without changing its form (see consideration in ref. [10]). An expression for a relativistic kinetic energy in half-momentum terms also takes an "absolute" form, since

$$E_p = m \operatorname{ch} \chi_p = m + \frac{2\vec{\pi}_p^{\,2}}{m} . \qquad (2.14)$$

3. "ABSOLUTE" FORM OF QUASIPOTENTIAL EQUATION IN THE COULOMB INTERACTION CASE

The Schrödinger integral equation, describing the internal motion of bound systems of two nonrelativistic particles with equal masses ($m_1 = m_2 = m$), is written in the form:

$$\left(\frac{\vec{p}^{\,2}}{2\mu} - \epsilon\right)\Psi(\vec{p}) = -\frac{1}{(2\pi)^3}\int V(\vec{p}-\vec{k})\Psi(\vec{k})\,d\vec{k}. \qquad (3.1)$$

Here $\vec{p}$, $\vec{k}$ are particle momenta in the centre mass system, $\mu = \frac{m}{2}$, $\epsilon = -\epsilon_b$, where ϵ_b is nonrelativistic binding energy. In the case of a Coulomb interaction the potential $V(\vec{p}-\vec{k})$ has the form:

$$V(\vec{p}-\vec{k}) = -\frac{e^2}{(\vec{p}-\vec{k})^2} . \qquad (3.2)$$

Using the Euclidean half-momentum of the particle $\vec{\pi}_{p,eucl}$ and the Euclidean half-transfer-momentum $\vec{\kappa}_{p,k,eucl}$, assume (3.1) with potential (3.2) in the form

$$\left(\frac{4\vec{\pi}^{\,2}_{p,eucl}}{m} + \epsilon_b\right)\Psi(\vec{p}) = \frac{1}{(2\pi)^3}\int \frac{e^2}{4\vec{\kappa}^{\,2}_{p,k,eucl}}\Psi(\vec{k})\,d\vec{k}. \qquad (3.1')$$

Let us turn to the relativistic case. Using relativistic expressions (2.12) and (2.14), and taking into account that the binding energy in the relativistic case is expressed over the total system energy $2E_q$ by the formula

$$E_b = 2m - 2E_q \qquad (3.3)$$

we write down (1.3) with quasipotential (1.5) in the form

$$\left(\frac{4\vec{\pi}_p^2}{m}+E_b\right)\Psi_q(\vec{p})=\frac{1}{(2\pi)^3}\int\frac{e^2}{4\vec{\kappa}^2_{p,k}}\Psi_q(\vec{k})d\Omega_{\vec{k}}. \qquad (3.4)$$

Comparing (3.1') and (3.4) we see that all the values in (3.4): quasipotential (1.5), volume element (1.4), and a relativistic free Green function (here and below $p=|\vec{p}|$ and so on)

$$G_{rel}(p,E_q)=\left(2\sqrt{\vec{p}^2+m^2}-2E_q\right)^{-1}=(2m\,\mathrm{ch}\,\chi_p-2E_q)^{-1} \qquad (3.5)$$

are relativistic geometric generalizations of corresponding nonrelativistic expressions: potential (3.2), the volume element $d\mathbf{k}$, and a nonrelativistic free Green function

$$G_{non}(p,\epsilon_b)=\left(\frac{p^2}{2\mu}+\epsilon_b\right)^{-1}=\left(\frac{4\vec{\pi}^2_{p,eucl}}{m}+\epsilon_b\right). \qquad (3.6)$$

This sets us thinking that solutions (3.4) can represent a geometrical generalization of nonrelativistic Coulomb wave functions.

Equation (3.1) with potential (3.2) by means of a partial expansion

$$\Psi(\vec{p})=\psi_\ell(p)Y_{\ell m}(\vec{n}\ \ )=\frac{1}{p}g_\ell(p)Y_{\ell m}(\vec{n}_p) \qquad (3.7)$$

is reduced to a one-dimensional integral equation for the function $g_\ell(p)$:

$$\left(\frac{p^2}{2\mu}+\epsilon_b\right)g_\ell(p)=\frac{e^2}{2\pi^2}\int_0^\infty Q_\ell\left(\frac{p^2+k^2}{2pk}\right)g_\ell(k)dk, \qquad (3.8)$$

where Q_ℓ is the Legendre function of a second kind. In the case $\ell=0$ with the help of explicit form

$$Q_0(z)=\frac{1}{2}\ln\frac{z+1}{z-1} \qquad (3.9)$$

integrating by parts in (3.6) we transform it:

$$\left(\frac{p^2}{2\mu}+\epsilon_b\right)g_0(p)=$$

$$=\frac{e^2}{4\pi^2}p\int_0^\infty\frac{2p}{p^2-k^2}dk\int_k^\infty g(k')dk'. \qquad (3.10)$$

Substitution of the known expression for a nonrelativistic wave function of a ground state into (3.10)

$$g_{1,0}(p) = \frac{p}{(p^2/2\mu + \epsilon_b)^2} = pG^2_{non}(p,\epsilon_b) \tag{3.11}$$

leads to the condition

$$1 = \frac{e^2}{4\pi^2}\int_0^\infty \frac{dk}{(k^2/2\mu+\epsilon_b)} \equiv \frac{e^2}{4\pi^2}\int_0^\infty G_{non}(k,\epsilon_b)ds_{k,eucl} \tag{3.12}$$

determining a binding energy of a ground state

$$\epsilon_b^{(1)} = \left(\frac{e^2}{4\pi}\right)^2 \frac{\mu}{2}. \tag{3.13}$$

Further, the fact that equation (3.10) for the function $\psi_0(p) \equiv \frac{1}{p} g_0(p)$ can be written down in the form

$$G^{-1}_{non}(p,\epsilon_b)\psi_0(p) = \\ = \frac{e^2}{4\pi^2} P \int_0^\infty \frac{ds_{k,eucl}}{G^{-1}_{non}(p,\epsilon_b) - G^{-1}_{non}(k,\epsilon_b)} \int_{k'=k}^\infty \psi_0(k')dG^{-1}_{non}(k',\epsilon_b) \tag{3.14}$$

plays an important role.

4. SOLUTION OF THE KADYSHEVSKY EQUATION

Consider equation (1.3) with quasipotential (1.5). Using a partial expansion

$$\Psi_q(\vec{p}) = \psi_{q,\ell}(p)Y_{\ell m}(\vec{n}_p) \equiv \frac{1}{p}\phi_{q,\ell}(p)Y_{\ell m}(\vec{n}_p) \tag{4.1}$$

we obtain for the function $\phi_{q,\ell}(p)$ a one-dimensional equation

$$(2E_p - 2E_q)\phi_{q,\ell}(p) = \\ = \frac{e^2}{4\pi^2}\int_0^\infty Q_\ell\left(\frac{E_pE_k - m^2}{pk}\right)\phi_{q,\ell}(k)\frac{m\,dk}{E_k}. \tag{4.2}$$

In the case $\ell=0$ due to (3.9) equation (4.2) with parametrization (2.5) takes the form:

$$(2m\,\mathrm{ch}\,\chi_p - 2E_q)\phi_{q,0}(p) = = \frac{e^2}{4\pi^2}\int_0^\infty \ln\left|\frac{\mathrm{sh}\left(\frac{\chi_p}{2}+\frac{\chi_k}{2}\right)}{\mathrm{sh}\left(\frac{\chi_p}{2}-\frac{\chi_k}{2}\right)}\right| \phi_{q,0}(k)\,m\,d\chi_k . \tag{4.3}$$

Integrating by parts in (4.3), we take into account that $\frac{d}{dx}\ln|x| = P\frac{1}{x}$ /19/. Then this equation for the $\psi_q(p)\equiv\psi_{q,0}(p)$ in terms of relativistic free Green function (3.5) is written in the form:

$$G^{-1}_{rel}(p,E_q)\psi_q(p) = = \frac{e^2}{4\pi^2}P\int_{k=0}^\infty \frac{ds_k}{G^{-1}_{rel}(p,E\ )-G^{-1}_{rel}(k,E\ )}\int_{k'=k}^{\infty}\psi_q(k')dG^{-1}_{rel}(k',E_q). \tag{4.4}$$

The integral over the $ds_k = md\chi_k$ in (4.4) analogous of the one over the $ds_{k,eucl}$ in (3.14) is understood as a Cauchy principal value. Now, due to the fact that (3.5) and (2.7) are the relativistic geometric generalizations of (3.6) and (2.8) we obtain that relativistic partial equation (4.4) is a geometric generalization of (3,14) and both the equations have an absolutely identical structure.

By analogy with (3.11) we find solution (4.4) in the form:

$$\psi_q(p) = G^2_{rel}(p,E_q). \tag{4.5}$$

We substitute (4.5) into (4.4) and integrate over $dG^{-1}_{rel}(k',E_q)$. Then with the help of the algebraic identity

$$[G^{-1}_{rel}(p,E_q)-G^{-1}_{rel}(k,E_q)]^{-1}G_{rel}(k,E_q) = = G_{rel}(p,E_q)\{[G^{-1}_{rel}(p,E_q)-G^{-1}_{rel}(k,E_q)]^{-1}+G_{rel}(k,E_q)\} \tag{4.6}$$

and accounting that (for instance, see ref. [20]), as in the nonrelativistic case,

$$P\int_0^\infty \frac{ds_k}{G^{-1}_{rel}(p,E_q) - G^{-1}_{rel}(k,E_q)} = \tag{4.7}$$
$$= P\int_0^\infty \frac{d\chi_k}{2\,ch\,\chi_p - 2\,ch\,\chi_k} \equiv 0$$

we arrive at the relation:

$$1 = \frac{e^2}{4\pi^2}\int_{k=0}^\infty G_{rel}(k,E_q)ds_k\,. \tag{4.8}$$

Introducing the parametrization $E_q = m\cos\chi_q$ we represent (4.8) in the form of the algebraic equation:

$$1 = \frac{e^2}{4\pi^2}\,\frac{\pi-\chi_q}{2\pi\sin\chi_q} = \frac{e^2}{4\pi}\,\frac{\pi - \mathrm{Arccos}\dfrac{E_q}{m}}{2\pi\sqrt{1-\dfrac{E_q^2}{m^2}}} \tag{4.9}$$

determining a binding energy of a ground state of the relativistic Coulomb system.

Formula (4.8) represents a geometric generalization of (3.12), as a substitution of the Euclidean half-momentum $\pi_{k,э}$ for the relativistic one π_k of the binding energy ϵ_{CB} for (3.3); and (2.8), for (2.7). Therefore it is evident that the E_{CB}, defined from (4.8) in the nonrelativistic limit turns into (3.13).

Construct now solutions for equation (4.4), corresponding to excited states*. Note that by (4.6) differentiation over E_q one more additional identity can be obtained:

$$[G^{-1}_{rel}(p,E_q) - G^{-1}_{rel}(k,E_q)]^{-1} G^{r}_{rel}(k,E_q) =$$
$$= [G^{-1}_{rel}(p,E_q) - G^{-1}_{rel}(k,E_q)]^{-1} G^{r}_{rel}(p,E_q) + \tag{4.10}$$
$$+ \sum_{i=1}^{r} G^{i}_{rel}(p,E_q) G^{r-i+1}_{rel}(k,E_q).$$

*A nonrelativistic spectrum problems for (3.14) can also be solved by the method stated below.

Insert the following polynomial over $G_{rel}(p,E_q)$

$$\psi_q^{(n)}(p) = \sum_{r=1}^{n} r B_r^{(n)} G_{rel}^{r+1}(p,E_q) \tag{4.11}$$

into equation (4.4).

There $B_r^{(n)}$ are some coefficients not yet defined. By integration in the r.h.s. of (4.4) and using (4.10), (4.7), we transform (4.4) to:

$$\sum_{r=1}^{n} r B_r^{(n)} G_{rel}^{r}(p,E_q) = \\ = \sum_{r=1}^{n} G_{rel}^{r}(p,E_q) \sum_{i=r}^{n} B_i^{(n)} F_{i-r+1}(E_q), \tag{4.12}$$

where

$$F_m(E_q) = \frac{e^2}{4\pi} \int_{k=0}^{\infty} G_{rel}^{m}(k,E_q)\,ds_k. \tag{4.13}$$

It is evident that function (4.11) is a solution of equation (4.4), if the coefficients $B_r^{(n)}$ satisfy the algebraic system of equations

$$r B_r^{(n)} = \sum_{i=r}^{n} B_i^{(n)} F_{i-r+1}(E_q); \quad (r = n, n-1, \dots, 1). \tag{4.14}$$

One of these equations (at $r = n$):

$$n = \frac{e^2}{4\pi^2} \int_{k=0}^{\infty} G_{rel}(k,E_q)\,ds_k \tag{4.15}$$

represents a condition of quantization, that is, defines the eigenvalue of the energy E_q, corresponding to function (4.11). Assuming $E_q = E_q^{(n)} = m\cos\chi_q^{(n)}$, condition (4.15) can be represented in the explicit form:

$$n = \frac{e^2}{4\pi} \frac{\pi - \chi_q^{(n)}}{2\pi \sin\chi_q^{(n)}}. \tag{4.16}$$

To define the wave function $\psi_q^{(n)}$ it is necessary to know the coefficients $B_r^{(n)}$. It is easy to see that with the help of the system of equations (4.14) (at $r \neq n$), in which the E_q is fixed by condition (4.15), the coefficients $B_{n-1}^{(n)}, \dots, B_1^{(n)}$ can be expressed through one value $B_n^{(n)}$ that cannot be defined from the integral equation and is fixed by an additional condition of the normalization type.

5. CONCLUSION

Thus there has been obtained a solution of the integral quasipotential Kadyshevsky equation (1.3) for a composite system of two relativistic particles in the case of a quasi-potential, taken in the form of a propagator of a massless meson exchange (1.5). The principal point here was the transformation of the nonrelativistic and relativistic equations with the Coulomb interaction to the "absolute" form, when the constituent values differ only by a geometric nature.

It has been determined that in the case $\ell = 0$ partial equations (relativistic and nonrelativistic ones) are written down in terms of only one variable - a free Green function and have the same structure. It is shown that relativistic wave functions and condition of quantization, written in $G_{рел}(p,E_q)$ terms have the form absolutely analogous to the nonrelativistic one.

Further publications will be devoted to the problems of solutions for the case $\ell \neq 0$; to the account of the Coulomb quasipotential dependence on the system total energy, as well as to the consideration of other quasipotentials.

The authors express their gratitude to V.G.Kadyshevsky, A.D.Linkevich, A.V.Sidorov, I.L.Solovtsov and S.G.Shulga for usefull discussions and interest to work.

REFERENCES

1. Schwinger J. Proc.Nat.Acad.Sci., USA, 1951, 37, p.452.
2. Salpeter E., Bethe H. Phys.Rev., 1951, 54, p.1232.
3. Logunov A.A., Tavkhelidze A.N. Nuovo Cimento, 1963, 29, p.380.
4. Logunov A.A. et al. Nuovo Cimento, 1963, 30, p.134; Logunov A.A., Tavkhelidze A.N., Khrustalev O.A. Phys. Lett., 1963, 4, p.325; Arbuzov B.A. et al. JINR, P2-1318, Dubna, 1963; Sov. Journ. ZETF, 1964, 46, p.122.
5. Kadyshevsky V.G. Nucl.Phys., 1968, B6, p.125.
6. Kadyshevsky V.G. Sov. Journ. ZETF, 1964, 46, p.654; Sov. Journ. ZETF, 1964, 46, p.872; Sov. Journ. Doklady AN SSSR, 1965, 160, p.573.
7. Kadyshevsky V.G., Mir-Kasimov R.M., Skachkov N.B. Nuovo Cimento, 1968, 55A, p.233.

8. Kadyshevsky V.G., Mir-Kasimov R.M., Skachkov N.B. Sov. Journ. "Particles and Nucleus", 1972, vol.2, No.3, p.635.
9. Skachkov N.B., Solovtsov I.L. Sov.Journ. "Particles and Nucleus", 1978, vol9, No.1, p.5.
10. Skachkov N.B. JINR, E2-7159, Dubna, 1973; JINR, E2-7333, Dubna, 1973; Sov.Journ. Theor. Mat. Fiz., 1975, 22, p.213; JINR, P2-12152, Dubna, 1979.
11. Varichak V. Phys.Zeitschrift, 1910, 11, p.585.
12. Klein F. About Geometrical Foundations of Lorentz Group. In: A New Ideas in Mathematics, 1909, No.5, SPB, p.144.
13. Kotelnikov A.P. Relativity Principle and Lobachevsky Geometry. In: "In memorium N.I.Lobatchevsky", 2, Izd. of KGU, Kazan, 1927, p.37-64.
14. Chernikov N.A. JINR, P-723, Dubna, 1961.
15. Fock V.A. The Theory of Space, Time and Gravitation. Gostekhizdat, M., 1955 (Published in English by Pergamon Press, Oxford, 1967).
16. Smorodinsky Ya.A. Sov.Journ.ZETF, 1962, 43, p.2217. Sov. Journ. "Atomnaya Energiya", 1963, 14, No.1, p. 110; Lectures in: "Problems of Elementary Particles Physics". Izd. of AN Arm.SSR, Erevan, 1963, vol. 3.
17. Robb A.A. Optical Geometry of Motion: a New View of the Theory of Relativity. Cambridge, 1911.
18. Chernikov N.A. Nauchn.Dokl. Vysshey Shkoly, 1958, 2, p.1958; 1959, 3, p.151; Sov. Journ.ZETF, 1957, 33, p.451; Sov.Journ. "Particles and Nucleus", vol4, No.3, p.733,1973. Lectures given at JINR Int. School in Theor. Phys. JINR, v.3, p.151, Dubna, 1964.
19. Gelfand I.M., Shilov G.E. Generalized Functions and Operations on Them. Fizmatgiz, M., 1958 (in Russian).
20. Gradshtein I.S., Ryzik I.M. Tables of Integrals, Sums, Series and Products. Fizmatgiz, M., 1962.

Received by Publishing Department
on November 13 1979.