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Квантовая хромодинамика как микроскопическая теория сверхтекучести

Показано, что квантовая хромодинамика может быть построена по аналогии с микроскопической теорией сверхтекучести /Боголюбов,1947г./ Представлены все элементы самосогласованного описания неабелевых калибровочных полей как сверхтекучей системы. Выведены уравнения для глюонного бозе-конденсата и найдены спектры кооперативного возбуждения и квазичастиц. Рассмотрен точно решаемый пример "ненаблюдаемых" и асимптотически свободных цветных квазичастиц.

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Quantum Chromodynamics as a Microscopic Theory of Superfluidity

The infrared catastrophe in quantum chromodynamics is shown to lead to an analog of the microscopic theory by Bogolubov (1947).

By the interpretation of charge in electrodynamics as a field theory it is assumed that the coloured (charged) gluon plays in QCD three roles: source of a field, singular "Coulomb" field, and "photon". The problem of description of stationary quantum states for singular fields is solved by introducing a new type of excitation in QFT, cooperative topological excitation of the singular Bose-condensate of gluons.

The gluon Bose-condensate is shown to be of the potential nature throughout except for singular points-vortices and is energetically favourable, i.e., it possesses properties of the superfluid state. Spectra are found for the cooperative excitation and quasiparticles in the field of a cylindrical-symmetric Bose-condensate. In this case the coloured quasiparticles are nonobservable (do not fly of "Coulomb" bags) and are asymptotically free at small distances.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

The classification of hadrons[1,2] and phenomenology of strong interactions at short distances[3] have led to the modern idea on hadrons based on the non-Abelian gauge theory of coloured quarks and gluons called the quantum chromodynamics (QCD)[4]. Historically, the non-Abelian theory was put forward by Yang and Mills as a natural generalization of electrodynamics.

The methods of calculation in QCD are based on the perturbation theory, applicable at short distances (asymptotic freedom) and inapplicable at large distances because of the unlimited growth of the coupling constant (infrared catastrophe)[5]. The removal of the infrared catastrophe is the most important problem of QCD. The solution of this problem involves the explanation of unobservability of coloured particles, the description of the spectrum and static parameters of hadrons.

In statistical physics, the unlimited growth of the coupling constant in the infrared region gives rise, as a rule, to a strongly coupled system of particles (superfluid Bose-condensate). The escape of particles from such a Bose-condensate is not favourable energetically and under certain external conditions the dynamics of the whole system is of a cooperative nature. (The system is excited as a unique object). The consideration of the motion of the individual initial particle in such a system contradicts the principal ideas of quantum mechanics [6].

The first solved problem with the infrared catastrophe is a model of the nonideal Bose-gas [7]. As is shown in the known work by Bogolubov [7] (1947), in which the microscopic theory of superfluidity was created, any arbitrarily weak interaction between particles in this model leads, because of infrared divergences, to the above-described superfluid state of the system.

In order to be convinced of the "infrared catastrophe" of the theory of non-Abelian fields and of the possibility of a superfluid state, it is sufficient, following [7], to make a qualitative analysis of the Lagrangian density of the Yang-Mills fields

$$\mathcal{L} = -\frac{1}{4}\left[\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right) + \left(g\varepsilon^{abc} A_\mu^b A_\nu^c\right)\right]^2 .$$

For the fields slowly changing in space the second term which describes the interaction of fields exceeds the first term at arbitrarily small fixed values of the coupling constant. As a result, there appears a system of strongly interacting fields which, generally, can have both the coupled Bose-condensate and the cooperative excitation.

The existence of the singular gluon Bose-condensate in chromodynamics is also testified, besides the infrared divergence, by the consistent analogy of the Yang-Mills theory with classical electrodynamics. The latter proceeds from an idea that each charge excites the electric field in space [8]. In accordance with this idea the coloured (charged) gluon should play in the theory three roles: the field source, singular Coulomb field, and photon (that is to say, the charged gluon should emit itself).

In my work [9] it was shown that the hypothesis of the singular Bose-condensate and cooperative excitation can be realized in the Yang-Mills quantum theory under a definite choice of the dynamical variables. In this paper these ideas are further developed by using the analogy with the microscopic theory of superfluidity and the spectrum of quasiparticle excitations is obtained for the Bose-condensate in the class of cylindrically symmetric functions.

2. Basic Assymptions and Results

The microscopic description of a superfluid system in statistical physics consists of the following steps /7/:

1) The theoretical construction is based on the hypothesis of the existence of strongly coupled Bose-condensate which should be substantiated qualitatively by the infrared instability of the conventional perturbation theory;

2) The wave function (or the Hamiltonian) of this condensate is calculated which describes the cooperative dynamics of the system . (Note that the main difference of the superfluid Bose-condensate from the usual condensate of free particles is the existence of the cooperative excitations of the superfluid system as a whole);

3) By the Bogolubov transformations the Bose-condensate is separated from the local excitations above the Bose-condensate (quasiparticles), the condition of stability of the new perturbation theory being imposed;

4) The spectrum of quasiparticles is calculated;

5) Within a self-consistent scheme the energetic advantageousness is proved for the cooperative stationary states of the Bose-condensate without quasiparticles.

We shall present all steps of the self-consistent description for the quantum theory of non-Abelian gauge fields.

1) First of all it should be noted that in quantum field theory (QFT) there is no concept of the Bose-condensate, as the ground state of QFT is commonly accepted to be a state with no particles. However, if we assume the existence of cooperative excitations of the fields as a whole, then the Bose-condensate can be understood as a physical medium carrying this excitation and described by a singular c -numerical field *). Note that the very statement of the problem of description of stationary quantum states for singular fields differs completely from that done within the usual QFT, which deals with the class of re-

*) Functions throughout differentiable in R (3) and vanishing at spatial infinity will be called regular, otherwise, singular.

gular functions[10]. The regularity of fields is one of the main postulates of QFT, necessary for the finiteness of the energy of stationary states. The regularity of fields is implicitly assumed in the canonical quantization of non-Abelian gauge fields which leads to the Faddeev-Popov functional integral[11].

2) The canonical quantization of non-Abelian fields (the construction of Hamiltonian, choice of dynamical variables) is performed by using the generalized-Hamiltonian Dirac formalism[12] (see the monograph by Slavnov and Faddeev[11], p. 79). However, the method of construction of the Hamiltonian in relativistic theory has been suggested by Dirac for systems with the finite number of degrees of freedom and, as Dirac emphasizes ([12], pp. 12, 29), the transition to the infinite number of degrees of freadom is formal. Therefore for systems with the assumed cooperative dynamics, i.e.,where the transition for the infinite number of degrees of freedom should be accompanied with a qualitative change,the use of the Dirac method contradicts the initial assumptions and leads to an incomplete description of the system.

A simple example of incompleteness of the quantization method in the class of regular functions[11] is the quantum electrodynamics in one spatial dimension (QED_2). On the one hand, the QED_2 does not contain transverse degrees of freedom and, consequently, has no dynamical variable in accordance with the usual quantization scheme[11]. On the other hand, it is clear from the physical point of view that the electrodynamics in one dimension describes a homogeneous (i.e.,singular) electric field between plane-parallel-capacitor plates, separated by an infinite distance. It can be shown[9] that such a field is the cooperative variable that remains after eliminating the transeverse degrees of freedom and is not allowed for by the quantization in the class of regular functions[11].

While quantizing the gauge fields by the Dirac method, one at first defines dynamical variables by establishing commutators and then separates physical from nonphysical variables by implicitly solving the equations of constraint and auxiliary gauge conditions[11,12].

Within our assumption on the Bose-condensate we consider it better at first to separate physical from nonphysical variables by explicitly solving the equations of constraint /13/ *), and then to define the dynamical part of physical variables and their commutation relations. (The construction of the Hamiltonian in such a variant of quantization appears a more involved procedure than in the Dirac formalism and includes, in particular, an analog of the Bogolubov transformations for a unique separation of gauge fields into the stationary Bose-condensate and quasiparticles /9/). In the variant there is no need to use the gauge conditions as starting equations. The gauge conditions result from the explicit solution of the equations of constraint-classical equations for field components with the canonical momentum, equal identically to zero. Note that the classical second-order differential equations used are strictly defined only outside the field singularities.

In the theory of differential equations /14/ there is a standard procedure of separating the degrees of freedom which describe the cooperative excitation of the system as a whole (the so-called zero vibration modes). Mathematically, the inclusion of cooperative degrees of freedom means that while solving explicitly the inhomogeneous equation of constraint one should include solutions of the homogeneous equation with coefficients which play the role of zero modes. It is to be stressed that the condition of the existence of cooperative variables is the singularity of non-Abelian fields. In the case of regular fields coefficients of these modes in the Lagrangian become zero, as they are total derivatives, and we arrive at a theory equivalent to the standard perturbation theory /11,12/.

What is the dynamical nature of cooperative excitations of the Yang-Mills fields? Answering this question we follow the relativity principle: the dynamical description of any system is the construction of irreducible representations of the invariance group of the considered system. The gauge-invariance group of the Yang-Mills theory G is topologically disconnected and represents a product of the connected component G_0 by the

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*) As Polubarinov has shown /13/, in electrodynamics such an approach is equivalent to the usual one /11/ in the Coulomb gauge.

cyclic group of integers Z [15]. The relativity principle as applied to the Yang Mills theory introduces, in addition to transverse physical fields, a dynamical variable transforming by a representation of the group Z [9]. This variable is a topological quantity directly connected with that is called the Pontryagin index for regular fields. The topology of the configurational space of all dynamical variables of non-Abelian fields is a topology of the cylinder (the first note of such a kind is due to Faddeev[16]); the cooperative variable, introduced in solving the equations of constraint and normalized by the Pontryagin index, describes rotation of the "cylinder".

As a result, the usual Lagrangian, describing the transverse degrees of freedom, is completed with a "cooperative" Lagrangian, which depends only on the total derivatives of fields, i.e., on the field singularities and is stable under the "regular" variations of fields. It is rather difficult to give such a Lagrangian any physical meaning in classical field theory. On the other hand, the quantization of the cooperative variable and transition to the stationary state with a definite energy and momentum of rotation of the "cylinder" means, mathematically, the subtraction of the action function of the singular Bose-condensate from the usual action function of transverse fields.

3) Owing to this subtraction, the presence of field singularities does not contradict the finiteness and stability of the quantum perturbation theory above the singular Bose condensate, if one requires the condition of potentiality of the Bose-condensate (electric and magnetic fields are proportional to the potential gradient) and charge quantization. We have here in fact an analog of the Lagrange theorem in hydrodynamics conserving stationarity of the potential fluid[6]. It is shown[17,18,19] that there exist singular solutions to the equatinns of potentiality describing "Coulomb" bags.

4) In this paper the spectrum of quasiparticles is calculated in a field of the cylindrically symmetric Bose-condensate. Quasiparticles appear to be unobservable at long distances and asymptotically free at short distances.

5) Stable quasiparticle excitations of a system may increase only the energy, therefore the state of "Coulomb" bags is energetically favourable.

The physical picture of QCD at long distances is equivalent to the theory of superfluid quantum liquid.

3. Derivation of Equations for the Bose-Condensate

Let us show the derivation of equations for the Bose-condensate[9]. Starting quantities of the theory are functionals of action and "Pontryagin index" *)

$$S[A]=\int L\,dt \quad ; \quad L=-\frac{1}{4}\int d^3x\, F^a_{\mu\nu}F^a_{\mu\nu}\,, \tag{1}$$

$$\nu[A]=\int \dot{N}\,dt \quad ; \quad \dot{N}=\frac{g^2}{64\pi^2}\int d^3x\,\varepsilon_{\mu\nu\rho\lambda}F^a_{\mu\nu}F^a_{\rho\lambda}\,, \tag{2}$$

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\,\varepsilon^{abc}A^b_\mu A^c_\nu\,. \tag{3}$$

Fields A^a_0 have no canonical momentum and the equations of constraint are classical equations for field A_0 [13]

$$(\nabla^2 A_0)^a-(\nabla_i\,\partial_0 A_i)^a=0\,, \tag{4}$$

where

$$\nabla_i^{ab}\equiv\nabla_i^{ab}(A_i)=\delta^{ab}\partial_i+g\,\varepsilon^{abc}A^c_i\quad;\quad \nabla^2=\nabla_i\nabla_i\,.$$

The general solution of eq. (4)

$$A^a_0=\dot{c}\,\Phi^a+\left(\frac{1}{\nabla^2}\nabla_i\,\partial_0 A_i\right)^a \tag{5}$$

contains a solution of the homogeneous equation

$$(\nabla^2\Phi)^a=0\,; \tag{6}$$

$\dot{c}=\frac{dc(t)}{dt}$ is the zero mode of the operator ∇^2 (i.e., cooperative variable).

Inserting (5) into eq.(1) we obtain the Lagrangian

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*) Fermions are included in Appendix A.

$$L = L_{(T)} + L_{(coop)}\ ,$$

$$L_{(T)} = \int d^3x\ \frac{1}{2}\left[(E_i^{(T)a})^2 - (B_i^a)^2\right]\ , \tag{7}$$

$$L_{(coop)} = \int d^3x \left[\frac{1}{2}\dot{c}^2(\nabla_i \Phi)^2 - \dot{c}(\nabla_i \Phi)^a E_i^{(T)a}\right]\ , \tag{8}$$

where

$$E_i^{(T)a} = \partial_0 A_i^a - \left(\nabla_i \frac{1}{\nabla^2}\nabla_j \partial_0 A_j\right)^a\ ; \quad B_i^a = \frac{1}{2}\varepsilon_{ijk} F_{jk}^a\ . \tag{9}$$

Fields $E^{(T)}, B$ obey the equations

$$\nabla_i^{ab} E_i^{(T)b} \equiv 0 \qquad ; \quad \nabla_i^{ab} B_i^b \equiv 0\ . \tag{10}$$

Lagrangian (8), owing to conditions (10), (6), depends only on field singularities.

Let us identify the Goldstone mode $\dot{c}$ in (8) with the topological variable $\dot{N}$, inserting the general solution for A_0 (3) into the Pontryagin index (2)

$$\dot{N} = \dot{c}\,\frac{g^2}{8\pi^2}\int d^3x\,(\nabla_i \Phi)^a B_i^a - \dot{\mathcal{N}}(A)\ . \tag{11}$$

Here $\dot{\mathcal{N}}$ is a functional of gauge fields. Expressing the Lagrangian (8) $L_{(coop)}$ in terms of $\tilde{N} = (N + \mathcal{N})$ using (11) we obtain

$$L_{(coop)} = \frac{(\dot{\tilde{N}})^2}{2}\left(\frac{8\pi^2}{g^2}\right)^2 \frac{1}{\langle B\rangle^2} - \dot{\tilde{N}}\,\frac{\langle E\rangle}{\langle B\rangle}\,\frac{8\pi^2}{g^2}\ , \tag{12}$$

where

$$\langle D\rangle^2 = \left[\int d^3x\, D_i^a (\nabla_i \Phi)^a\right]^2 \Big/ \int d^3x\,(\nabla\Phi)^2\ ; \quad D_i^a = E_i^a,\ B_i^a\ . \tag{13}$$

(Transformation properties of the dynamical variables are discussed in Appendix A). The Hamiltonian corresponding to Lagrangian (12) does not depend on $\dot{\mathcal{N}}$:

$$H_{(coop)}(P) \equiv L_{(coop)}(P) = \frac{1}{2}\left[P^2\left(\frac{g^2}{8\pi^2}\right)^2 \langle B\rangle^2 - \langle E\rangle^2\right] \tag{14}$$

with

$$P = \delta L_{(coop)}(\dot{N}) / \delta \dot{N}\ .$$

We quantize the variable N, $[N,p]=i$ requiring the state vector Ψ to transform by the representation of the cyclic group Z : $\Psi(N+1)=e^{i\theta}\Psi(N)$ and find the spectrum of momenta of the "rotator"

$$p = 2\pi k + \theta, \tag{15}$$

where k is the number of the Brillouin zone; θ, the quasimomentum.

Finally, we get the following effective action

$$S_{eff} = S_{(r)} + S_{(coop)} = \int dt\, L_{eff} = \int dt \left(L_{(r)} + L_{(coop)}(p) \right), \tag{16}$$

$$L_{eff} = \frac{1}{2}\left[\int d^3_x E^2 - \langle E\rangle^2\right] - \frac{1}{2}\left[\int d^3_x B^2 - \rho^2 \langle B\rangle^2\right], \tag{17}$$

$$\rho^2 = (2\pi k + \theta)^2 \left(\frac{g^2}{8\pi^2}\right)^2.$$

Let us represent the gauge field A by a sum of the singular background field $\underset{\sim}{b}$ (Bose-condensate) and regular field $\underset{\sim}{a}$, having zero boundary conditions at the singularities (quasiparticles)

$$S_{(coop)}(\underset{\sim}{b} + \underset{\sim}{a}) = S_{(coop)}(\underset{\sim}{b}), \tag{18}$$

so that we have the following expansion of S_{eff} in powers of $\underset{\sim}{a}$:

$$S_{eff}(\underset{\sim}{a} + \underset{\sim}{b}) = S_{eff}(\underset{\sim}{b}) + S'_{(r)}(\underset{\sim}{b})\cdot\underset{\sim}{a} + \frac{1}{2!} S''_{(r)}(\underset{\sim}{b})\cdot\underset{\sim}{a}^2 + \dots$$

(The procedure of quantization and, in particular, the variational derivative are well defined only for the regular field $\underset{\sim}{a}$). The conditions of "finiteness" $S_{eff}(\underset{\sim}{b})_{Euclead} = 0$ and "stability" $S'_{(r)}(\underset{\sim}{b})\cdot\underset{\sim}{a} = 0$ may be satisfied provided that in (16)

$$\rho^2 = 1, \tag{19}$$

$$\int d^3_x\, \mathcal{D}^2 = \langle \mathcal{D}\rangle^2 \quad ; \quad \mathcal{D} = E, B. \tag{20}$$

Then from the Cauchy-Bunyakovsky inequality we find that the background fields $E(\underset{\sim}{b})$, $B(\underset{\sim}{b})$ are of the potential nature

$$E(\underset{\sim}{b}) \sim \nabla(\underset{\sim}{b})\Phi \quad ; \quad B(\underset{\sim}{b}) \sim \nabla(\underset{\sim}{b})\Phi . \tag{21}$$

The first of eqs. (21) means the stationarity of the Bose-condensate and the second coincides with the stationary self-dual equation[17-19]

$$B(\underset{\sim}{b}) = \gamma \nabla(\underset{\sim}{b})\Phi \tag{22}$$

up to the proportionality coefficient γ which can be removed by the change $\Phi \to \Phi'/\gamma$. Note that for $\gamma = -\rho$ (19), (17) the mean expectation value of the operator $\dot{c}$ in (5) equals zero.

4. Quasiparticle Spectrum for the Cylindrically Symmetric Superfluid State

Cylindrical - symmetric solutions of the self-dual stationary equation are found in papers[17,18] for the group SU(2) and in ref.[19] for a wide class of the Lie groups (including SU(3)).

For the illustration of the quasiparticle spectrum consider here the periodic singular solutions which describe the "Coulomb" bags for group SU(2)

$$\underset{\sim}{b}_\mu^a = (\Phi^a, \underset{\sim}{b}_i^a), \tag{23}$$

$$\Phi^a = \frac{m}{g} n^a \left(ctg(mr) - 1/mr \right), \tag{24}$$

$$\underset{\sim}{b}_i^a = \frac{m}{g} \varepsilon_{ial} n^l \left(1/\sin(mr) - 1/mr \right), \tag{25}$$

where $r = \sqrt{x_i^2}$, $n^a = \frac{x^a}{r}$, m is a solution parameter with the dimension of mass.

Consider an equation for a scalar coloured field in the same class of cylindrically symmetric functions *)

$$[\nabla_\mu(\underset{\sim}{b}_\mu)]^{ac} [\nabla_\mu(\underset{\sim}{b}_\mu)]^{cd} U^d = 0 . \tag{26}$$

With the substitution

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*) In contrast to ref.[17], we consider the real fields and solve relativistic equations.

$$U^a = \sum_\ell \frac{n^a}{r}\left(\Psi_\ell(r)\,\hat{\xi}_\ell^{(+)} e^{iE_\ell t} + \Psi_\ell(r)\,\hat{\xi}_\ell^{(-)} e^{-iE_\ell t}\right) \quad (27)$$

($\hat{\xi}_\ell$ are coefficients of the expansion over eigenfunctions) we obtain the analog of the Schrödinger equation for the particle in the Pöschl-Teller potential[20]

$$\left\{-\frac{\partial^2}{\partial r^2} + \frac{m^2}{2}\left[\frac{1}{\sin^2(\frac{mr}{2})} + \frac{1}{\cos^2(\frac{mr}{2})}\right]\right\}\Psi_\ell(r) = E_\ell^2\,\Psi_\ell(r). \quad (28)$$

To solve it, it is sufficient to consider one "cell" of the periodic potential. The solution and spectrum look as follows:

$$\Psi_\ell(r) = C(\ell)\left[\sin(\tfrac{mr}{2})\cos(\tfrac{mr}{2})\right]^2 {}_2F_1\left(-\ell, 4+\ell; 5/2; \sin^2(\tfrac{mr}{2})\right), \quad (29)$$

$$E_\ell^2 = m^2(2+\ell)^2 \;; \quad \ell = 0, 1, 2, 3, \ldots, \quad (30)$$

where $C(\ell)$ is defined by the normalization condition: $\int_0^{\pi m} dr\, \Psi_{\ell_1}(r)\Psi_{\ell_2}(r) = \delta_{\ell_1,\ell_2}$. Also it can be readily verified that the vector equation

$$\left[\nabla_\mu^2(b_\mu)\right]^{ac} a_\nu^c + 2g\,\varepsilon^{adc} F_{\nu\mu}^d(b_\mu)\,a_\mu^c = 0 \quad (31)$$

for pure longitudinal fields $a_\mu = \nabla_\mu(b_\mu)U$ reduces to eq. (28). The spectrum of eqs. (26), (31) for the eigenvalues $\nabla_\mu^2 U = -\varkappa^2 U$ can be obtained by the shift $E^2 \to E^2 - \varkappa^2$.

5. Analogy with the Superfluid Helium in a Rotating Bucket

The physical picture arising from the assumption of a singular coupled Bose-condensate of non-Abelian fields and its cooperative motion as a whole (rigid-body rotation) resembles the superfluid helium in a rotating bucket[21] in many aspects.

Recall[21] that the helium at a temperature below the boiling point is a two-component fluid consisting of a) superfluid Bose-condensate throughout potential except for singular points-vortices and b) weakly interacting quasiparticles, i.e., excitations above the Bose-condensate. In a rotating bucket rotation of the whole superfluid Bose-condensate is energetically favourable.

However, the condition of such a cooperative motion is the formation of vortices homogeneously distributed over the bucket. Analogously, the condition of existence of the cooperative degree of freedom (rotation of a "cylinder") in the Yang-Mills theory is the presence of singularities ("vortices") of non-Abelian fields (see formula (8)).

The theory of non-Abelian fields differs from the theory of superfluid helium in the description of the dynamics of the superfluid state. The assumption of London on the existence of a wave function describing the coherent state of the superfluid (superconductor) as a whole is replaced by the assumption on the cooperative topological variable.

One should mention also the different physical meaning of topological quantities of the compared theories. The Pontryagin index in the Yang-Mills theory,in contrast to the circulation of helium vortices, is not the conserved quantity of the theory characterizing stationary states of the system. The Pontryagin index is therefore treated as a topological cooperative degree of freedom and the conserved quantity is that canonically conjugated to the cooperative variable ("torque" of the "cylinder").

Parameters of the most energetically favourable singular configuration of the superfluid helium are defined by the external (boundary) conditions and internal characteristics of vortices, their energy and circulation. For the calculation of the energy of vortices the helium atomic structure is important. However, if the liquid is ideal, the energy of vortices is infinite.

The energy of "vortices" of non-Abelian fields coincides in form with the energy of helium vortices $\int d^3x\,(\nabla\phi)^2$ and is also infinite. (There is an analogy with the infinite Coulomb energy of electron). We are still far from knowing the "atomic" structure of physical non-Abelian fields, therefore the parameters of vortices (dimension of the lattice m) and the most optimal configuration may apparently be determined by experiment. The theoretical ambiguity is fully due to the boundary conditions of potentiality equations.

Because of the infinite vortex energy (see (8) and (20)) the cooperative degree of freedom in the classical theory of non-Abelian fields has no direct physical meaning, like the classical theory of superfluidity and superconductivity.

The torque of a cylindrical configurational space of non-Abelian fields $2\pi k+\theta$ is tightly related to the coupling constant (see formulae (17) and (19)):

$$\alpha = \frac{g^2}{4\pi} = \left| \left(k+\frac{\theta}{2\pi}\right)^{-1} \right| \tag{32}$$

with k integer and $\theta \in (-\pi, \pi)$.

It should be noted that the quantization of charge (32) is necessary in order to simultaneously satisfy the requirements of finiteness of the action in the Euclidean space

$$\int d^3_x E^2 - \langle E \rangle^2 = 0 \quad ; \quad \int d^3_x B^2 - \rho^2 \langle B \rangle^2 = 0$$

and stability of the theory (i.e.,the Bose-condensate should obey classical equations out of singularities).

The stopping of the "cylinder" $(k=\theta=0)$ is equivalent to the infinitely large coupling constant. We have here the "infrared catastrophe" and "asymptotic freedom" in the simplest version. The limit $g \to 0$ does not coincide with the Abelian theory because of nonanalyticity of the superfluid Bose-condensate in coupling constant.

We have considered the Bose-condensate in the class of cylindrically symmetric functions, when non-Abelian "vortices" form the periodic spatial lattice of dimension $\sim \frac{1}{m}$ ("Coulomb" bags). The spectrum of quasiparticles in this case is discrete and tunnelling from one cell to another is forbidden. The lattice with barriers impenetrable for quasiparticles is an example of the confinement with asymptotical freedom at short distances. The limit of small distances corresponds to the infinite increase of the lattice cell $(m \to 0)$. So, we, finally, consider the theory in one cell, in which all the singularities are at infinity and are unimportant for the dynamics. In this case the standard perturbation theory formulated by the Faddeev-Popov method/11/ is valid.

6. Conclusion

The idea of electrodynamics that each charge excites a field leads to the singular gluon Bose-condensate in QCD. The quantum stationary states of singular fields are described by introducing the topological cooperative excitation of the Bose-condensate. We have shown that nonobservability of coloured quasiparticles can be a result of superfluid properties of vortices of the Bose-condensate.

The comparison with known physical systems of this type (superfluid helium in a rotating bucket) shows that it may happen to be impossible theoretically, without experimental facts, to define the parameters of gluon "vortices", i.e., the dimension and spatial configuration of gluon bags. In other words, the non-Abelian theory (even without quarks) like electrodynamics is a logically incomplete theory that testify to the unexhausted, great physical content of the quantum chromodynamics.

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Appendix A

Inclusion of the fermions and transformation properties of the dynamical variables

Let us apply the proposed method of quantization to the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \bar{\psi} \not{\partial} \psi + j^a_\mu A^a_\mu \;; \quad j^a_\mu = g \bar{\psi} \frac{\tau^a}{2} \gamma_\mu \psi$$

and obtain the following effective Lagrangian

$$L'_{eff} = L_{eff} + L_Q \,,$$

where L_{eff} is given by eq.(16), and

$$\mathcal{L}_Q = -\bar{\psi}^{(T)} \not{\partial} \psi^{(T)} - j^{(T)a}_i A^{(T)a}_i + j^{(T)a}_0 \left(\frac{1}{\nabla^2(T)}\right)^{ab} j^{(T)b}_0 .$$

All the Lagrangians depend on purely transverse fields

$$\hat{A}^{(T)}_k = U(A)(\hat{A}_k + \partial_k) U^{-1}(A) \;; \quad \hat{A} = g A^a \frac{\tau^a}{2i} \,,$$

$$\psi^{(T)} = U^{-1}(A) \psi \;; \quad \nabla(T) \equiv \nabla(A^{(T)}) \,,$$

where the matrix $U(A)$ is defined by the equation

$$\frac{\partial}{\partial t} U(A) = \frac{1}{\nabla^2} \nabla_j \left(\frac{\partial}{\partial t} \hat{A}_j\right)$$

with boundary condition

$$U(A)\big|_{t=-\infty} = v(\vec{x}) \; ; \; v(\vec{x}) \in G = G_0 \times Z .$$

The matrices $v(\vec{x})$ define the gauge ambiguity of the theory and should be given in the class of regular functions. The space of matrices $v(\vec{x})$ with the condition $\lim_{|\vec{x}| \to \infty} v(\vec{x}) = 1$ is topologically disconnected. Each matrix is characterized by the index n - the degree of mapping of $R(3)$ into SU(2) which indicates how frequently the space $R(3)$ has turned about SU(2) (or SU(N)). The topological variable (11) has the form of the total time variable

$$N(t) = c(t)\left[\frac{g^2}{8\pi^2} \int d^3x\, \nabla\varphi \cdot B\right] - \frac{g^2}{16\pi^2} \int d^3x\, \varepsilon_{ijk} \left(\partial_i A_j^a A_k^a + \frac{g}{3} \varepsilon^{abc} A_i^a A_j^b A_k^c\right)$$

since the coefficient of $c(t)$ depends only on the stationary Bose-condensate. It can be readily verified /16/ that under the transformations of transverse fields

$$\hat{A}^{(T)\prime} = v^{(n)}(\vec{x})^{-1} \left(\hat{A}^{(T)} + \partial_i\right) v^{(n)}(\vec{x})$$

the quantity $N(t)$ as a functional of fields $A^{(T)}$ changes in a covariant manner with respect to the group Z :

$$N(t)' = N(t) + n ,$$

where n is an integer.

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