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S.G.Mikhov

# A NONLINEAR REALIZATION OF THE CONFORMAL SUPERSYMMETRY

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Михов С.Г.　　　　　　　　　　　　　　　　**E2 - 12872**

Нелинейная реализация конформной суперсимметрии

С помощью метода, ранее развитого для конформной группы, получен класс нелинейных реализаций супреконформной группы. Эти нелинейные трансформационные законы при сужении на подгруппу Лоренца оказываются линейными. Все остальные преобразования суперконформной группы в действительности линейны, но неоднородны. При этом линейные однородные члены в трансформационных законах могут быть получены как частные случаи ранее известных линейных представлений суперконформной группы при некоторых специальных выборах параметров представлений. Получены также суперконформно-ковариантные производные как нелинейно, так и линейно преобразующихся полей.

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Mikhov S.G.　　　　　　　　　　　　　　　　**E2 - 12872**

A Nonlinear Realization of the Conformal Supersymmetry

In the present paper using a method originally developed for the conformal group a class of nonlinear realizations of the superconformal group is obtained. The transformation laws for these realizations, when restricted to the Lorentz group, are linear. All other transformations from the superconformal group are in fact linear but nonhomogeneous, which is known to take place in the conformal group also. It turned out also that the linear homogeneous parts of the transformation laws can be obtained from the previously known linear representations of the superconformal group for a specific choice of the parameters labelling these representations. The superconformally covariant derivatives of the nonlinear fields as well as the superconformally covariant derivatives of the previously known linearly transforming fields are constructed.

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1. The aim of the present paper is to introduce a nonlinear realization of the superconformal group. In several preceding papers[1,2,3] were obtained two series of linear representations of the same group labelled by the set of real numbers $d$ , $z$ , $p$ , $q$. In what follows we shall make use of these representations, so that it is necessary to recall certain notations used there. By $M_{\mu\nu}$ , $P_\mu$ , $K_\mu$ and $D$ we denote the generators of the conformal group (the generators of the Lorentz group, the translations, the special conformal transformations and dilatations, respectively), while $S^{\pm}_{\alpha}$ , $T^{\pm}_{\alpha}$

$$\frac{1}{2}(1\pm i\gamma^5)_{\alpha\beta}\, S^{\pm}_{\beta} = S^{\pm}_{\alpha} \qquad \frac{1}{2}(1\pm i\gamma^5)_{\alpha\beta}\, T^{\pm}_{\beta} = T^{\pm}_{\alpha} \tag{1.1}$$

are two sets of spinor generators and $\pi$ is a pseudoscalar generator. The $\gamma$-matrices are taken in the Majorana representation

$$\gamma^{\mu}_{\alpha\beta} = (\gamma^{0}\gamma^{\mu}\gamma^{0}) \qquad \sigma^{\mu\nu}_{\alpha\beta} = (\gamma^{0}\sigma^{\mu\nu}\gamma^{0})_{\beta\alpha}$$
$$\gamma^{5}_{\alpha\beta} = -\gamma^{5}_{\beta\alpha} \qquad (\gamma^{5})^{2} = -1 \tag{1.2}$$

and the metric is $g_{\mu\mu} = (-,+++)$. For the commutation relations of the generators we refer to paper[1]. We shall construct the nonlinear realizations in the space of functions of $(x_\mu, \theta^{+}_{\alpha}, \xi^{-}_{\alpha})$, where $x_\mu$ are the coordinates in Minkowski space and $\theta^{+}_{\alpha}, \xi^{-}_{\alpha}$ are two sets of anticommuting Grassman variables and satisfy

$$\frac{1}{2}(1+i\gamma^{5})_{\alpha\beta}\,\theta^{+}_{\beta} = \theta^{+}_{\alpha} \qquad \frac{1}{2}(1-i\gamma^{5})_{\alpha\beta}\,\xi^{-}_{\beta} = \xi^{-}_{\alpha} . \tag{1.3}$$

The class of nonlinear realizations is fixed by the condition that when restricted to the subgroup spanned over the generators $M_{\mu\nu}$, $P_\mu$, $S_a^+$ and $T_a^-$ one obtains the linear transformation law for a field from one of the series of representations introduced in [1,2,3]. The method that is used is a simple generalization of a well-known technique [4,5]. For the purpose consider the quantity

$$\psi(x,\Theta^+,\xi^-;\sigma,r,\Phi,\phi^+,\phi^-)=e^{i\sigma D+ir\pi}\,e^{i\Phi^\mu K_\mu}\times$$
$$\times\, e^{i\phi^+\gamma^\circ T^+ + i\phi^-\gamma^\circ S^-}\,e^{i\theta^+\gamma^\circ S^+ + i\xi^-\gamma^\circ T^-}\,e^{ix^\mu P_\mu} \tag{1.4}$$

where $\sigma$, $r$, $\Phi^\mu$, $\phi_a^+$ and $\phi_a^-$ are functions of the generalized coordinates $X_A=(x_\mu,\Theta_a^+,\xi_a^-)$. It is obvious that $\psi$ belongs to the superconformal group (more precisely to the quotient space of the superconformal group and the Lorentz group) and therefore $(\sigma,r,\Phi,\phi^+,\phi^-)$ belong to a subspace of the group parameter space. So that under the action of the right regular representation of the group on $\psi$, they transform according to the group composition law. If one adopts the exponential parametrization with the ordering (1.4) and the convention that the elements of the Lorentz group should stay to the left to all other exponents one can define then

$$\psi(X_A;\sigma(X),r(X),\Phi(X),\phi^+(X),\phi^-(X))g\equiv$$
$$\equiv\exp[i\omega_{\mu\nu}M^{\mu\nu}]\psi(X';\sigma'(X'),r'(X'),\Phi'(X'),\phi'^+(X'),\phi'^-(X')), \tag{1.5}$$

where

$$\omega_{\mu\nu}=\omega_{\mu\nu}(g;X;\sigma,r,\Phi,\phi^+,\phi^-). \tag{1.6}$$

Formula (1.5) defines the transformation laws of the fields $\sigma$, $r$, $\Phi^\mu$, $\phi_a^+$, $\phi_a^-$. In order to obtain the explicit form of this transformation laws one must produce the necessary commutations in the l.h.s. of (1.5) (for the commutators of the exponents see Appendix). One can guess that this procedure keeps the arguments of the functions $\sigma$, $r$, $\Phi^\mu$, $\phi_a^+$ and $\phi_a^-$ in the l.h.s. unchanged. Let for simplicity denote by $H_j$ the components of any of the latter fields and by

$$(T_g X)_A = X'_A \tag{1.7}$$

the transformed coordinates.



It is readily seen that one can write down the transformation law for the fields $H_j$ in the form

$$H'_j(X)=F_j\,[\,g;H_k((T_{g^{-1}}X))], \tag{1.8}$$

where the function $F_j(g;H)$ denotes the group composition law. Thus one has a correct transformation law. From the definition (1.4) for the quantity $\psi$ and the fact that $P_\mu$, $K_\mu$, $S_a^\pm$, $T_a^\pm$, $D$ and $\pi$ with respect to the Lorentz group are vectors, spinors, a scalar and a pseudoscalar respectively, one can easily guess that so are $X_\mu$, $\Phi_\mu$, $\Theta_a^+$, $\xi_a^-$, $\phi_a^+$, $\phi_a^-$, $\sigma$ and $r$. It is evident also that the abelian transformations spanned over the generators $P_\mu$, $S_a^+$ and $T_a^-$ should coincide with the linear transformation laws from papers [1,3]. So, in what follows attention is paid to the case when $g$ is an element of one of the abelian subgroups corresponding to the generators $D$, $\pi$, $K^\mu$, $T_a^+$ and $S_a^-$ only. If one denotes the latter generators by $O_i$, it appears then that the transformation law corresponding to the group element $g=e^{i\alpha_j O_j}$ (there is no summation over the repeated indices) for the components $H_j$ of any of the fields $\sigma$, $r$, $\Phi_\mu$, $\phi_a^+$ and $\phi_a^-$ can be compactly written as

$$H'_j=\Delta_H(\alpha_i)V_{jk}(\omega_{\mu\nu})H_k(T_{g^{-1}}X)+R_{H_j}, \tag{1.9}$$

where $V_{jk}$ is unity for $\sigma$ and $r$ and the vector and spinor matrix representation of the Lorentz group for $\Phi_\mu$ and $\phi_a^\pm$, respectively, whose parameter $\omega_{\mu\nu}$ is defined by eq. (1.5). $\Delta_H(\alpha)$ is a weight factor. We shall not write down the transformations of $x_\mu$, $\Theta_a^+$ and $\xi_a^-$ since they are well known [3]. In what follows the explicit expressions are written down for $\omega_{\mu\nu}$, $\Delta_H$, and $R_{H_j}$. Thus, one has for the dilatations $e^{i\lambda D}$

$$\omega_{\mu\nu}=0;\quad R_{\Phi_\mu}=R_{\phi_a^+}=R_{\phi_a^-}=R_r=0.\quad R_\sigma=-\lambda$$
$$\Delta_{\Phi_\mu}=e^{\lambda};\quad \Delta_{\phi\pm}=e^{\pm\frac{1}{2}\lambda}\quad \Delta_\sigma=\Delta_r=0. \tag{1.10}$$

The corresponding quantities for the transformations $e^{i\delta\pi}$ are

$$\omega_{\mu\nu}=0;\quad R_{\Phi_\mu}=R_{\phi_a^+}=R_{\phi_a^-}=R_\sigma=0\qquad R_r=-\delta$$

$$\Delta_{\Phi_\mu} = \Delta_\sigma = \Delta_\tau = 0; \quad \Delta_{\phi_a^+} = \Delta_{\phi_a^-} = e^{i\delta} . \tag{1.11}$$

For the special conformal transformations $\exp[ia^\mu K_\mu]$ using the notations

$$\rho(a,x) \equiv 1 + 2a\cdot x + a^2x^2; \quad a^\mu\gamma_\mu = \hat{a} \tag{1.12}$$

one has

$$\omega_{\mu\nu} = [(a\cdot x)^2 - a^2x^2]^{-1/2}(a_\mu x_\nu - a_\nu x_\mu)\ln\frac{1 + a\cdot x + \sqrt{(a\cdot x)^2 - a^2x^2}}{\sqrt{\rho(a,x)}} \tag{1.13}$$

$$\Delta_\Phi = \rho^{-1}(a,x) \qquad \Delta_{\phi_a^+} = \rho^{-1/2}(a,x) \qquad \Delta_{\phi_a^-} = \rho^{1/2}(a,x) \tag{1.14}$$

$$\Delta_\sigma = \Delta_\tau = 0$$

$$R_{\Phi_\mu} = -\rho^{-1}(a,x)(a_\mu + x_\mu a^2); \quad R_\sigma = R_\tau = 0 \tag{1.15}$$

$$R_{\phi_a^+} = -(\hat{a}\phi^-)_a (x', \Theta'^+, \xi_a'^-); \quad R_{\phi_a^-} = 0.$$

For the transformations $e^{i\beta^-\gamma^\circ s^-}$ it is useful to introduce the notation

$$\Sigma_1(\beta^-, \xi^-) = 1 + 16i\beta^-\gamma^\circ\xi^- \tag{1.16}$$

in order to write down

$$\omega_{\mu\nu} = 8i\beta^-\gamma^\circ\sigma_{\mu\nu}\xi^- \tag{1.17}$$

$$\Delta_{\Phi_\mu} = \Sigma_1^{-1/2}; \quad \Delta_{\phi_a^+} = \Sigma_1^{-1}; \quad \Delta_{\phi^-} = \Sigma_1^{-1/2}; \quad \Delta_\sigma = \Delta_\tau = 0 \tag{1.18}$$

$$R_{\Phi_\mu} = R_{\phi_a^+} = 0; \qquad R_{\phi_a^-} = -\Sigma_1^{-1}\beta_a^- \tag{1.19}$$

$$R_\sigma = \frac{1}{2}\ln\Sigma_1 ; \qquad R_\tau = -\frac{3}{4}\, i\ln\Sigma_1 .$$

And at the end introducing the notations

$$\Sigma_2(\beta^+, \Theta^+) = 1 + 16i\beta^+\gamma^\circ\Theta^+ \tag{1.20}$$



and

$$\Sigma_3(\beta^+, \Theta^+, \xi^-) = \Sigma_2(\beta^+, \Theta^+) + 16i\beta^+\gamma^\circ\hat{x}\xi^-) \tag{1.21}$$

one can write

$$\omega_{\mu\nu} = 8i[\beta^+\gamma^\circ\sigma_{\mu\nu}\Theta^+ + \beta^+\gamma^\circ\sigma_{\mu\nu}\hat{x}\xi^-] + 128\Theta^+\gamma^\circ\sigma_{\mu\nu}\hat{x}\xi^- \tag{1.22}$$

$$\Delta_{\Phi_\mu} = \Sigma_3^{-1/2}\Sigma_2; \quad \Delta_{\phi_a^+} = \Sigma_3^{-1}\Sigma_2^{1/2}; \quad \Delta_{\phi_a^-} = \Sigma_3^{-1/2}\Sigma_2^{-1/2} \tag{1.23}$$

$$\Delta_\sigma = \Delta_\tau = 0$$

$$R_{\Phi_\mu} = -8i\,\Sigma_3^{3/2}\Sigma_2^{-1/2}\beta^+\gamma^\circ\gamma_\mu\xi^-; \quad R_{\phi_a^-} = \Sigma_3^{-1}(\hat{x}\beta^+)_a \tag{1.24}$$

$$R_{\phi_a^+} = 8i\,\Sigma_3^{-1/2}(\beta^+\gamma^\circ\gamma^\nu\xi^-)(\gamma_\nu\phi^-)_a(x', \Theta'^+, \xi'^-) - \Sigma_3^{-1/2}\Sigma_2^{-1/2}\beta_a^+$$

$$R_\sigma = \frac{1}{2}\ln\Sigma_3 - \ln\Sigma_2 ; \qquad R_\tau = -\frac{3}{4}\, i\ln\Sigma_3 .$$

Thus one can see that the linear homogeneous terms can be obtained from the ordinary linear homogeneous representations described in paper [8] if one fixes the numbers d , z , p and q to be

$$\begin{aligned} &d_\Phi = -1, \quad z_\Phi = 0, \quad p_\Phi = q_\Phi = 1 \\ &d_{\phi^+} = \frac{1}{2}, \quad z_{\phi^+} = -1, \quad p_{\phi^+} = 1, \quad q_{\phi^+} = 0 \\ &d_{\phi^-} = -\frac{1}{2}, \quad z_{\phi^-} = -1, \quad p_{\phi^-} = 0, \quad q_{\phi^-} = 1 \\ &d_\sigma = z_\sigma = p_\sigma = q_\sigma = d_\tau = z_\tau = p_\tau = q_\tau = 0. \end{aligned} \tag{1.25}$$

Considering formulae (1.15) and (1.24) one can see that $\phi_a^+$ and $\phi_a^-$ transform according to a triangular representation of the special conformal transformations and the transformations spanned over the generator $T_a^+$. One may hope, at first, that a substitution $\psi_a^+ = \phi_a^+ - \Phi^\mu(\gamma_\mu\phi^-)_a$ can change the situation. However, this substitution is equivalent to a reordering of the initial form (1.4) and it would then appear that $\psi_a^+$ and $\Phi_\mu$ shall transform according to a triangular representation in that case.

2. Having constructed a nonlinear realization it is natural to try to introduce covariant derivatives. However, one can not follow the explicit procedure of paper [5] in the

case of the superconformal group, since there would appear technical difficulties. In what follows a simple modification of the technique described in paper [7] is used. Consider the differential form $(d\psi)\psi^{-1}$, where $\psi$ is the quantity (1.4). If now $O_j$ denotes any generator of the conformal superalgebra one has then

$$(d\psi)\psi^{-1} = i\sum_j W_j(\sigma,\tau,\Phi,\phi^{\pm};d\sigma,d\tau,d\Phi,d\phi^{\pm})O_j\ , \tag{2.1}$$

where $W_j$ are certain differential forms of the fields $\sigma$, $\tau$, $\Phi_\mu$, $\phi_a^+$, $\phi_a^-$ and the "coordinates" $x_\mu$, $\Theta_a^+$ and $\xi_a^-$. Having in mind the transformation law (1.5) one can easily see that $(d\psi)\psi^{-1}$ is transformed as follows

$$(d\psi')\psi'^{-1} = e^{-i\omega_{\mu\nu}M^{\mu\nu}}(d\psi)\psi^{-1}e^{i\omega_{\mu\nu}M^{\mu\nu}} + (d\,e^{-i\omega_{\mu\nu}M^{\mu\nu}})\,e^{i\omega_{\mu\nu}M^{\mu\nu}}\ . \tag{2.2}$$

So, except for the differential form $W_{\mu\nu}$ corresponding to the Lorentz generators, all other forms $W_j$ are Lorentz covariant quantities. One can calculate the explicit expressions for the forms $W_j$ using the formulae of the Appendix. Then the differentials of the fields $d\sigma$, $d\tau$, $d\Phi_\mu$, $d\phi_a^\pm$ can be expressed in terms of the differentials $dx_\mu$, $d\Theta_a^+$ and $d\xi_a^-$ of the generalized coordinates. It turns out however that these differentials are not superconformally covariant. Nevertheless one can introduce implicitly the covariant quantities, $dY_\mu$, $d\eta_a^+$ and $d\zeta_a^-$ by means of the formulae

$$dx_\mu = dY_\mu - 8i\,d\eta^+\gamma^\circ\gamma_\mu\phi^- \tag{2.3}$$

$$d\Theta_a^+ = (1+16i\phi^-\gamma^\circ\xi^-)d\eta_a^+ - dY^\mu(\gamma_\mu\xi^-)_a \tag{2.4}$$

$$d\xi_a^- = d\zeta_a^- + \Phi^\mu(\gamma_\mu d\eta^+)_a\ . \tag{2.5}$$

Then it is obvious that each of the differential forms $W_j$ can be decomposed as follows

$$W_j = dY^\mu W_{\mu;j} + (d\eta^+\gamma^\circ)_a W^+_{a;j} + (d\zeta^-\gamma^\circ)_a W^-_{a;\gamma}\ , \tag{2.6}$$

where $W_{\mu;j}$, $W^+_{a;j}$ and $W^-_{a;j}$ are certain forms that contain field derivatives. One can then see that the forms $W_\mu(P)$ $W_a^+(S^+)$ and $W_a^-(T^-)$ corresponding to the generators $P_\mu$, $S_a^+$ and $T_a^-$ coincide with $e^{-\sigma}dY_\mu$, $e^{-\frac{1}{2}\sigma-i\tau}d\eta_a^+$ and $e^{\frac{1}{2}\sigma-i\tau}$



respectively. Field derivatives are contained in the forms $W_j$ corresponding to the rest of the generators of the superconformal group. The stand point now is to consider the forms $W_{\mu;j}$, $W^+_{a;j}$ and $W^-_{a;j}$ (introduced by eq. (2.6) corresponding to the generators $D$, $\pi$, $K_\mu$, $S_a^-$ and $T_a^+$ to be the corresponding covariant derivatives of the fields $\sigma$, $\tau$, $\Phi_\mu$, $\phi_a^-$ and $\phi_a^+$, respectively.

Then introducing the notations

$$\Delta_\mu = \partial_\mu - \xi^-\gamma^\circ\gamma_\mu\gamma^\circ\frac{\partial}{\partial\Theta^+} \tag{2.7}$$

$$\Delta_a^+ = (1+16i\phi^-\gamma^\circ\xi^-)(\gamma^\circ\frac{\partial}{\partial\Theta^+})_a + 8i(\gamma^\mu\phi^-)_a\partial_\mu - \Phi_\mu(\gamma^\mu\gamma^\circ\frac{\partial}{\partial\xi^-})_a \tag{2.8}$$

$$\Delta_a^- = (\gamma^\circ\frac{\partial}{\partial\xi^-})_a \tag{2.9}$$

one has for the covariant derivatives explicitly

$$D_\mu\sigma = \Delta_\mu\sigma + 2\Phi_\mu \tag{2.10}$$

$$D_\mu\tau = \Delta_\mu\tau \tag{2.11}$$

$$D_\mu\Phi_\nu = \Delta_\mu\Phi_\nu - 2\Phi_\mu\Phi_\nu + g_{\mu\nu}\Phi^2 \tag{2.12}$$

$$D_\mu\phi_a^+ = \Delta_\mu\phi_a^+ - \Phi^\nu\Delta_\mu(\gamma_\nu\phi^-)_a + 2\Phi^\nu(\sigma_{\mu\nu}\phi^+)_a - \Phi_\mu\phi_a^+ \tag{2.13}$$

$$D_\mu\phi_a^- = \Delta_\mu\phi_a^- + (\gamma_\mu\phi^+)_a \tag{2.14}$$

$$D_a^+\sigma = \Delta_a^+\sigma + 8i\phi_a^+ + 8i\Phi^\mu(\gamma_\mu\phi^-)_a \tag{2.15}$$

$$D_a^+\tau = \Delta_a^+\tau - 12\phi_a^+ + 12\Phi^\mu(\gamma_\mu\phi^-)_a \tag{2.16}$$

$$D_a^+\Phi_\mu = \Delta_a^+\Phi_\mu + 16i\Phi^\nu(\sigma_{\mu\nu}\phi^+)_a - 8i\phi_a^+\Phi_\mu - 16i\Phi_\mu\Phi^\lambda(\gamma_\lambda\phi^-)_a + 8i\Phi^2(\gamma_\mu\phi^-)_a \tag{2.17}$$

$$\begin{aligned} D_a^+\phi_\beta^+ = {} & \Delta_a^+\phi_\beta^+ - \Phi^\mu\Delta_a^+(\gamma_\mu\phi^-)_\beta + 16i\Phi^\mu(\gamma_\mu\phi^-)_a\phi_\beta^+ - \\ & -4i(\sigma^{\mu\nu}\phi^+)_a(\sigma_{\mu\nu}\phi^+)_\beta + 4i\phi_a^+\phi_\beta^+ - \\ & -16i\Phi^\mu(\gamma_\mu\phi^-)_a\Phi^\lambda(\gamma_\lambda\phi^-)_\beta \end{aligned} \tag{2.18}$$

$$D_a^+\phi_\beta^- = \Delta_a^+\phi_\beta^- - 16i\Phi^\mu(\gamma_\mu\phi^-)_a\phi_\beta^- \tag{2.19}$$

$$D_a^-\sigma = \Delta_a^-\sigma - 8i\phi_a^- \tag{2.20}$$

$$D_\alpha^- r = \Delta_\alpha^- r - 12\phi_\alpha^- \tag{2.21}$$

$$D_\alpha^- \Phi_\mu = \Delta_\alpha^- \Phi_\mu + 16 i(\sigma_{\mu\nu}\phi^-)_\alpha \Phi^\nu + 8i\phi_\alpha^- \Phi_\mu \tag{2.22}$$

$$D_\alpha^- \phi_\beta^+ = \Delta_\alpha^- \phi_\beta^+ - \Phi^\mu \Delta_\alpha^- (\gamma_\mu \phi^-)_\beta + 16 i \phi_\alpha^+ \phi_\beta^- - \tag{2.23}$$
$$-4i\phi_\alpha^- \Phi^\mu (\gamma_\mu\phi^-)_\beta - 4i(\sigma^{\mu\nu}\phi^-)_\alpha (\gamma_\lambda \sigma_{\mu\nu}\phi^-)_\beta \Phi^\lambda$$

$$D_\alpha^- \phi_\beta^- = \Delta_\alpha^- \phi_\beta^- - 4i\phi^-\gamma^0\phi^-\gamma^0(1+i\gamma^5)_{\alpha\beta} \ . \tag{2.24}$$

where $D_\mu$ denotes vector covariant derivative and $D_\alpha^\pm$ are spinor covariant derivatives and

$$\frac{1}{2}(1 \pm i\gamma^5)_{\alpha\beta} D_\beta^\pm = D_\alpha^\pm \ . \tag{2.25}$$

It must be noted that these derivatives are superconformally covariant with respect to both indices despite the fact that $\Phi_\mu$ and $\phi_\alpha^\pm$ are not covariant themselves.

Now in order to obtain the superconformally covariant derivatives of a superfield $u_j$ with arbitrary Lorentz structure that transforms according to a representation of one of the series introduced in papers /1,2,3/ we make use of the differential form $W_{\mu\nu}$ that corresponds to the generator $M_{\mu\nu}$. According to formula (2.2) the quantity $W_{\mu\nu}M^{\mu\nu}$ transforms as follows

$$W'_{\mu\nu}M^{\mu\nu} = W_{\mu\nu} e^{-i\omega_{\lambda\rho}M^{\lambda\rho}} M_{\mu\nu} e^{i\omega_{\lambda\rho}M^{\lambda\rho}} + \tag{2.26}$$
$$+ (d e^{-i\omega_{\lambda\rho}M^{\lambda\rho}})\, e^{i\omega_{\lambda\rho}M^{\lambda\rho}} \ .$$

The transformation law of an arbitrary superfield $u_j$ can be written again in the form (1.9) but with different values for $d$, $z$, $p$ and $q$. But then since the matrix $V_{ij}$ appearing in (1.9) is a representation of the Lorentz group, one can see that the transformation law of the differential of a superfield acquires an additive term that is quite alike the second term in (2.26) except for the fact that instead of $M_{\lambda\rho}$ one has the generators of the corresponding representation of the Lorentz group. So one can use the differential form $W_{\mu\nu}$ in order to cancel this additive term in the latter transformation law.

So one can write down the covariant differential for the field in the form



$$(Du)_j = du_j + iW_{\mu\nu}(\Sigma^{\mu\nu}u)_j + (Ru)_j \ , \tag{2.27}$$

where $\Sigma^{\mu\nu}$ denotes the generators of the corresponding representation of the Lorentz group, while the last term $(Ru)_j$ must be chosen in such a way that the differentials of the weight factors in the transformation law (1.9) should be cancelled. Introducing once more the covariant quantities $d\Upsilon_\mu$, $d\eta_\alpha^+$ and $d\zeta_\alpha^-$ and following exactly the same procedure as in the previous case one can finally obtain the covariant derivatives of the superfield $u_j$ in the form

$$D_\mu u_j = \Delta_\mu u_j + 2i\Phi^\nu(\Sigma_{\mu\nu}u)_j - 2d_u\Phi_\mu u_j \tag{2.28}$$

$$D_\alpha^+ u_j = \Delta_\alpha^+ u_j + 8i(\sigma^{\mu\nu}\phi^+)_\alpha(\Sigma_{\mu\nu}u)_j -$$
$$-8i\Phi^\lambda(\gamma_\lambda\sigma^{\mu\nu}\phi^-)_\alpha(\Sigma_{\mu\nu}u)_j - 8i(d_u+3/2\,z_u)\phi_\alpha^+ u_j - \tag{2.29}$$
$$-8i(d_u - 3/2\,z_u)\Phi^\mu(\gamma_\mu\phi^-)_\alpha u_j$$

$$D_\alpha^- u_j = \Delta_\alpha^- u_j + 8i(\sigma^{\mu\nu}\phi^-)_\alpha(\Sigma_{\mu\nu}u)_j + \tag{2.30}$$
$$+8i(d_u - 3/2\,z_u)\phi_\alpha^- u_j \ ,$$

where $d_u$ and $z_u$ are the values of the numbers $d$ and $z$ for the field $u_j$. One can prove by direct computation that formulae (2.28) determine indeed superconformally covariant quantities.

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## APPENDIX

Here we give a list of necessary formulae for the commutators of the exponents. For brevity we write down explicitly only those commutators, which can not be easily written down from general considerations.

$$e^{i\Theta^\pm\gamma^0 S^\pm}\, e^{iz\pi} = e^{iz\pi}\, e^{ie^{\pm iz}\Theta^\pm\gamma^0 S^\pm}$$

$$e^{i\xi^\pm\gamma^0 T^\pm}\, e^{iz\pi} = e^{iz\pi}\, e^{ie^{\mp iz}\xi^\pm\gamma^0 T^\pm} \tag{A.1}$$

$$e^{i\Theta^+\gamma^o S^+}\, e^{i\Theta^-\gamma^o S^-} = e^{i\Theta^-\gamma^o S^-}\, e^{i\Theta^+\gamma^o S^+}\, e^{-8\Theta^-\gamma^o\gamma_\mu \Theta^+ P^\mu}$$

$$e^{i\xi^-\gamma^o T^-}\, e^{i\Theta^-\gamma^o S^-} = e^{i(1+16i\Theta^-\gamma^o\xi^-)^{-1}\Theta^-\gamma^o S^-} \times$$

$$\times\, e^{-8\xi^-\gamma^o\sigma^{\mu\nu}\Theta^- M_{\mu\nu}}\, e^{-\frac{1}{2}\ln(1+16i\xi^-\gamma^o\Theta^-)(D-3/2i\pi)} \times \qquad (A.2)$$

$$\times\, e^{i(1+16i\xi^-\gamma^o\Theta^-)^{-1}\Theta^-\gamma^o T^-}$$

$$e^{ix_\mu P^\mu}\, e^{ia_\mu K^\mu} = e^{i\frac{a^\mu + x^\mu a^2}{\rho(a,x)}K_\mu}\, e^{-i\ln\rho(a,x)D} \times$$

$$(A.3)$$

$$\times\, e^{i\left[\ln\frac{1+a\cdot x+\sqrt{(a\cdot x)^2-a^2x^2}}{\rho(a,x)}\right]\frac{a_\mu x_\nu - a_\nu x_\mu}{\sqrt{(a\cdot x)^2-a^2x^2}}M^{\mu\nu}}\, e^{i\frac{x^\mu + a^\mu x^2}{\rho(a,x)}P_\mu}$$

$$e^{i\Theta^\pm\gamma^o S^\pm}\, a^{ia^\mu K_\mu} = e^{ia^\mu K_\mu}\, e^{i[\Theta^\pm\gamma^o S^\pm - \Theta^\pm\gamma^o \hat{a} T^\mp]}$$

$$e^{ix_\mu P^\mu}\, e^{i\xi^\pm\gamma^o T^\pm} = e^{i\xi^\pm\gamma^o T^\pm}\, e^{i\xi^\pm\gamma^o \hat{x} S^\mp}\, e^{ix^\mu P_\mu},$$

$$e^{i\xi^-\gamma^o T}\, e^{i\beta^+\gamma^o T^+} = e^{i\beta^+\gamma^o T^+}\, e^{i\xi^-\gamma^o T^-}\, e^{-8\beta^+\gamma^o\gamma^\nu\xi^- K_\nu} \qquad (A.4)$$

$$e^{i\Theta^+\gamma^o S^+}\, e^{i\beta^+\gamma^o T^+} = e^{i(1+16i\beta^+\gamma^o\Theta^+)^{-1}\beta^+\gamma^o T^+} \times$$

$$\times\, e^{\frac{1}{2}\ln(1+16i\beta^+\gamma^o\Theta^+)(D+3/2i\pi)}\, e^{8\beta^+\gamma^o\sigma_{\mu\nu}\Theta^+ M^{\mu\nu}} \times$$

$$\times\, e^{i(1+16i\beta^+\gamma^o\Theta^+)^{-1}\Theta^+\gamma^o S^+}$$



All these formulae can be proved using the commutators of the generators of the superconformal group.

REFERENCES


1. Molotkov V., Petrova S.G., Stoyanov D.Ts. TMPh, 1976, 26, p.188.
2. Aneva B.L., Mikhov S.G., Stoyanov D.Ts. TMPh, 1976, 27, p.307.
3. Aneva B.L., Mikhov S.G., Stoyanov D.Ts., TMPh, 1977, 31, p.177.
4. Coleman S., Wess G., Zumino B. Phys.Rev., 1969, 177, p.2239.
5. Salam A., Strathdee J. Phys.Rev., 1969, 184, p.2247.
6. Ogievetsky V.I. Acta Univ. Wratislaw., No. 207, X-th Winter School of Theor. Phys. in Karpacz.
7. Volkov D.V. Particles and Nucleus, 1973, 4, No.3.



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