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Объединенный институт ядерных исследований БИБЛИОТЕКА

Михов С.Г., Стоянов Д.Ц.

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О бозонизации безмассовой спинорной электродинамики

Предложен метод построения полей, которые преобразуются по линейному представлению некоторой группы Ли, из полей, преобразующихся нелинейно под действием той же самой группы. Этот метод применяется для построения спинорного поля с помощью антисимметрического тензорного поля. Данная "бозонизация" спинорного поля используется для переформулировки уравнений безмассовой спинорной электродинамики в терминах нелинейных тензорных полей. При этом оказывается, что уравнение Дирака сводится к определению электромагнитного вектор-потенциала в терминах нелинейных тензорных полей и к условию сохранения тока спинорных полей. Это условие обеспечивает самосогласованность данной теории.

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On the Bosonization of the Massless Spinor Electrodynamics

A method for constructing a field transformed according to a linear representation of a Lie group out of fields transformed nonlinearly under the same group is proposed. This procedure is used in order to construct spinor fields out of tensor ones. Such a "bosonization" of the spinor field is used to reformulate the massless spinor electrodynamics in terms of nonlinear tensor fields. It appears in this formulation that the Dirac equation is reduced to a definition of the electromagnetic vector potential in terms of the nonlinear tensor fields and to the current conservation playing the role of a consistency condition for this formulation.

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INTRODUCTION

It is generally accepted that one can construct tensor fields out of spinor ones, but not on the contrary. The origin of this statement lies on the fact[1] that one is usually restricted to deal with linear irreducible representations of the Lorentz group, and the tensor product of integer spin representations of this kind does not contain half integer spin representations. However, the two-dimensional models give counter example[1,2,3]. The latter are remarkable for the fact that the tensor fields, in terms of which one can reproduce the spinor field, are not homogeneous representations of the Lorentz group. Some recent works of Luther et al.[4] give rise to hopes that in the realistic four-dimensional case the construction of spinors out of tensors can prove to be useful too. In the present paper we propose a correct mathematical formulation for such a construction and use it to reformulate the classical equations of the massless spinor electrodynamics.

2. FORMULATION OF THE BOSONIZATION ANSATZ

Let G be an N-parametric Lie group and g_i, $i=1,2,...,N$ are the parameters of the group element $g \in G$ in a given parametrization. The space Z of the parameters of the group G is simple homogeneous under the action of the left regular representation of the group, so that it induces a certain representation in Z. The latter is written in the form

$$Z'_i = F_i(g_k; z_\ell), \qquad z'_i, z_\ell \in Z, \tag{2.1}$$

where F_i is the group composition law in the group G. As it is well known[5,6] formula (2.1) introduces a nonlinear realization of the group G.

On the other hand, let $U(g)$ be a linear representation of the group G acting in a M-dimensional space H_M

$$h_a = U_{ab}(g) h_b \,, \tag{2.2}$$

where h_a are the coordinates of a given element $h \in H_M$.

Consider now $z_i = z_i(x)$, $i=1,\dots,N$ be an N-component field defined over a space $M(x \equiv \{x_\mu\} \in M)$, where a linear representation $\Lambda_\mu{}^\nu(g)$ of the group G is acting. Under the action of the group G the fields z_i transform according to

$$z'_i(x) = F_i[g_k; z_\ell(\Lambda_\mu{}^\nu(g^{-1})x_\nu)], \tag{2.3}$$

where $F_i(g,z)$ is the function (2.1). We then introduce the field

$$\psi_a(x) = U_{ab}(z_i(x))u_b \,, \tag{2.4}$$

where u_b, $b=1,\dots,M$ are M numbers while $U_{ab}(z_i)$ is the matrix determined by eq. (2.2) in which the group parameters are replaced by the field $z_i(x)$. We suppose that under the action of the group G, the transformed field $\psi'_a(x)$ should be written in the same form (2.4) by means of the transformed fields $z'_i(x)$:

$$\psi'_a(x) = U_{ab}(z'_i(x))u_b \,. \tag{2.5}$$

Here in fact it must be stressed that u_b are supposed to be scalars under the action of the group G. Now one can easily see that with the assumption (2.5) for the transformed $\psi'_a(x)$ the form of the field $\psi_a(x)$ is transformed according to the linear representation (2.2). The following sequence of equations

$$\begin{aligned}\psi'_a(x) &= U_{ab}(z'_k)u_b = U_{ab}(F(g,z))u_b = \\ &= U_{ab}(g)U_{bc}(z_k)u_c = U_{ab}(g)\psi_b(x')\end{aligned} \tag{2.6}$$

proves this statement.

Now we are ready to formulate the bosonization ansatz. For the purpose we fix the group G to be the Lorentz group, and denote by $\alpha_{\mu\nu}$ its parameters while by $V_{\rho\beta}(\alpha_{\mu\nu})$ we denote the spinor representation of the given group. Then according to our previous statement, the field

$$\psi_\rho(x) = V_{\rho\beta}(\alpha_{\mu\nu}(x)) \qquad \alpha_{\mu\nu} = -\alpha_{\nu\mu} \tag{2.7}$$

is a spinor field. According to formula (2.7) we can consider the nonlinear field $\alpha_{\mu\nu}(x)$ instead of the spinor field $\psi_\rho(x)$. Under transformations belonging to the Lorentz group, the fields $\alpha_{\mu\nu}(x)$ transform as follows

$$\alpha'_{\mu\nu}(x) = F_{\mu\nu}(g; \alpha_{\mu\nu}(\Lambda_\lambda{}^\rho(g^{-1})x_\rho)), \tag{2.8}$$

where the function $F_{\mu\nu}(g_{\rho\sigma}; \alpha_{\lambda\tau})$ is the group composition law of the Lorentz group and $\Lambda_\lambda{}^\rho(g)$ is the vector representation of the same group. We shall call formula (2.7) bosonization of the spinor field.

Now we shall use the ansatz (2.7) in order to formulate the Dirac equation for a massless spinor field interacting with an electromagnetic vector potential in terms of the nonlinear tensor field $\alpha_{\mu\nu}(x)$. As is well known for the massless case the γ^5-invariance implies that the original equation is decomposed onto the following two equations

$$i\gamma^\mu \partial_\mu \psi_\pm(x) = e\gamma^\mu A_\mu(x)\psi_\pm(x) \tag{2.9}$$

(We choose the metric to be $g_{\mu\mu} = (+,-,-,-)$), where the quantities $\psi_{\pm\rho}(x)$ are defined by

$$\psi_{\pm\rho}(x) = \frac{1}{2}(1 \pm i\gamma^5)_{\rho\beta}\psi_\beta(x) = P_{\pm\rho\beta}\psi_\beta(x); \quad (\gamma^5)^2 = -1. \tag{2.10}$$

As is well known the Lorentz group has two irreducible unequivalent representations with spin 1/2. The fields $\psi_+(x)$ and $\psi_-(x)$ distinguish between these two representations. Each of these two fields can be expressed in terms of the nonlinear tensor fields $\alpha^+_{\mu\nu}(x)$ and $\alpha^-_{\mu\nu}(x)$, respectively, according to (2.7). Now we shall substitute eq. (2.7) for $\psi_{\pm\rho}(x)$ into eq. (2.9). For the analysis of the L.H.S. of eq. (2.9) we use the first differential equation of Lie, namely

$$\frac{\partial V^\pm_{\rho\beta}}{\partial \alpha^\pm_{\mu\nu}} = \frac{i}{2} S^{\mu\nu}{}_{\sigma\lambda}(\alpha^\pm)(\Sigma^{\pm\sigma\lambda})_{\rho\gamma} V^\pm_{\gamma\beta} \,, \tag{2.11}$$

where $(\Sigma^{\pm\sigma\lambda})_{\rho\gamma}$ are the generators of the irreducible spinor representations of the Lorentz group and $S^{\mu\nu}{}_{\sigma\lambda}$ are the structure functions of the group. It is well known that they are real and do not depend on the representation.

In order to go further we must calculate the quantity $(\gamma^\mu \Sigma^{\pm\rho\tau})_{\sigma\lambda}$. For the purpose we first write the generators $(\Sigma^{\pm\rho\tau})_{\sigma\lambda}$ in the following convenient form

$$(\Sigma^{\pm\rho\tau})_{\sigma\lambda} = (\sigma^{\rho\tau} P_\pm)_{\sigma\lambda}. \qquad (2.12)$$

Then it is easy to see that

$$(\gamma^\mu \Sigma^{\pm\rho\tau})_{\sigma\lambda} = 2i\, \Pi^{\pm\rho\tau;\mu\delta} (\gamma_\delta P_\pm)_{\sigma\lambda}, \qquad (2.13)$$

where

$$\Pi^{\pm\mu\nu}{}_{\lambda\rho} = \frac{1}{4}\{(\delta^\mu{}_\lambda \delta^\nu{}_\rho - \delta^\mu{}_\rho \delta^\nu{}_\lambda) \pm i\epsilon^{\mu\nu}{}_{\lambda\rho}\},$$
$$\epsilon^{0123} = -\epsilon_{0123} = 1. \qquad (2.14)$$

One can easily see that $\Pi^\pm$ are the projection matrices in the space of antisymmetric tensors, that is

$$\Pi^\pm \Pi^\mp = 0 \qquad \Pi^\pm \Pi^\pm = \Pi^\pm, \qquad (2.15)$$

$$\Pi^+ + \Pi^- = I \qquad I^{\mu\nu}{}_{\lambda\rho} = \frac{1}{2}(\delta^\mu{}_\lambda \delta^\nu{}_\rho - \delta^\mu{}_\rho \delta^\nu{}_\lambda). \qquad (2.16)$$

On can see that the matrices $\Pi^\pm$ satisfy in addition the following duality condition.

$$\frac{1}{2}\epsilon^{\mu\nu}{}_{\alpha\beta}\Pi^{\pm\alpha\beta}{}_{\lambda\rho} = \frac{1}{2}\epsilon^{\alpha\beta}{}_{\lambda\rho}\Pi^{\pm\mu\nu}{}_{\alpha\beta} = \mp i\,\Pi^{\pm\mu\nu}{}_{\lambda\rho}. \qquad (2.17)$$

So that the corresponding projections have the same duality properties.

We will produce the calculations for sign " + " only, since they are analogous for sign "-". So in view of eqs. (2.11) and (2.13) we have for the L.H.S. of eq. (2.9)

$$-\frac{i}{2}(\partial^\mu a^+_{\nu\lambda}) S^{\nu\lambda}{}_{\rho\sigma}(a^+)\Pi^{+\rho\sigma}{}_{\mu\tau}\gamma^\tau \psi_+(x).$$

If one takes into account the following identity for the matrices

$$\sigma^{\mu\nu} P_\pm = \Pi^{\pm\mu\nu}{}_{\lambda\rho}\sigma^{\lambda\rho} \qquad (2.18)$$

one can see that it implies the following identity for the structure functions

$$\Pi^{+\mu\nu}{}_{\alpha\beta} S^{\alpha\beta}{}_{\lambda\rho} = S^{\mu\nu}{}_{\alpha\beta}\Pi^{+\alpha\beta}{}_{\lambda\rho}. \qquad (2.19)$$

Equation (2.19) gives the possibility for the following definition

$$T^{\mu\nu}{}_{\lambda\rho} = 2\Pi^{+\mu\nu}{}_{\alpha\beta} S^{\alpha\beta}{}_{\lambda\rho}. \qquad (2.20)$$

So that if we introduce the selfdual quantities

$$\omega^+_{\mu\nu} = 2\Pi^+_{\mu\nu}{}^{\lambda\rho} a^+_{\lambda\rho} \qquad i{*\omega}^+_{\mu\nu} = \omega^+_{\mu\nu}, \qquad (2.21)$$

where

$${*\omega}^+_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu}{}^{\lambda\rho}\omega^+_{\lambda\rho}. \qquad (2.22)$$

We obtain finally

$$\{\frac{i}{4}(\partial^\nu \omega^+{}_{\lambda\rho}) T^{\lambda\rho}{}_{\nu\mu} + eA_\mu\}(\gamma^\mu \psi_+)_a = 0. \qquad (2.23)$$

And therefore we can write down

$$eA_\mu = -\frac{i}{4}(\partial^\nu \omega^+_{\lambda\rho}) T^{\lambda\rho}{}_{\nu\mu}. \qquad (2.24)$$

Now, however, we have a problem, the electromagnetic potential is a real function, while the R.H.S. of (2.24) is, generally speaking, a complex quantity. So that in fact eq. (2.24) is decomposed onto the following two equations:

$$eA_\mu = \frac{1}{4}(\partial^\nu {*a}^+_{\lambda\rho}) S^{\lambda\rho}{}_{\nu\mu}(a^+), \qquad (2.25)$$

$$(\partial^\nu a^+_{\lambda\rho}) S^{\lambda\rho}{}_{\nu\mu}(a^+) = 0. \qquad (2.26)$$

Equation (2.26) is a consistency condition which we will call condition for reality.

If one produces the analogous calculations for $a^-_{\mu\nu}(x)$ one obtains the same expression for eq. (2.26), while the R.H.S. of (2.25) has the opposite sign. So that one can compactly write

$$eA_\mu = \pm\frac{1}{4}(\partial^\nu {*a}^\pm_{\lambda\rho}) S^{\lambda\rho}{}_{\nu\mu}(a^\pm). \qquad (2.27)$$

So that having in mind that $A_\mu(x)$ is equal for both ψ_+ and ψ_- a consistency condition equating the R.H.S. of (2.27) appears. Now, it is evident why $a^+_{\mu\nu}$ should not be equal to $a^-_{\mu\nu}$.

3. A CONVENIENT PARAMETRIZATION. REALITY AND CURRENT CONSERVATION. TRANSFORMATION PROPERTIES

In order to obtain explicit formulae, it is necessary to introduce a concrete parametrization. Using the tensors $\omega^{\pm}_{\mu\nu}$ as parameters of the Lorentz group we can write the matrix of eq. (2.7) in the form

$$V^{\pm}_{\alpha\beta}(\omega^{\pm}_{\mu\nu}(x)) = \frac{1}{4} L^{\pm}_{\mu\nu}(x)[(g^{\mu\nu} - 2i\sigma^{\mu\nu})P_{\pm}]_{\alpha\beta}, \tag{3.1}$$

where

$$L^{\pm}_{\mu\nu}(x) = g_{\mu\nu}a^{\pm}(x) - \frac{1}{2}\omega^{\pm}_{\mu\nu}(x); \quad \pm i * \omega^{\pm}_{\mu\nu}(x) = \omega^{\pm}_{\mu\nu}(x), \tag{3.2}$$

$$a^{\pm 2}(x) + \frac{1}{16}\omega^{\pm}_{\mu\nu}(x)\omega^{\pm\mu\nu}(x) = 1. \tag{3.3}$$

In what follows we make a wide use of the matrices $L^{\pm}_{\mu\nu}(x)$. So it is necessary to give some of their properties, that follow from (3.3) and the self duality of $\omega^{\pm}_{\mu\nu}(x)$. We have

A. $L^{\pm}_{\mu\nu}(x)$ are complex orthogonal matrices, i.e.,

$$L^{\pm}_{\mu\nu} L^{\pm\lambda\nu} = \delta_{\mu}^{\lambda}. \tag{3.4}$$

This follows immediately from the fact that a self- or anti-selfdual tensor is proportional to an orthogonal matrix. Then eq. (3.3) does the rest of the job.

B. $L^{+}_{\mu\nu}$ commutes with $L^{-}_{\mu\nu}$, as well as $L^{+}_{\mu\nu}$ commutes with $\bar{L}^{+}_{\mu\nu}$ and $L^{-}_{\mu\nu}$ with $\bar{L}^{-}_{\mu\nu}$. This follows from the fact that the product of a selfdual and antiselfdual is a symmetric tensor.

C. As a corollary to properties A and B we have the following statement. The matrix defined by

$$\Lambda_{\mu}^{\ \nu} = L^{+}_{\mu\lambda}\bar{L}^{\lambda\nu} \tag{3.5}$$

belongs to the vector representation of the Lorentz group. It is evident that all elements of the proper Lorentz group can be represented in that way and vice versa.

Now we can calculate the structure functions $S^{\mu\nu}_{\ \ \lambda\rho}$ in terms of the parameters $\omega_{\mu\nu}$. We consider in what follows the case $\omega^{+}_{\mu\nu}$ only, since for $\omega^{-}_{\mu\nu}$ the considerations are

analogous. So that if we express $\omega^{+}_{\mu\nu}$ in terms of $L^{+}_{\mu\nu}$ through eq. (3.12) and substitute into (2.25) and (2.26), we obtain

$$eA_{\mu} = -\operatorname{Im}[L^{+}_{\mu\nu}\partial_{\lambda}L^{+\lambda\nu}], \tag{3.6}$$

$$L^{+}_{\mu\nu}\partial_{\lambda}L^{+\lambda\nu} + \bar{L}^{+}_{\mu\nu}\partial_{\lambda}\bar{L}^{+\lambda\nu} = 0. \tag{3.7}$$

Now we can use the fact that $L^{+}_{\mu\nu}$ and $\bar{L}^{+}_{\mu\nu}$ commute to write eq. (3.7) in the form

$$(\partial^{\lambda}L^{+}_{\lambda\nu})\bar{L}^{+\nu\mu} + (\partial^{\lambda}\bar{L}^{+}_{\lambda\nu})L^{+\nu\mu} = 0. \tag{3.8}$$

If we substitute eq. (3.1) into (2.7) and use the current definition we obtain

$$j^{+}_{\mu} = \bar{\psi}_{+}\gamma_{\mu}\psi_{+} = L^{+}_{\mu\nu}\bar{L}^{+\nu\lambda}\ \bar{u}\gamma_{\lambda}P_{+}u = \Lambda_{\mu}^{\ \lambda}\ \bar{u}\gamma_{\lambda}P_{+}u. \tag{3.9}$$

Therefore the current conservation will read

$$(\partial^{\mu}L^{+}_{\mu\nu})\bar{L}^{+\nu\lambda} + L^{+}_{\mu\nu}\partial^{\mu}\bar{L}^{+\nu\lambda} = 0. \tag{3.10}$$

One can now use the identity

$$L^{+}_{\mu\nu}\partial^{\mu}\bar{L}^{+\nu\lambda} = (\partial^{\mu}\bar{L}^{+}_{\mu\nu})L^{+\nu\lambda} \tag{3.11}$$

to see that the current conservation coincides with the reality condition (3.8). To consider the Lorentz transformation properties we write the transformation law for the matrices $L^{+}_{\mu\nu}$ which is linear

$$L'^{+}_{\mu\nu}(x) = L^{+}_{\mu\lambda}(\beta)L^{+\lambda}_{\ \ \nu}(x'), \tag{3.12}$$

where $L^{+}_{\mu\lambda}(\beta)$ is defined analogously to (3.2)

$$L^{+}_{\mu\nu}(\beta) = g_{\mu\nu}\rho - \frac{1}{2}\beta^{+}_{\mu\nu}; \quad \beta^{+}_{\mu\nu} = \beta_{\mu\nu} + i{*}\beta_{\mu\nu}, \tag{3.13}$$

$$\rho^{2} + \frac{1}{16}\beta_{\mu\nu}\beta^{\mu\nu} = 1,$$

$$x'_{\mu} = x_{\lambda}\Lambda^{\lambda}_{\ \ \nu}(\beta) = x_{\lambda}L^{+\lambda\rho}(\beta)\bar{L}^{+}_{\rho\mu}(\beta), \tag{3.14}$$

and $\beta_{\mu\nu}$ is the parameter of the Lorentz transformation.

We must prove now that the R.H.S. of (3.6) is a Lorentz vector. For the purpose we consider the quantity $\partial^\mu L^+_{\mu\nu}$. Having in mind that $L^+_{\mu\nu}$ and $\bar{L}^+_{\mu\nu}$ are mutually commuting we have

$$(\partial^\mu L^+_{\mu\nu})(x))' = \partial^\mu L^+_{\mu\nu}{}'(x) = \bar{L}^+_{\nu\lambda}(\beta)(\partial_\mu L^{+\mu\lambda})(x'). \tag{3.15}$$

Substituting eq. (3.15) into the R.H.S. of eq. (3.6) we see that it transforms as a vector field.

4. FURTHER DISCUSSION OF THE REALITY CONDITION

For our purpose the form (3.8) of the reality condition is most convenient. It is obvious that the two terms of this equation are complex conjugate one to another. Therefore, we can write it in the form

$$(\partial^\lambda L^+_{\lambda\nu})\bar{L}^{+\nu\mu} = i f^{+\mu}(x), \tag{4.1}$$

where $f^{+\mu}$ are four real functions. Having in mind the transformation laws (3.12) and (3.15) as well as orthogonality of the matrices $L^+_{\mu\nu}$, we see that the L.H.S. of (4.1) despite of its vector index is in fact a set of four scalars. The functions $f^{+\mu}$ determine the arbitrariness of the problem. The fact that they are scalars implies that there exist stringent conditions on their choice. It is most probable that they should be constants. In what follows we confine ourselves to that case. So we have the conditions

$$\partial^\mu L^+_{\mu\nu}(x) = i\bar{L}^+_{\nu\mu}(x) f^{+\mu} . \tag{4.2}$$

This is a system of linear differential equations and it can be explicitly resolved. However, we first rewrite it in a more transparent form. For the purpose we introduce the quantities h^+, g^+ and $F^+_{\mu\nu}$ by means of the formulae

$$a^+ = \frac{1}{2}[(1+i)h^+ - (1-i)g^+], \tag{4.3}$$

$$\omega^+_{\mu\nu} = \frac{1}{2}[(1+i)F^+_{\mu\nu} - (1-i)*F^+_{\mu\nu}]. \tag{4.4}$$

If we now separate the real and immaginary parts of eqs. (4.2) we will obtain the following system of equation*

$$\nabla^{+\mu} F^+_{\mu\nu} = 2\nabla^-_\nu h^+ , \tag{4.5}$$

$$\nabla^{-\mu} *F^+_{\mu\nu} = 2\nabla^+_\nu g^+ , \tag{4.6}$$

where

$$\nabla^{\pm\mu} = \partial^\mu \pm f^{+\mu} . \tag{4.7}$$

Equations (4.5) and (4.6) look quite alike the Maxwell equations in the presence of electric and magnetic currents, except for the fact that instead of the usual derivatives we have the differential operators (4.7). And it is immediate to try to introduce as usually a system of vector potentials $B^{+\mu}$ and $D^{+\mu}$. So we write $F^+_{\mu\nu}$ in the form

$$F^+_{\mu\nu} = \nabla^-_{[\mu} B^+_{\nu]} + *(\nabla^+_{[\mu} D^+_{\nu]}), \tag{4.8}$$

where the star denotes as usually the dual tensor. As is well known the form (4.8) is invariant under the following two kinds of gauge transformations. First these are the "ordinary" gauge transformations

$$D^+_\mu \to D^+_\mu + \nabla^+_\mu \tilde{\phi}^+ , \tag{4.9}$$

$$B^+_\mu \to B^+_\mu + \nabla^-_\mu \phi^+ . \tag{4.10}$$

And then we have gauge transformations of the kind

$$B^+_\mu \to B^+_\mu + B^+_{0;\mu} , \tag{4.11}$$

$$D^+_\mu \to D^+_\mu + D^+_{0;\mu} , \tag{4.12}$$

where $B^+_{0;\mu}$ and $D^+_{0;\mu}$ satisfy the equation

$$\nabla^-_{[\mu} B^+_{0;\nu]} + *(\nabla^+_{[\mu} D^+_{0;\nu]}) = 0. \tag{4.13}$$

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* Remark. The system (4.2) can be written exactly in the same form (4.5)-(4.6) in the case, when $f^{+\mu}$ are functions of the coordinates

So that we have even more arbitrariness in the determination of $B^{\pm}_{\mu}$ and $D^{\pm}_{\mu}$ from eq. (4.5) and (4.6). So that there exists a problem whether eq. (3.3) and the condition following from (2.27) are compatible with the linear differential system of equations (4.5) and (4.6) with the ansatz (4.8). We do not consider this problem here, although, it is necessary to mention that there exists a quite simple choice for $B^{\pm}_{\mu}$ and $D^{\pm}_{\mu}$ (being constant vectors multiplied by $\exp(Q\cdot x)$, where $Q^2=f^2$) that satisfy all the conditions of the problem. We do not write here explicitly this example since it appeared that in this case the electromagnetic vector potential turned out to be a pure gauge. Nevertheless, it means that eq. (4.2) is compatible with the nonlinear conditions, and one might hope that there exist nontrivial solutions also.

Now having in mind eqs. (3.6) and (4.2) we find for the explicit form

$$eA_\mu = -L^+_{\mu\nu}\bar{L}^{+\nu\lambda} f^+_\lambda = -\Lambda_\mu^{\ \lambda} f^+_\lambda . \qquad (4.14)$$

And therefore we have also (see (3.10))

$$\partial^\mu A_\mu = 0, \qquad (4.15)$$

$$A^\mu A_\mu = f^2 . \qquad (4.16)$$

So that the Maxwell equation reads

$$\Box A_\mu = j_\mu . \qquad (4.17)$$

On the other hand having in mind the explicit form (3.9) for the current, we see that it is quite similar to A_μ , although not necessarily equal. So that one can think of a situation, where the electromagnetic potential A_μ is proportional to the current j_μ (this condition will obviously imply one more relation between the matrices $L^+_{\mu\nu}$ and $L^-_{\mu\nu}$)

$$j_\mu = a A_\mu . \qquad (4.18)$$

And therefore eq. (4.17) will then read

$$(\Box - a) A_\mu = 0. \qquad (4.19)$$

If we choose $a = -m^2$ we obtain finally an effective massive vector field.

So we see that, in principle, in that particular case ($f^{+\mu}(x) = f^{+\mu} = \text{const}$) starting from a nonlinear system of equations for interacting massless spinor and massless vector fields we are able to obtain finally a transverse massive free vector field that satisfies in addition the nonlinear boundary condition (4.2).

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