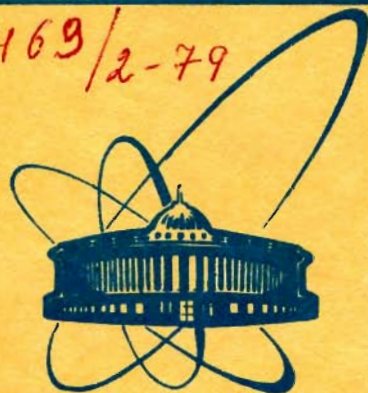


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IN HADRON-NUCLEUS INTERACTIONS**

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**KNO-SCALING  
IN HADRON-NUCLEUS INTERACTIONS**

*Submitted to ЯФ*

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**E2 - 12823**

**KNO-скейлинг в адрон-ядерных взаимодействиях**

В рамках модели каскада лидирующей частицы с учетом флуктуации энергетических потерь налетающей частицы получено соотношение, связывающее KNO-функции в  $hA$ - и  $hN$ -столкновениях. Вид KNO-функции в  $hA$ -взаимодействиях слабо зависит от атомного номера  $A$ . Получены аналитические выражения для первых пяти нормированных моментов в  $hA$ -взаимодействиях. Расчеты с помощью этих выражений показывают, что величины  $C_p^{hA}$  мало отличаются от аналогичных величин  $C_p^{hN}$ , а их различие растет с ростом порядка моментов.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1979

Alaverdyan G.B. et al.

**E2 - 12823**

**KNO-Scaling in Hadron-Nucleus Interactions**

The relation connecting KNO-functions in  $hA$ - and  $hN$ -collisions is obtained in the framework of the cascade model of a leading particle with consideration of energy loss fluctuation of the incident particle. The analytical expressions for the first five normalized moments in  $hA$ -interactions are given. The calculation results show that difference between the quantities  $C_p^{hA}$  and  $C_p^{hN}$  is small, but this difference increases with growth of moments order.

The investigations has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1979

## 1. INTRODUCTION

One of the interesting properties of hadron-nucleus (hA) interactions is the fact that the multiplicity distribution of the particles produced in them satisfies the KNO-scaling hypothesis <sup>1/</sup>, i.e.,  $\frac{1}{\sigma} \frac{d\sigma}{dn} = \frac{1}{\bar{n}_{hA}} \psi\left(\frac{n}{\bar{n}_{hA}}\right)$ , the KNO-function  $\psi\left(\frac{n}{\bar{n}_{hA}}\right)$  having the form independent of the incident particle energy and being practically identical to that in hadron-nucleon (hN) collisions.

The property noted here is frequently considered as an argument against the cascade type models (including the cascade model of a leading particle) and as an indication to the existence of some collective mechanism <sup>2,3/</sup> of the simultaneous hadron interaction with all nucleons in a nucleus or with some of them. The sequence of arguments is approximately the following.

According to the cascade model of a leading particle (CMLP), the mean multiplicity of shower particles ( $\bar{n}_{hA}$ ) increases with the atomic number roughly as a mean number of inelastic collisions  $\bar{\nu}$  (it is true in the case of energy-independent mean multiplicity  $\bar{n}_{hN}$  in hN-interactions).

Dispersion squared for general reasons should be also proportional to the mean collision number

$$D_{hA}^2 \approx \bar{\nu} D_{hN}^2 . \quad (1)$$

Therefore in the model of successive collisions the ratio

$$\frac{D_{hA}^2}{\bar{n}_{hA}^2} = \frac{1}{\bar{\nu}} \frac{D_{hN}^2}{\bar{n}_{hN}^2} \quad (2)$$

should decrease with the atomic number growth. But it is found experimentally to be constant and approximately equal to the corresponding ratio in the hN-interactions.

In this consideration scheme the supposition (1) is incorrect as it does not take into account correlations of particles produced on different centres (nucleons). Having this correction in mind (and supposing the parameters of multiple production in hN interactions to be energy independent) one should replace relation (1) by the following one

$$D_{hA}^2 = D_{hN}^2 \bar{\nu} + (\nu^2 - \bar{\nu}^2) \bar{n}_{hN}^2 \quad (3)$$

and instead of (2) one obtains

$$\frac{D_{hA}^2}{\bar{n}_{hA}^2} = \frac{1}{\bar{\nu}} \frac{D_{hN}^2}{\bar{n}_{hN}^2} + \frac{D_{\nu}^2}{\bar{\nu}^2}, \quad (4)$$

where  $D_{\nu}^2 = \nu^2 - \bar{\nu}^2$ .

Since the right-hand side of eq. (4) contains both the first term decreasing with A growth and a growing with A ratio of the distribution squared dispersion by collision number to mean collision number squared; the behaviour of expression (4) with the atomic number variation turns out to be rather complicated.

Results calculated according to eq. (4) are shown in fig.1 in comparison with the corresponding experimental data and predictions of the expression (2).

Behaviour of the curves corresponding to eqs.(4) and (2) is seen to be qualitatively (opposite), the first curve being closer to the experimental value although differing notably from it. This discrepancy cannot be considered as an argument against CMLP since the result (4) is obtained under a simplifying assumption of the energy independent multiple production characteristics in hN-collisions. As it will be shown the energy dependence, when taken into account, changes notably many of the results, in particular diminished the ratio  $R_A = \bar{n}_{hA} / \bar{n}_{hN}$  from  $\bar{\nu}$  to its experimental value.

Numerical results corresponding to eq. (4) with  $\bar{n}_{hN}$  energy dependence taken into account are also presented in fig.1 (see curve (b)).

Their argument with the experimental data is seen to be notably better, therefore it is surely interesting to study the characteristics of the distribution in a number of par-

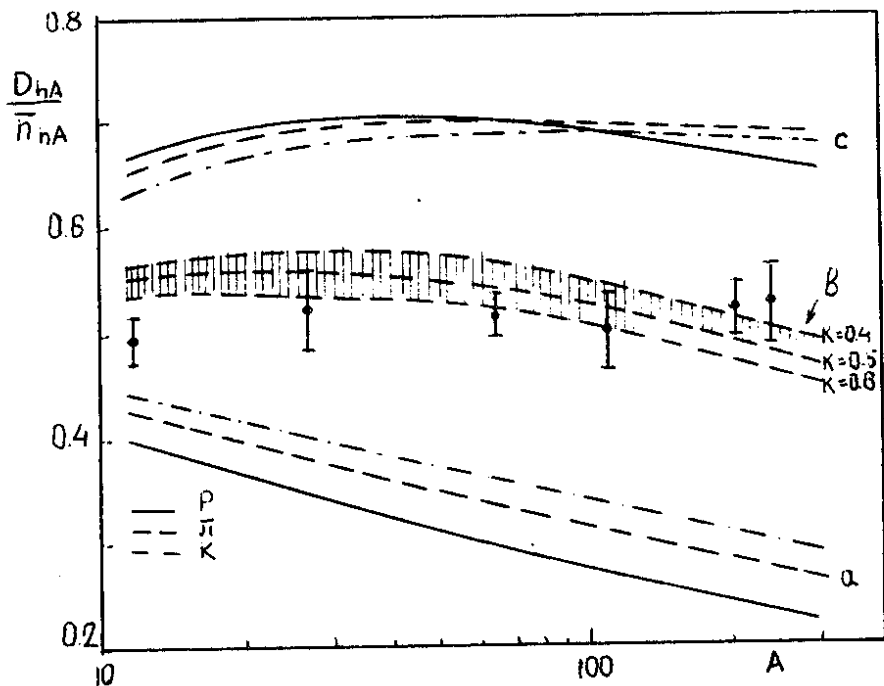


Fig. 1. The ratio  $D_{hA} / \bar{n}_{hA}$  versus  $A$ . Points are the experimental data of ref. [7]. The curves: a - calculated value by expression (2), b - by (4), the correlation was taken into account; c - by (4) the correlation effect and energy dependence of  $\bar{n}_{hN}$  were taken into consideration.

ticles produced in hA-interactions in the framework of CMLP with a minimum of simplifying assumptions. Such a study can lead to the following results.

1. If the distribution in the number of produced particles in hN-interactions is supposed to obey KNO-scaling hypothesis and  $\bar{n}_{hN}$  is assumed to depend on energy according to power law, CMLP leads automatically to KNO-scaling in hA-interactions.

2. Normalized moments  $C_p^{hA} = \overline{n^p} / \bar{n}^p$  calculated with taking into account the real energy dependence of  $\bar{n}_{hN}$  change weakly with the target nucleus atomic number variation and are close to the corresponding values in hN-collisions.

2. CONNECTION BETWEEN KNO-  
FUNCTIONS IN hA- AND  
hN-INTERACTIONS

Let us consider the process of incident hadron inelastic collision with nucleons of nucleus A.

For the sake of simplicity we assume the distribution  $\frac{1}{\sigma} \frac{d\sigma}{dn}$  by a number of particles produced in each collision to be not discrete but continuous (the final result does not depend on this assumption). In this case the resulting distribution by the number of particles produced in  $\nu$ -fold collision can be written in the following form

$$\frac{1}{\sigma} \frac{d\sigma^{(\nu)}}{dn}(E, n) = \int \prod_{i=1}^{\nu} \frac{1}{\sigma} \frac{d\sigma}{dn_i}(E_i, n_i) dn_i \delta(n - \sum_{i=1}^{\nu} n_i), \quad (5)$$

where  $E_i$  is the energy of hadron before  $i$ -th collision. Taking into account that

$$\frac{1}{\sigma} \frac{d\sigma}{dn}(E_i, n_i) = \frac{1}{\bar{n}_{hN}(E)} \psi\left(\frac{n_i}{\bar{n}(E_i)}\right) \quad (6)$$

one can rewrite eq. (5) in the form

$$\bar{n}_{hA} \left( \frac{1}{\sigma} \frac{d\sigma}{dn}(E) \right)^{(\nu)} = \int \prod_{i=1}^{\nu} \psi(z_i) \delta\left(\frac{n}{\bar{n}_{hA}} - \sum_{i=1}^{\nu} \frac{\bar{n}(E_i) z_i}{\bar{n}_{hA}(E)}\right) dz_i, \quad (7)$$

where  $z_i = n_i / \bar{n}(E_i)$ .

Assuming that

$$\bar{n}(E) \sim \left(\frac{E}{E_0}\right)^\alpha \quad (8)$$

we obtain

$$\bar{n}_{hN}(E_i) = \bar{n}_{hN}(E) (f^\alpha)^{i-1}, \quad (9)$$

where  $f$  is the mean energy fraction carried by the leading particle after inelastic hN-collision.

Since under assumption (8) both the ratios  $R_A$  and  $\bar{n}_{hN}(E_i) / \bar{n}_{hN}(E)$  do not depend on energy we get finally

$$\left( \frac{\bar{n}_{hA}}{\sigma} \frac{d\sigma}{dn}(E, n) \right)^{(\nu)} = \psi^{(\nu)}(z) = \int \prod_{i=1}^{\nu} \psi(z_i) dz_i \delta\left(z - \sum_{i=1}^{\nu} \lambda_i z_i\right), \quad (10)$$

6 where  $\lambda_i = (f^\alpha)^{i-1} / R_A$ .

The resulting distribution averaged over all possible collision numbers is given by the following expression

$$\psi_{hA}(z) = \sum_{\nu=1}^A W_{\nu} \psi_{hA}^{(\nu)}(z), \quad (11)$$

where  $W_{\nu} = N_{\nu}(\sigma, A)/N(0, \sigma)$ ,

$$N_{\nu}(\sigma, A) = \frac{1}{\sigma \nu!} \int d^2b [\sigma T(b)]^{\nu} e^{-\sigma T(b)},$$

$$N(0, \sigma) = \frac{1}{\sigma} \int d^2b [1 - e^{-\sigma T(b)}] \quad - \text{are effective numbers.}$$

It satisfies evidently the KNO-scaling hypothesis.

Using the hN-interaction KNO-function parametrized as in ref. /5/

$$\psi(z) = \sum_{k=0}^3 d_k z^{2k+1} e^{-az} \quad (12)$$

one can in principle calculate quantity (11) for any finite  $\nu$ . However this procedure is obviously difficult and extremely cumbersome; it is hardly performable practically. For this reason it is desirable to develop a method that will permit to calculate and compare the distribution characteristics which on the one hand could reflect in sufficient completeness all the main properties of  $\psi(z)$  and on the other could be easily calculated. Low normalized moments of the distribution  $\psi(z)$  turn out to be such characteristics.

### 3. NORMALIZED MOMENTS $C_p$ IN hA-INTERACTIONS

The normalized moments of the distribution  $\psi(z)$  are determined as follows:

$$C_p^{hA} = \overline{n_{hA}^p} / \overline{n_{hA}}^p, \quad C_p^{hN} = \overline{n_{hN}^p} / \overline{n_{hN}}^p, \quad (13)$$

where  $\overline{n_{hA}^p} = \sum n^p \frac{1}{\sigma} \frac{d\sigma}{dn}$ .



Since  $\bar{n}_{hA}$  is given by

$$\bar{n}_{hA} = \bar{n}_{hN} R_A, \quad R_A = N(0, \sigma(1-f^\alpha)) / N(0, \sigma) \quad (14)$$

to calculate  $C_p^{hA}$  it is sufficient to calculate the  $\overline{n_{hA}^p}$ . We begin with the simplest case  $p=2$  and once more consider a  $\nu$ -fold term. Since a casual quantity namely the number of particles produced in this collision is additive  $n_\nu = \sum_{i=1}^{\nu} n_i$  the mean value of its square has the form

$$\overline{n_\nu^2} = \sum_{i=1}^{\nu} \overline{n_i^2} + 2 \sum_{i>k}^{\nu} \bar{n}_i \bar{n}_k, \quad (15)$$

where  $\overline{n_i^2} = \overline{n^2}(E_i) = C_2^{hN} \overline{n^2}(E_i)$ ,  $\bar{n}_i = \bar{n}_{hN}(E_i)$ .

Summing up the right part of eq. (15) and using the relation (9) one obtains

$$\frac{\overline{n_\nu^2}}{\bar{n}^2} = C_2^{hN} \frac{1-a_2^\nu}{1-a_2} + 2 \frac{a_1}{a_1-a_2} \left( \frac{1-a_1^\nu}{1-a_1} - \frac{1-a_2^\nu}{1-a_2} \right), \quad (16)$$

where we use the notation  $a_k = (f^\alpha)^k$ . Finally by averaging eq. (16) over the collision number one easily obtains the following simple expression for the mean squared number of particles produced in hA-interactions

$$\overline{n_{hA}^2} = \frac{\bar{n}_{hN}^2}{N(0, \sigma)} \left\{ C_2^{hN} N(0, \sigma_2) + \frac{2a_1}{a_1-a_2} [N(0, \sigma_1) - N(0, \sigma_2)] \right\}, \quad (17)$$

where  $\sigma_k = \sigma(1-a_k)$ ,  $C_2^{hN} = \overline{n_{hN}^2} / \bar{n}_{hN}^2$ .

Using eqs. (14) and (17) one finds the second normalized moment

$$C_2^{hA} = \frac{N(0, \sigma)}{N^2(0, \sigma_1)} \left\{ C_2^{hN} N(0, \sigma_2) + \frac{2a_1}{a_1-a_2} [N(0, \sigma_1) - N(0, \sigma_2)] \right\}. \quad (18)$$

It seems that this complicated combination of variables depending on nuclear density parameters, inelasticity coefficient in hN-interactions, parameter  $\alpha$  and at last on the values of  $C_2^{hN}$  is unlikely to be numerically close to the value of  $C_2^{hN}$  independently of the target-nucleus atomic

number. However paradoxical it may seem nevertheless it is so.

In fig.2 the value of  $C_2^{hA} / C_2^{hN} - 1$  is presented as a function of atomic number for three types of incident particles ( $\pi, K, p$ ) and for different values of inelasticity coefficients ( $K_N = 1-f$ ). The value of  $a$  was chosen to be 0.25. The deviation from zero is seen to be within 7%. The most close to zero values have been obtained at  $K_N = 0.6$ . For the sake of comparison the results of calculations with  $a=0$  which correspond to energy independent  $\bar{n}_{hN}$  are also presented. In this case the deviation from zero is seen to be considerably larger.

The next in complicity moment is the  $C_3^{hA}$ . To obtain it one is to calculate again the quantity  $\frac{n_{hA}^3}{n_{hA}^3}$  only. The contribution of the  $\nu$ -fold collision to the  $\bar{n}_{hA}^3$  is

$$\bar{n}_{\nu}^3 = \sum_{i=1}^{\nu} \bar{n}_i^3 + 3 \sum_{i>k}^{\nu} (\bar{n}_i^2 \bar{n}_k + \bar{n}_i \bar{n}_k^2) + 6 \sum_{i>k>\ell}^{\nu} \bar{n}_i \bar{n}_k \bar{n}_{\ell}. \quad (19)$$

Taking into account  $\bar{n}_i^p = C_p^{hN} \bar{n}_i^p$ ,  $\bar{n}_i = \bar{n}(E_i) = a_1^{i-1} \bar{n}(E)$  and carrying out the summation over  $i, k, \ell$  we get

$$\begin{aligned} \bar{n}_{\nu}^3 = & \bar{n}_{hN}^3(E) \left\{ C_3^{hN} \frac{1-a_3^{\nu}}{1-a_3} + 3C_2^{hN} \left[ \frac{a_1}{a_1-a_3} \left( \frac{1-a_1^{\nu}}{1-a_1} - \frac{1-a_3^{\nu}}{1-a_3} \right) + \right. \right. \\ & + \frac{a_2}{a_2-a_3} \left( \frac{1-a_2^{\nu}}{1-a_2} - \frac{1-a_3^{\nu}}{1-a_3} \right) \left. \right] + 6a_3 \left[ \frac{1-a_1^{\nu}}{(1-a_1)(a_1-a_2)(a_1-a_3)} + \right. \\ & \left. \left. + \frac{1-a_2^{\nu}}{(1-a_2)(a_2-a_1)(a_2-a_3)} + \frac{1-a_3^{\nu}}{(1-a_3)(a_3-a_1)(a_3-a_2)} \right] \right\}. \quad (20) \end{aligned}$$

After the averaging over  $\nu$  it acquires the form

$$\bar{n}_{hA}^3 = \frac{\bar{n}_{hN}^3(E)}{N(0, \sigma)} \left\{ C_3^{hN} N(0, \sigma_3) + 3C_2^{hN} \left[ \frac{N(0, \sigma_1) - N(0, \sigma_3)}{a_1 - a_3} a_1 + \right. \right.$$

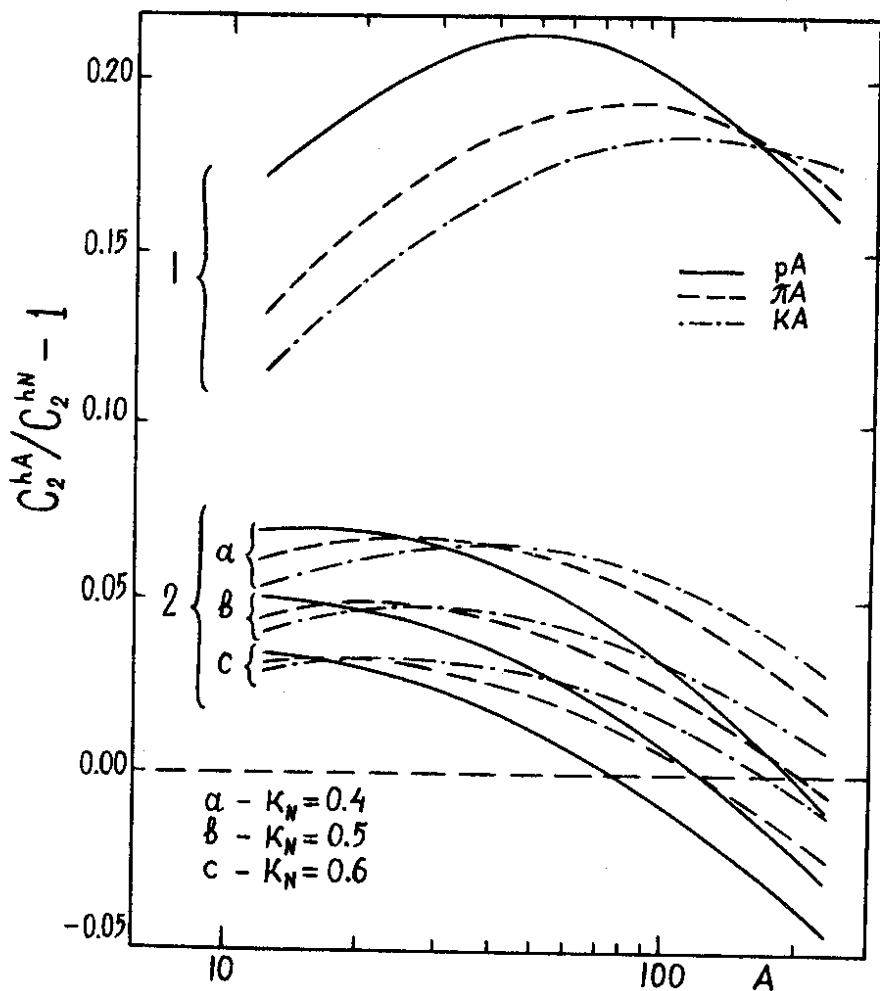


Fig.2. The quantities  $C_2^{hA}/C_2^{hN} - 1$  versus  $A$  with (1) and without (2) energy dependence of  $\bar{n}_{hN}$  for three types of incident particles:  $\pi$  (dashed line),  $k$  (dott-dashed line) and  $p$  (solid line) at different values of  $K_N$ .

$$\begin{aligned}
& + \frac{N(0, \sigma_2) - N(0, \sigma_3)}{a_2 - a_3} a_2 ] + 6 a_3 \left[ \frac{N(0, \sigma_1)}{(a_1 - a_2)(a_1 - a_3)} + \right. \\
& \left. + \frac{N(0, \sigma_2)}{(a_2 - a_1)(a_2 - a_3)} + \frac{N(0, \sigma_3)}{(a_3 - a_1)(a_3 - a_2)} \right] \}. \tag{21}
\end{aligned}$$

In order to obtain  $C_3^{hA}$  it is necessary to divide the expression (21) by  $(\bar{n}_{hN} R_A)^3$ .

Quantity  $C_3^{hA}/C_3^{hN} - 1$  calculated in this way is shown in fig.3. Values  $K_N = 0.6$  and  $\alpha = 0.25$  correspond again to the most close to zero values of  $C_3^{hA}/C_3^{hN} - 1$ . For  $\alpha = 0$  the difference from zero is rather large.

Analogously we can obtain also the closed expressions for other higher moments. We do not present the calculational details which are the same in principle as in the previous example but more complicated, and present only final expressions for moments  $C_4^{hA}$  and  $C_5^{hA}$ . For  $C_4^{hA}$  one obtains

$$\begin{aligned}
C_4^{hA} = & \frac{N^3(0, \sigma)}{N^4(0, \sigma_1)} \{ C_4^{hA} N(0, \sigma_4) + 4C_3^{hN} [a_1 G(1,4) + a_3 G(3,4)] + \\
& + 6(C_2^{hN})^2 a_2 G(1,4) + 12C_2^{hN} [a_3 G(1,2,4) + a_4 G(1,3,4) + \\
& + a_5 G(2,3,4)] + 24a_6 G(1,2,3,4) \} \tag{22}
\end{aligned}$$

and for  $C_5^{hA}$ :

$$\begin{aligned}
C_5^{hA} = & \frac{N^4(0, \sigma)}{N^5(0, \sigma_1)} \{ C_5^{hA} N(0, \sigma_5) + 5C_4^{hN} [a_1 G(1,5) + a_4 G(4,5)] + \\
& + 10C_2^{hN} C_3^{hN} [a_2 G(2,5) + a_3 G(3,5)] + 30(C_2^{hN})^2 [a_4 G(1,3,5) + \\
& + a_5 G(2,3,5) + a_6 G(2,4,5)] + 20C_3^{hN} [a_3 G(1,2,5) + a_5 G(1,4,5) + \tag{23}
\end{aligned}$$

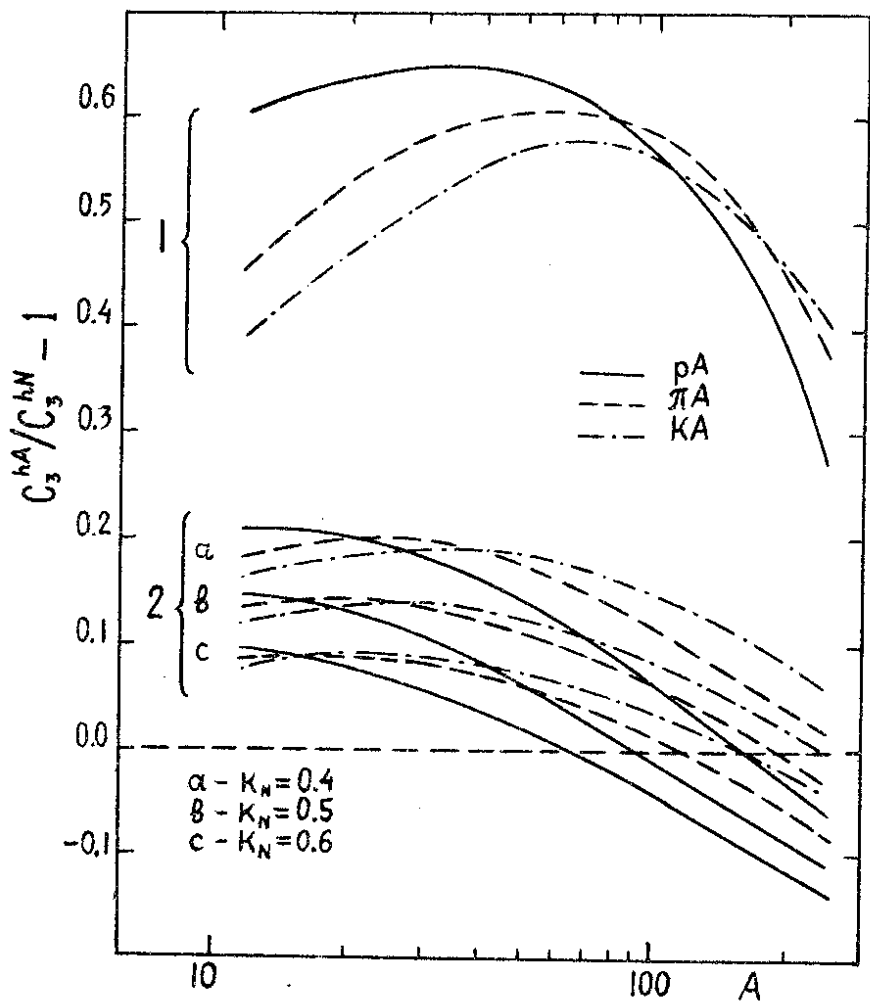


Fig.3. The same as in fig. 2, but for  $C_3^{hA}/C_3^{hN} - 1$ .

$$\begin{aligned}
 &+ a_7 G(3,4,5)] + 60 C_2^{hN} [a_6 G(1,2,3,5) + a_7 G(1,2,4,5) + \\
 &+ a_8 G(1,3,4,5) + a_9 G(2,3,4,5)] + 120 a_{10} G(1,2,3,4,5)\},
 \end{aligned}$$

where

$$G(i, k) = \frac{N(0, \sigma_i) - N(0, \sigma_k)}{a_i - a_k},$$

$$G(i, k, \ell) = \frac{N(0, \sigma_i)}{(a_i - a_k)(a_i - a_\ell)} + \frac{N(0, \sigma_k)}{(a_k - a_i)(a_k - a_\ell)} + \frac{N(0, \sigma_\ell)}{(a_\ell - a_i)(a_\ell - a_k)}$$

and so on.

The calculated results  $C_4^{hA}/C_4^{hN}$  and  $C_5^{hA}/C_5^{hN}$  are given in fig.4.

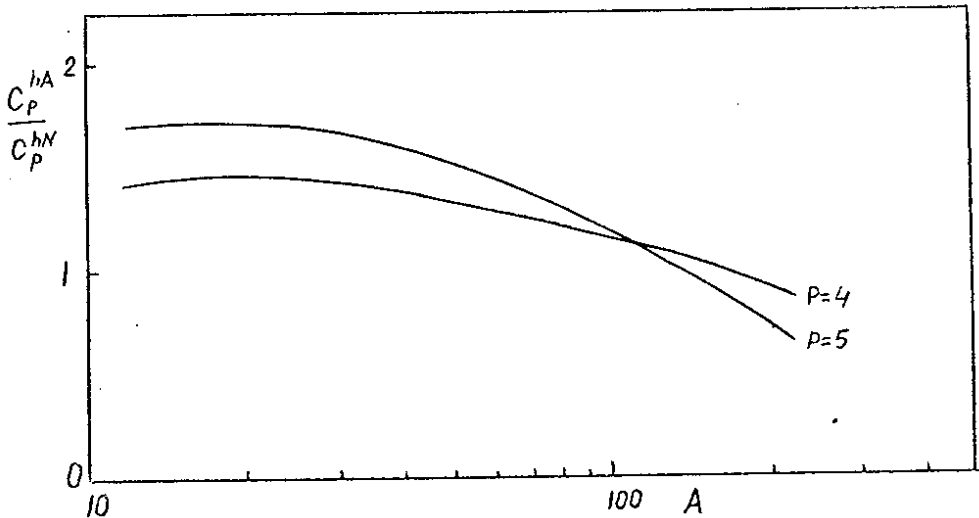


Fig.4. The ratios  $C_4^{hA}/C_4^{hN}$  and  $C_5^{hA}/C_5^{hN}$  versus  $A$ .

It is seen that the deviation of these ratios from unity becomes larger with the growth of the moment index  $p$  but remains either small or weakly dependent on  $A$ .

The negligible difference between low moments of functions  $\psi(z)$  for hA- and hN-interactions implies that these functions differ unimportantly in the region of not very large values of the argument.

4. THE CALCULATION OF THE  
LEADING PARTICLE ENERGY  
LOSS FLUCTUATION IN  
SUCCESSIVE COLLISIONS

As it was mentioned above the coincidence (or almost coincidence) of low moments  $C_p^{hA}$  and  $C_p^{hN}$  calculated in the framework of CMLP is the re-

sult of an accidental, almost perfect compensation of various effects each of them alone leading to violation of this equality and the compensation being true only when definite parameter values used in our calculations are chosen. Furthermore it was supposed that in each collision a leading particle loses a discrete portion of its energy.

A question can arise whether this supposition is too restrictive and whether the results will change significantly if we remove this limitation and take into account the energy loss fluctuations in the successive collision.

It is possible to show that the effect of energy loss fluctuation can be taken into account if we use the effective inelasticity coefficient  $K_{eff}$  instead of the usual one  $K_N = 1 - f$ .

$$K_{eff} = 1 - f_{eff} = 1 - \left( \frac{f}{f + (1-f)a} \right)^{1/a}$$

Such procedure leads to the trivial replacement of

$$C_k N(0, \sigma_k) \quad \text{by} \quad C_k N(0, \tilde{\sigma}_k), \quad \text{where} \quad \tilde{\sigma}_k = \frac{\sigma_k a}{1 - \beta + k a}. \quad \text{As}$$

for other terms in eqs. (18), (21), (22) and (23) things are more complicated. To calculate them it is insufficient to know the hadron energy dependence of the  $n^k$  moments only. A simple example  $\nu = 2$  clarifies this notion. Let us consider the term

$$\sum_{i > k} \bar{n}_i \bar{n}_k \tag{24}$$

in expression (15).

Let particle  $h$  of energy  $E$  lose some fraction of its energy on colliding with the first nucleon so that before the second collision it has the energy  $E_2 = XE$ . The mean multiplicity of this second collision is clear to be defined by this energy value, i.e.,  $\bar{n}_2 = \bar{n}(XE)$ . However the mean multiplicity in the first collision depends not only on the initial energy  $E_1 = E$  but also on the energy fraction lost in this collision. It is evident that for small  $X$  the multiplicity is greater than that for large values of  $X$ .

At  $X=1$  particles cannot be produced in the first collision due to the law of energy conservation.

Therefore  $\bar{n}_1 = \bar{n}(E, X)$  should be written in eq. (24) before averaging it over energy loss fluctuations with

$$\frac{1}{\sigma} \frac{d\sigma}{dX} = (1+\beta)X^\beta, \text{ which is the leading particle spectrum}$$

after the first collision.

Thus the value (24) in the case  $\nu = 2$  should be written as

$$\frac{1}{\sigma} \frac{d\sigma}{dX} \bar{n}(E, x) \bar{n}(EX) dX. \quad (25)$$

To calculate expression (25) the information about  $E, X$  - dependence of differential multiplicity  $\bar{n}(E, X)$  (for higher moments also about  $n^p(E, X)$ ) is required. The simplest assumption about the type of this dependence that agrees with the experimental data is the following<sup>5/</sup>

$$\bar{n}^p(E, X) = \bar{n}^p(M_X^2), \quad (26)$$

where  $M_X$  is  $X$ -system mass in the process  $hN \rightarrow hX$ . Within the limit  $E \gg m_N$  the relation  $M_X^2 = 2m_N(1-X)E$  is true.

Assuming the dependence (26) and using the normalization condition

$$\int \frac{1}{\sigma} \frac{d\sigma}{dX} \bar{n}^p(E, X) dX = \bar{n}^p(E)$$

one finds

$$\bar{n}^p(E, X) = \frac{\Gamma(2+p\alpha+\beta)}{\Gamma(2+\beta)\Gamma(1+p\alpha)} \bar{n}^p(E)(1-X)^{\alpha p}. \quad (27)$$



With the required  $X$ -dependence of the differential multiplicity characteristics fixed in this way one easily calculates the moments  $C_p^{hA}$  with the leading particle energy loss fluctuations taken into consideration. Not difficult but cumbersome calculations show that the mean multiplicity is not shifted numerically. It is natural as the effective inelasticity coefficient is defined by the very condition that mean multiplicity calculated with the energy loss fluctuations taken into account coincides with its corresponding value in the case of energy discrete loss.

As for the other moments their numerical values vary weakly. It is interesting to note that in the processes corresponding to a fixed collision number the normalized moments  $C_p^{(\nu)} = \overline{n_p^\nu} / \overline{n_p}^\nu$  differ stronger (and to the opposite side) from the analogous moments  $C_p^{hN}$ .

It is obvious that  $C_p^{(1)} = C_p^{hN}$  (see fig.5) one should mention that an accurate experimental analysis of the KNO-functions in hA- and hN-interactions shows the difference in higher moments to be systematically  $C_p^{hA} / C_p^{hN} > 1$  and this difference is growing with index  $p$  increase.

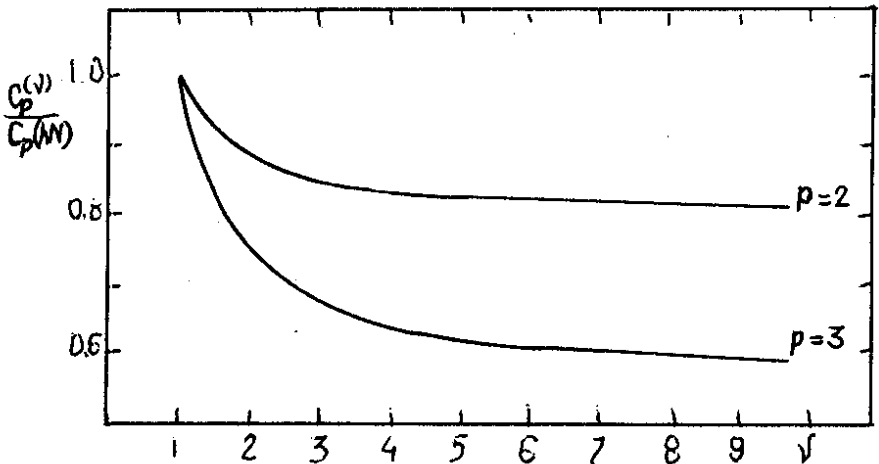


Fig.5. The ratio  $C_p^{(\nu)} / C_p^{hN}$  versus the collision number  $\nu$ .

In fig.6 the emulsion data are compared with the results of our calculations.

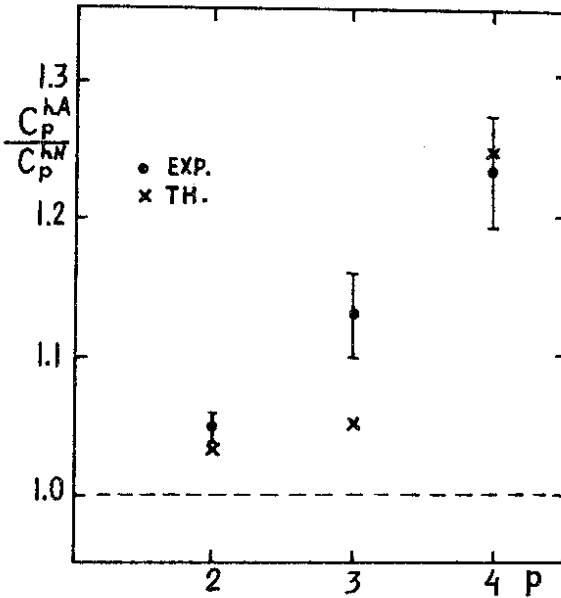


Fig.6. The ratio  $C_p^{hA} / C_p^{hN}$  versus  $p$ . Points - experimental data of ref. <sup>8/</sup>, x - the results of theoretical calculation.

For further verification of CMLP it is interesting to measure experimentally the values of  $C_p^{(p)}$  corresponding to a definite leading particle collision number. However this programme is impracticable. Nevertheless the experiments can be carried out in the selection of events which not corresponding to a definite collision number, do not contain at least the small collision number contributions. For example in the framework of emulsion technique it corresponds to events with any number of strong-ionizing particles (the so-called  $g$ -particles) exceeding a definite value.

An experiment of type <sup>8/</sup> can also be carried out in which the events with large transverse momentum transfer are under investigation. It is an equivalent to the selection of events with a large collision number. If in such processes the cha-

racteristics of produced particles are studied parallelly to the leading hadron detection it corresponds in a sense to the study of a system of particles produced in collisions with large elementary collision number.

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