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G.B.Alaverdyan,[1] A.S.Pak,[2] A.V.Tarasov, Ch.Tseren,
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[1] Yerevan State University, Yerevan.
[2] IHEP, Academy of Sciences Kazakh SSR, Alma-Ata.

Алавердян Г.Б. и др. **E2 - 12822**

Распределение вторичных частиц по быстротам в адрон-ядерных взаимодействиях

В модели каскада лидирующей частицы с учетом флуктуаций энергетических потерь лидирующей частицы в актах последовательных столкновений с нуклонами ядра, исследованы быстротные распределения вторичных частиц в адрон-ядерных взаимодействиях.

Показано, что с ростом атомного номера ядра мишени центр распределения по быстроте сдвигается в сторону малых быстрот.

Модель хорошо воспроизводит как энергетическую так и A-зависимость быстротных распределений.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1979

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Rapidity Distributions of Secondary Particles in Hadron-Nucleus Collisions

In the framework of the cascade model of a leading particle the rapidity distributions of secondary particles in the hadron-nucleus interactions are considered. The energy loss fluctuations of leading particles in the successive collisions were taken into account.

It is shown that the center of y-distribution is displaced towards small rapidity with target nucleus atomic number growth. The model well reproduces the energy and A-dependence of the rapidity distributions.

The investigations has been performed at the Laboratory of Nuclear Problems, JINR.

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It was found experimentally that the rapidity distributions of charged secondary particles produced in hadron-nucleon (hN) collisions have the following properties in the lab. frame:

a) They are practically symmetric with respect to the point $y = \bar{y}_{hN}(E) = \frac{1}{2} \ln \frac{E + P_h + m_N}{E - P_h + m_N}$, which is called the centre of the distribution. In the case when $h = p$ this symmetry is exact.

b) The distributions corresponding to different values of the incident hadron energy coincide within the experimental errors in the target fragmentation region, i.e., in the rapidities' region $y \approx 0$.

At the same time as concerns the rapidity distribution of the secondary particles in the hadron-nucleus (hA) collisions it is found (within the experimental errors) that:

a*) At the fixed beam energy the centre of the distribution ($\bar{y}_{hA}$) is displaced from $\bar{y}_{hN}$ towards small y region.

b*) In the target fragmentation region these distributions are independent of the incident hadron energy, and the ratio of the nucleus spectrum to the nucleon spectrum (R_y) is of the order of the mean inelastic collision number $\bar{\nu} = A\sigma / \sigma_{hA}^{prod}$.

c) In the beam fragmentation region the spectrum ratio R_y is close to unity.

d) R_y as a function of $\bar{\nu}$ is independent of the type of incident particle or of the nucleus target atomic number /1/.

In the present paper it is shown that the cascade model of a leading particle /2,3/ (CMLP) reproduces all the listed peculiarities of the secondary particle rapidity spectra in the hN -collisions.

Connection between the rapidity distributions of particles produced in hA - and hN -interactions in CMLP is analogous to that between the mean multiplicities in these processes and is given by the following expression:

$$\left(\frac{dn}{dy}\right)_{hA} = \sum_{n=1}^{A} W_n \sum_{i=1}^{n} \left(\frac{dn}{dy}(E_k)\right)_{hN}, \qquad (1)$$

where $\left(\frac{dn}{dy}\right)_{hA}$ and $\left(\frac{dn}{dy}\right)_{hN}$ are the distributions in hA- and hN -collisions, respectively. $W_n(\sigma, A) = N_n(\sigma, A)/N(0, \sigma)$ is the probability of the n-fold inelastic collision of the leading hadron with the intranuclear nucleons. $N_n(\sigma, A)$ and $N(0, \sigma)$ are effective numbers /4/, E_k is the leading particle energy before k-th collision, taking into account the energy loss fluctuations it is defined as $E_k = E\, f_{eff}^{k-1}$,

$f_{eff} = \left(\frac{f}{f + \alpha(1-f)}\right)^{1/\alpha}$, and f is the mean energy fraction carried by the leading hadron in the inelastic hN -collision.

From ratio (1), A-dependence of $W_n(\sigma, A)$ values and properties a) and b) of the distribution on the nucleon follow the above-listed features of the rapidity spectra in the hA -interactions. Consider each point in more detail.

Let us estimate numerically the shift of the rapidity distribution centre in hA -collisions with respect to its position in hN -collisions at the same energy of the incident particle. At $E \gg m_h$ and $y_{hN}(E) \simeq \frac{1}{2}\ln\frac{2E}{m_N}$ we obtain:

$$\bar{y}_{hA}\, \bar{n}_{hA} = \frac{1}{2} \sum_{n=1}^{A} W_n \sum_{k=1}^{n} \bar{n}_{hN}(E_k) \ln\frac{2E_k}{m_N}. \qquad (2)$$

In the case of the power parametrization of the energy dependence of the mean secondary particle multiplicity in hN--collisions $(\bar{n}_{hN}(E) \sim (E/E_0)^{\alpha})$ expression (2) can be written in the form:

$$\bar{y}_{hA}\bar{n}_{hA} = \frac{1}{2}\bar{n}_{hA}\ln\frac{2E_0}{m_N} + \frac{1}{2}\frac{\partial}{\partial\alpha}\bar{n}_{hA}. \qquad (3)$$

Taking into account that

$$\bar{y}_{hN} = \frac{1}{2}\ln\frac{2E_0}{m_N} + \frac{1}{2}\frac{\partial}{\partial\alpha}\ln\bar{n}_{hN}(E)$$

one has finally for the distribution centre shift

$$\Delta\bar{y} = \bar{y}_{hA}(E) - \bar{y}_{hN}(E) = \frac{1}{2}\frac{\partial}{\partial\alpha}\ln R_A =$$

$$= -\frac{1+\beta}{2\alpha(1+\alpha+\beta)}\;\frac{N(0,\sigma_1)-N_1(\sigma,A)}{N(0,\sigma_1)} = \mathrm{const}(E), \tag{4}$$

where

$$R_A = \bar{n}_{hA}/\bar{n}_{hN},\ \sigma_1 = \sigma\alpha/(1+\alpha+\beta),\ \beta = (2f-1)/(1-f).$$

Expression (4) shows that in the applied approach $\Delta\bar{y}$ depends on the target-nucleus atomic number and type of the incident particle only. This dependence is demonstrated in fig. 1, where the results of calculations for incident π-mesons and protons are shown calculated for three values of the inelasticity coefficient in hN-collisions ($K_N = 1 - f = 0.4$, 0.5 and 0.6).

In fig.2 the same characteristic $\Delta\bar{y}$ is presented as a function of the mean collision number $\bar{\nu}$. It is seen that the distribution centre shifts toward the small $\bar{y}$ region with the increase of the nucleus-target atomic number and the value of the shift depends on $\bar{\nu}$ only.

However experimental data [1,4] (these are essentially the only data obtained at high energies with the use of "pure" targets) are of insufficient rapidity resolution to extract with adequate accuracy experimental quantities $\Delta\bar{y}_{exp}$ which could be compared to the theoretical ones (4). Straight forward comparison of the experimental and theoretical distributions $(\frac{dn}{dy})_{hA}$ was hampered by the choice of a definite parametrization of $(\frac{dn}{dy})_{hN}$ during the calculations. In our calculations we assumed

$$\left(\frac{dn}{dy}\right)_{hN} = \int \frac{E}{\sigma}\,\frac{d\sigma}{d^3P}\,d^2P_\perp, \tag{5}$$

where the invariant cross-section of the secondary particle production (pions mainly) was taken in the form

$$E\frac{d\sigma}{d^3P} = f(X_R, \vec{P}_\perp) = f_1(X_R)\cdot f_2(\vec{P}_\perp), \tag{6}$$

$\vec{P}_\perp$ and $X_R = P/P_{max}$ are transverse momentum and radial scale variable, defined within the c.m. of hN frame.

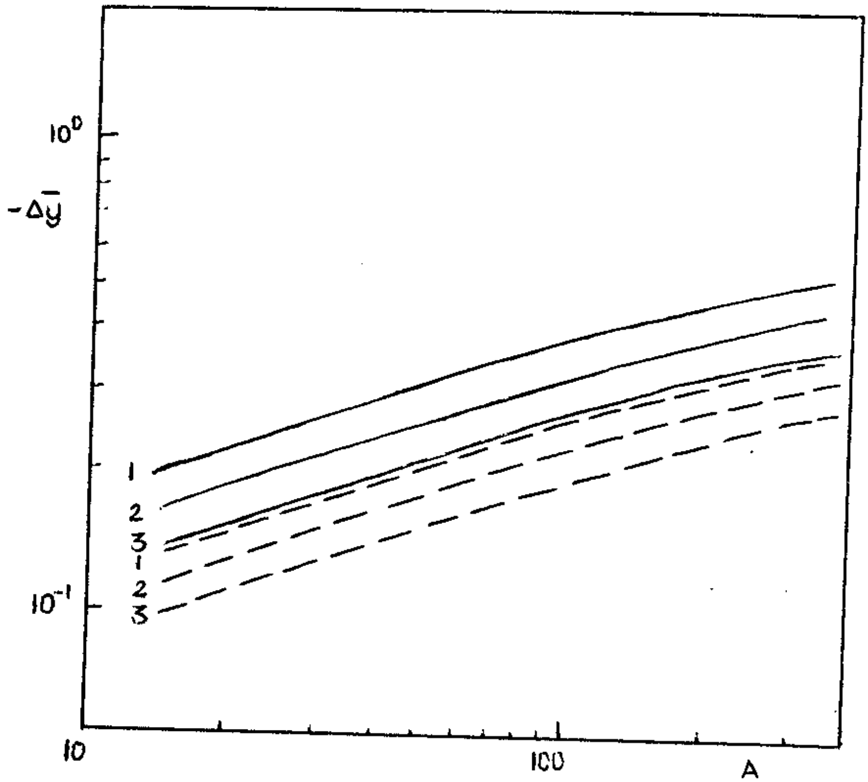

Fig.1. $\overline{\Delta y}$ versus the target nucleus atomic number. The solid and dashed lines are for pA-and πA-interactions, respectively. Indexes 1,2,3 correspond to the calculation results with K_N=0.6, 0.5 and 0.4.

To simplify the calculations the dependences f_1 and f_2 were changed as follows

$$f_1(X_R) = C(1 - X_R)^k / X_R^{\lambda} ,$$

$$f_2(\vec{P}_\perp) = \frac{\sigma}{2\pi \langle P_\perp^2 \rangle} \exp - \frac{\sqrt{6}\, P_\perp}{\langle P_\perp^2 \rangle^{0.5}} , \quad \langle P_\perp^2 \rangle^{0.5} \approx 0.4\ \mathrm{GeV/c},$$

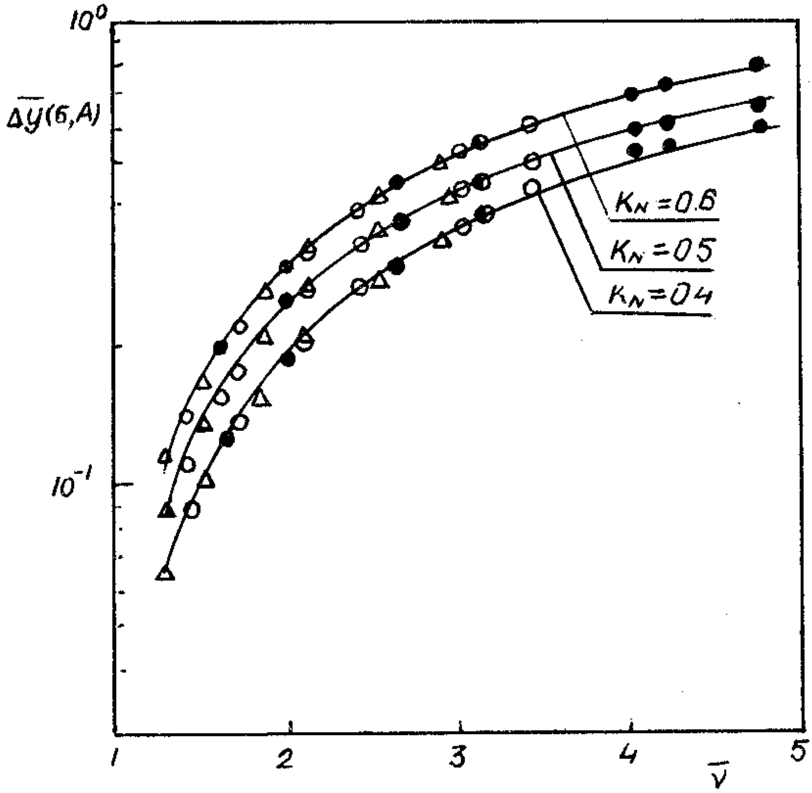

Fig.2. The same as in Fig.1, but $\overline{\Delta y}$ versus the variable $\bar{\nu}$. The points • , O , Λ correspond to three types of the incident particles (π, K, p). The lines were drawn to guide the eye.

where parameters C , k and λ were determined by fitting the experimental data [5] according to mean multiplicity at $5 < E < 3000$ GeV ($C = 1.37$, $k = 4$, $\lambda = 0.1$). The results of these calculations are presented in figs.3,4 together with the experimental data [1] . The model is seen to satisfactory reproduce both energetic and A-dependences of the rapidity distributions. It should be noted that the data in ref. [1] are given not for definite A but for ν values ob-

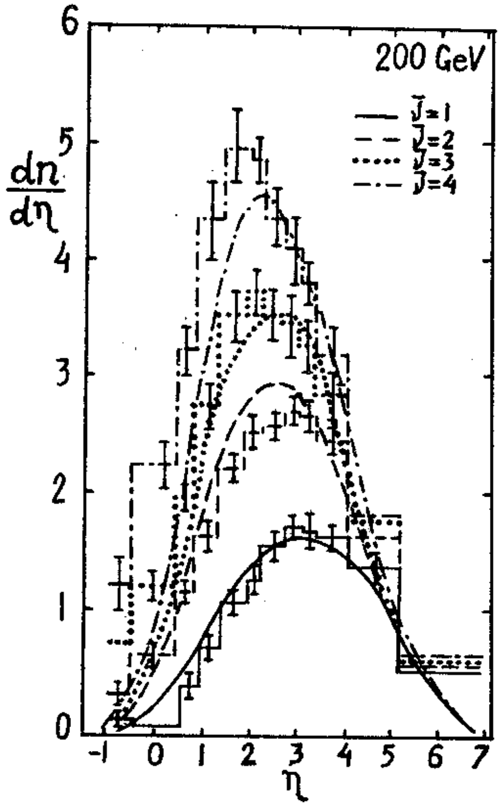

Fig.3. The secondary particles rapidity distributions in pA--interactions at 200 200 GeV as a function of the mean inelastic collision number $\bar{\nu}$. The experimental data from /1/.

tained from A-distributions by means of inter- (extra) polation. This fact obviously contributes some uncertainty to the results and hampers the comparison. Furthermore, defining the mean collision number $\bar{\nu}$ the authors /1,4/ use $\sigma_{hA}^{in} = \sigma_{hA}^{tot} - \sigma_{hA}^{el}$ instead of σ_{hA}^{prod} which should be used in this case.

That is why some divergencies between the calculated results and experimental data, in particular for the imaginary nucleus with A=320 (corresponding to $\bar{\nu}$=4) cannot be considered as an argument against the CMLP.

Let us consider now the consequences of the ratio (1) and properties a) and b). Firstly, irrespective of the specific parametrization of $(\frac{dn}{dy})_{hN}$ one obtains directly that

$R_y = (\frac{dn}{dy})_{hA} / (\frac{dn}{dy})_{hN} \to 1$ at $y \to y_{max}$, i.e., in the beam fragmentation region.

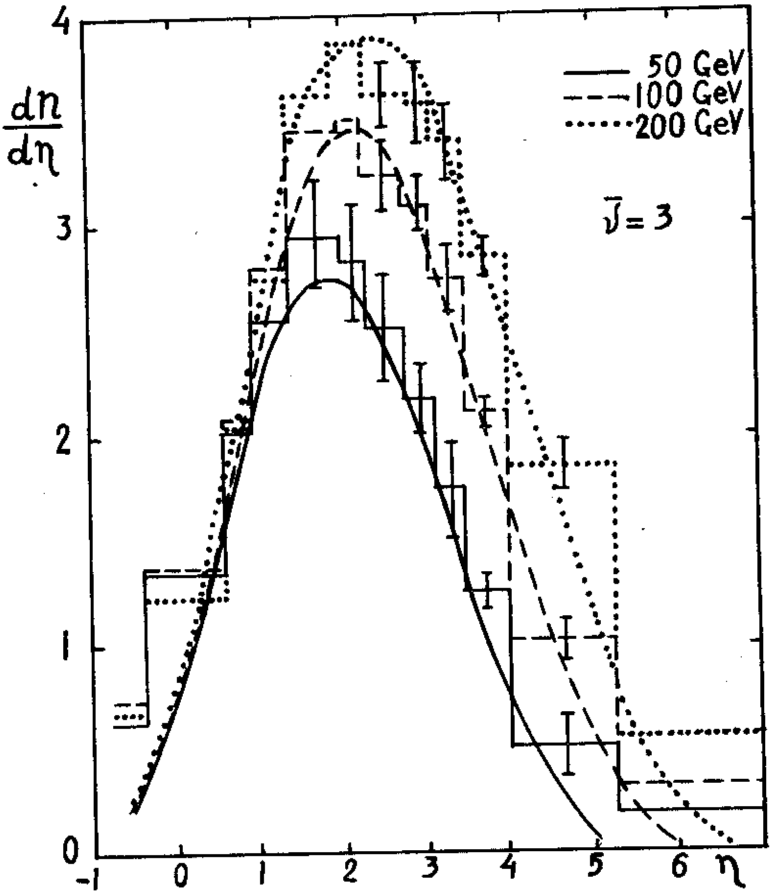

Fig.4. The rapidity distributions at different energies of incident protons. The experimental data of ref. /1/.

Secondly, $R_y \to \bar{\nu}$ at $y \to y_{min}$, i.e., in the target fragmentation region. Thus at these limiting rapidity R_y value satisfies the experimentally found in ref./1,4/ " $\bar{\nu}$ -scaling" condition. There is no strict analytical expression for R_y in the intermediate rapidity region $y_{min} < y < y_{max}$ which would show " $\bar{\nu}$ -scaling" explicitly. However it is satisfied numerically with the experimental accuracy of the same order as in the case of R_A.

So, CMLP reproduces qualitatively the main regularities of the R_y behaviour. Nevertheless, comparing numerical values R_y (exp) and R_y (theor) one finds a considerable divergence (R_y(theor) < R_y(exp)) in the small y region. One of the trivial reasons that may result in the R_y -value increase in the small y region is undoubtedly the interaction of slow secondary particles which are not damped by the traling effect. This interaction cannot result in the notable growth of the total multiplicity but can cause some pumping of particles to the small y region.

Other causes of this R_y augmentation are also possible. One of them is the scattering of leading particles in their transverse momentum after at least one inelastic collision in the nucleus. This circumstance has not been taken into account in the considerations above. However, this effect is inessential at least at very high energies since the transverse momentum of leading particles is restricted after n collisions ($\langle P_\perp \rangle \sim \sqrt{n} \cdot 0.6$ GeV/c).

Lastly the form of the rapidity distribution (pseudorapidity distribution to be exact since all the experimental nuclear target data are presented as $\frac{dn}{d\eta} \approx \frac{dn}{dy}$, $\eta = -\ln \mathrm{tg} \frac{\theta}{2}$) can change (irrespective of the beam energy) due to the fermi-motion of the intranuclear nucleons.It is evident that only the nucleon fermi-motion in the plane defined by the momentum of the incident particle and that of the observed produced particle can notably (the effetcs of the order v_F -fermi velocities) affect the probability of finding a particle at a definite angle to the beam direction. The transverse motion of nucleons of the target-nucleus results in effects of the order of $\langle v_F^2 \rangle$ which numerically are considerably smaller. The fermi-motion in the beam direction spreads the boundary of the y distribution in the region $\Delta y \sim v_F$. Moreover a sharp change of $(\frac{dn}{dy})_{hN}$ results in the fact that near the boundary ($y \approx 0$) the "pumping in" effect (when a nucleon of the nucleus and hadron h move in the opposite directions) dominates the "pumping out" effect (when they move in the same direction).

The fermi-motion in the direction orthogonal to the beam momentum results in the fact that in a nucleon at rest system the produced particle moving in the laboratory system, say strictly backward, has an angle different from 180°, though the probability of the latter is noticeably higher.

The masses of both produced particles (it was supposed that they are all pions) and incident hadrons (at $E \gg m_h$)

were neglected in the numerical estimations. This makes it possible to use the formulae of transforming the angle between two light rays in two different Lorentz frames moving one respectively to the other.

$$\mathrm{ctg}\,\theta_{1(2)} = \frac{\mathrm{ctg}\,\theta_{2(1)}(1+\beta\cos\alpha)+\beta\sin\alpha}{\sqrt{1-\beta^2}}, \qquad (7)$$

where $\beta = v_F / c$; v_F is the nucleon fermi velocity; $\theta_{1(2)}$ - angles between the hadron and the produced particle momenta in the lab.system and in the rest system of nucleon; α is the angle between the beam direction and the nucleon fermi-momentum.

The result of substitution of (7) into (1) and averaging according to the fermi-motion is presented in fig.5. It

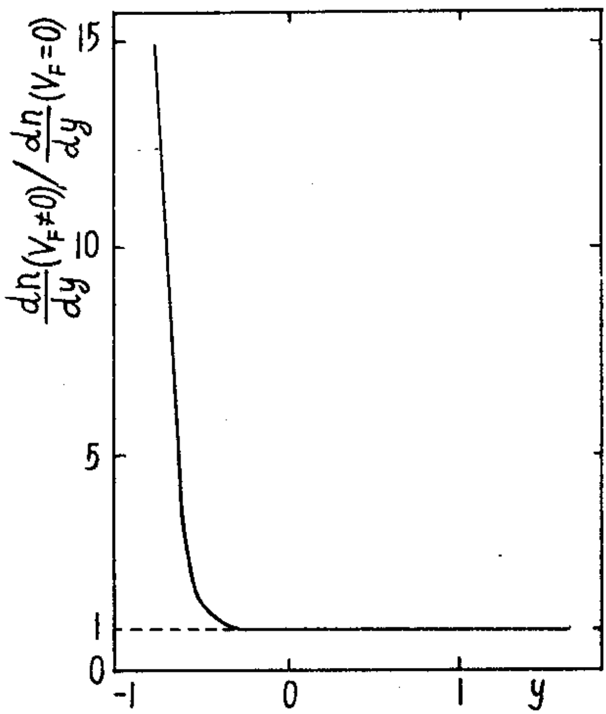

Fig.5. The ratio of rapidity distributions calculated with and without fermi-motion as a function of y .

shows that the consideration of the intranuclear motion can significantly change the value of R_y but in a very narrow y region near the boundary of the target fragmentation region. Note that the fermi-motion when being taken into account does not change the "$\bar{\nu}$ -scaling" properties of the R_y behaviour (it should be supposed of course that different nuclei have close fermi-distributions). Therefore it may turn out that in appreciating the role of cascading from R_y behaviour one must take accurate account of such small effects as fermi-motion and at not very high energies - the angular scattering of leading particles.

Here are some remarks on R_y behaviour in the beam fragmentation region ($y \sim y_{max}$). Experimental data presented in refs. /1,4/ and obtained on pure nuclear targets lead to the result $R_y \sim 1$ at $y \sim y_{max}$ in accordance with the model predictions.

However these data have been obtained after averaging over a rather wide pseudorapidity interval around $y \sim y_{max}$. Emulsion data /6/ which are of better rapidity resolution lead to a somewhat different result, namely $\lim_{y \to y_{max}} R_y < 1$,

the deviation from unity for heavy emulsion component (AgBr) being more than for light component (CNO). A possible explanation of this effect is the following.

The observed rapidity distributions of leading particles include not only the produced particles but also the remaining ones. Therefore before comparing with the data, the contributions corresponding to these leading particles should be added to the numerator and denominator of the R_y ratio.

$$\left(\frac{dn}{dy}\right)_{hA} = \left(\frac{dn}{dy}\right)^{prod}_{hA} + \left(\frac{dn}{dy}\right)^{Lid}_{hA} \tag{8}$$

$$\left(\frac{dn}{dy}\right)_{hN} = \left(\frac{dn}{dy}\right)^{prod}_{hN} + \left(\frac{dn}{dy}\right)^{Lid}_{hN} \tag{9}$$

where

$$\left(\frac{dn}{dy}\right)^{Lid}_{hN} = \frac{1}{\sigma_{hN}} \int \left(E\frac{d\sigma}{d^3p}\right)_{hN \to hX} d^2P_\perp$$

$$\left(\frac{dn}{dy}\right)^{Lid}_{hA} = \frac{1}{\sigma_{hA}} \int \left(E\frac{d\sigma}{d^3p}\right)_{hA \to hX} d^2P_\perp .$$

Since in the beam fragmentation region second terms in eqs. (8) and (9) dominate, the experimentally measured ratio of values (8) and (9) tends to the limit $\lim_{y \to y_{max}} R_y = W_1$.

The A atomic number dependence of the value W_1 is exactly the same as that of the experimentally measured (with large error however) $R_\eta |_{\eta=\eta_{max}}$.

Recent measurements at neutral FERMILAB beam [7] (neutrons mainly) tell that thė mentioned effect takes place also in the case of charged (produced) particles.

This result becomes understandable if one realizes that the decay products of quasi binary reactions

$$hN \to h^*N$$
$$\quad \quad \lfloor\!\!\to \pi^+\pi^- \ \ldots.$$

may contribute to the observed distributions.

Although the integral cross sections of such processes are not very large their contribution may be dominant in the narrow kinematical region $y \sim y_{max}$ (in the absence of the neutral leading particle contribution). Furthermore if the excited state h^* interacts with the nuclear matter with the same intensity as the incident hadron the result $R_\eta |_{\eta=\eta_{max}} < 1$ will also remain for the produced particles.

The difference between the two variants with changes and neutral leading particle respectively reveals itself in the difference of values $y = y_0$, in which case R_y becomes less than unity.

In the case of a charged leading particle this value y_0 should be less because the $hN \to h^*X$ cross section is smaller than that of the process $hN \to hX$. However the information necessary for the concrete numerical calculation is unfortunately absent presently.

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