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Функции Вайтмана в модели Тирринга

Обсуждаются некоторые свойства решения модели Тирринга. Поле Тирринга действует в пространстве Фока скалярного поля нулевой массы. Функции Вайтмана найдены в явной форме. Показано, что в пределе $\mu \to 0$ калибровочные симметрии модели, которые спонтанно нарушены при $\mu \neq 0$, восстанавливаются и поле приобретает определенный спин и масштабную размерность. Установлено соответствие с $2n$-точечными функциями решения Клайбера. Показано, что при целых и полуцелых значениях спина возникает обычная связь спина со статистикой.

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Hadjiivanov L.K., Stoyanov D.T. **E2 - 12816**

Wightman Functions in the Thirring Model

Some properties of the solution of the Thirring model, proposed by Hadjiivanov, Mikhov, Stoyanov /1/, and its vacuum expectation values are studied, the Thirring field acting in the Fock space of a free scalar field. The explicit form of the Wightman functions is found. It is shown that in the limit $\mu \to 0$ the two gauge symmetries which are spontaneously broken for $\mu \neq 0$ are restored and the field acquires fixed spin and scale dimension. The correspondence with the $2n$-point functions for Klaiber's solution is exhibited. It is shown that for integer or half-integer spin the common relation between spin and statistics arises.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Here we study some properties of the solution of the massless Thirring model constructed by Hadjiivanov, Stoyanov [2] and Hadjiivanov et al.[1]. It is well known that on the Thirring model which serves as an example for exactly solvable quantum field theory many hypotheses reflecting the development of the theory have been tested. There exist several different approaches to the model (see refs.[3-6]). Its most important features which make it pliable to investigation (besides the peculiarities of the two-dimensional space-time) are, first, the conformal invariance, and second, the existence of two gauge symmetries of first kind.

In 1977 Nakanishi insisting on the asymptotic completeness, proposed a solution of the model in terms of free massless scalar fields only. Actually, the possibility of constructing spin 1/2-fields from scalar ones has already been pointed out by Skyrme [8] (see also the rigorous papers by Streater and Wilde [9] and Streater [10]). Unfortunately, there are some contradictions in Nakanishi's papers (see in this connection ref.[11])

The right conformal transformation properties of the scalar fields (gradients of which are the conserved currents) and of the solution of the model have been found by Hadjiivanov et al.[1]. It has been shown (Aneva et al.[12]) that there exists in fact a one-parameter family of solutions and that the gauge symmetry is spontaneously broken.

2. THE THIRRING MODEL AND ITS OPERATOR SOLUTION

The classical Thirring model is based on the equation of motion

$$i\partial_\mu \gamma^\mu \psi(x) = -g J_\mu (x) \gamma^\mu \psi(x), \quad J_\mu = \bar{\psi} \gamma_\mu \psi , \tag{2.1}$$

which is invariant with respect to two groups of global gauge transformations:

$$\psi(x) \to e^{i\alpha}\,\psi(x) \tag{2.2}$$

and

$$\psi(x) \to e^{i\beta\gamma^5}\psi(x) \tag{2.3}$$

(we choose a standard basis for the γ-matrices with γ^5 diagonal:

$\gamma^\circ = \sigma_1,\ \gamma^1 = i\sigma_2,\ \ \gamma^5 \equiv \gamma^\circ\gamma^1 = -\sigma_3$,
where σ_i $(i=1,2,3)$ are the Pauli matrices). It follows from $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$, $g_{00} = -g_{11} = 1$ and the definition of γ^5 that $\gamma^5\gamma_\mu = -\epsilon_{\mu\nu}\gamma^\nu$, where $\epsilon_{\nu\mu} = -\epsilon_{\mu\nu}$, $\epsilon_{01} = 1$. As a result, we obtain that the conserved currents are connected by

$$J^5_\mu(x) = -\epsilon_{\mu\nu}\,J^\nu(x) \tag{2.4}$$

what implies that there exist real scalar fields $\phi, \tilde{\phi}$ and a real number λ such that

$$J_\mu(x) = \lambda\partial_\mu\,\phi(x) \qquad J_\mu^5(x) = \lambda\partial_\mu\,\tilde{\phi}(x)\,. \tag{2.5}$$

Because of the current conservation

$$\Box\,\phi(x) = 0 = \Box\,\tilde{\phi}(x) \tag{2.6}$$

and because of Eq. (2.4)

$$\partial_\mu\,\phi(x) = -\epsilon_{\mu\nu}\,\partial^\nu\,\tilde{\phi}(x)\,. \tag{2.7}$$

In the quantum Thirring model the usual commutation relations between the components of J_μ are in accordance with the canonical commutation relations of ϕ and $\tilde{\phi}$. It is shown by Hadjiivanov et al. [1] and Aneva et al. [12] that

$$\psi(x) = e^{i\beta\gamma^5\tilde{\phi}^-(x) - i\alpha\phi^-(x)}\,e^{i\beta\gamma^5\tilde{\phi}^+(x) - i\alpha\phi^+(x)}\,u \tag{2.8}$$

is a one-parameter family of solutions of the equation

$$i\partial_\mu\,\gamma^\mu\psi(x) = -g\gamma^\mu\,:J_\mu(x)\psi(x): \tag{2.9}$$

where u_i $(i=1,2)$ are arbitrary constants and

$$:J_\mu(x)\psi(x): \equiv \frac{\alpha+\beta}{2\pi}(\partial_\mu\,\phi^-(x)\psi(x) + \psi(x)\partial_\mu\phi^+(x)) \tag{2.10}$$

$\phi^{\pm}$ and $\tilde{\phi}^{\pm}$ are the frequency parts of ϕ and $\tilde{\phi}$. They satisfy the following commutation relations:

$$[\phi^{+}(x), \phi^{-}(y)] = D^{+}(x-y) = [\tilde{\phi}^{+}(x), \tilde{\phi}^{-}(y)], \tag{2.11}$$

$$[\phi^{+}(x), \tilde{\phi}^{-}(y)] = \tilde{D}^{+}(x-y) = [\tilde{\phi}^{+}(x), \phi^{-}(y)], \tag{2.12}$$

where

$$\begin{aligned} D^{+}(x) &= -\frac{1}{4\pi}\ln(-\mu^2 x^2 + i0\, x^{\circ}), \\ \tilde{D}^{+}(x) &= \frac{1}{4\pi}\ln\frac{x^{\circ} - x^{1} - i0}{x^{\circ} + x^{1} - i0} \end{aligned} \tag{2.13}$$

the parameter $\mu > 0$ (connected with the infrared regularization) has the dimension of mass. In the theory there are two charges proportional to

$$q = \int \partial_0 \phi(x)\, dx^1 = \frac{1}{\lambda}\int J_0(x)\, dx^1 = q^{+} + q^{-}, \tag{2.14}$$

$$\tilde{q} = \int \partial_0 \tilde{\phi}(x)\, dx^1 = \frac{1}{\lambda}\int J_0^5(x)\, dx^1 = \tilde{q}^{+} + \tilde{q}^{-}, \tag{2.15}$$

where $q^{\pm}$, $\tilde{q}^{\pm}$ correspond to the separation of ϕ, respectively $\tilde{\phi}$, into positive and negative frequency parts. From Eqs. (2.11), (2.12) we obtain

$$[q^{\pm}, \phi^{\mp}(x)] = -\frac{i}{2} = [\tilde{q}^{\pm}, \tilde{\phi}^{\mp}(x)], \tag{2.16}$$

$$[\tilde{q}^{\pm}, \phi^{\mp}(x)] = 0 = [q^{\pm}, \tilde{\phi}^{\mp}(x)]. \tag{2.17}$$

The operators $q^{\pm}$, $\tilde{q}^{\pm}$ commute with each other.

3. WIGHTMAN FUNCTIONS

Now we shall compute the Wightman functions corresponding to the operator solution (2.8). We shall restrict ourselves to the (n+m)-point functions of the form

$$\langle \psi_{j_1}(x_1)\psi_{j_2}(x_2)\dots\psi_{j_n}(x_n)\bar{\psi}_{j_{n+1}}(x_{n+1})\dots\bar{\psi}_{j_{n+m}}(x_{n+m})\rangle_0 \tag{3.1}$$

since the consideration of the most general Wightman functions causes unnecessary complications.

To compute the function (3.1), we use the operator formula $e^{A}e^{B} = e^{[A,B]}e^{B}e^{A}$, which is valid when the commutator

$[A, B]$ is proportional to the unit operator. The result is:

$$w^{(n,m)}_{j_1,\dots,j_{n+m}}(x_1,\dots,x_{n+m}) \equiv$$

$$\equiv \langle \psi_{j_1}(x_1)\dots\psi_{j_n}(x_n)\bar{\psi}_{j_{n+1}}(x_{n+1})\dots\bar{\psi}_{j_{n+m}}(x_{n+m})\rangle_0 = \tag{3.2}$$

$$= \prod_{k=1}^{n} u_{j_k} \prod_{\ell=n+1}^{n+m} \bar{u}_{j_\ell} \exp F(x_1,\dots,x_{n+m}|j_1,\dots,j_{n+m}),$$

where $F(x_1,\dots,x_{n+m}|j_1,\dots,j_{n+m})$ is a sum of three types of addends, namely

$$F(x_1,\dots,x_{n+m}|j_1,\dots,j_{n+m}) =$$

$$= \sum_{r=1}^{n-1}\sum_{s=r+1}^{n}\{-[\alpha^2+\beta^2(-1)^{j_r+j_s}]D^+(x_{rs})+\alpha\beta[(-1)^{j_r}+(-1)^{j_s}]\tilde{D}^+(x_{rs})\}+$$

$$+\sum_{r=1}^{n}\sum_{s=n+1}^{n+m}\{[\alpha^2-\beta^2(-1)^{j_r+j_s}]D^+(x_{rs})+\alpha\beta[-(-1)^{j_r}+(-1)^{j_s}]\tilde{D}^+(x_{rs})\}+$$

$$+\sum_{r=n+1}^{n+m-1}\sum_{s=r+1}^{n+m}\{-[\alpha^2+\beta^2(-1)^{j_r+j_s}]D^+(x_{rs})-$$

$$-\alpha\beta[(-1)^{j_r}+(-1)^{j_s}]\tilde{D}^+(x_{rs})\} \tag{3.3}$$

(here the identity $[\gamma^5_{(\ell)}]_{ik} = (-1)^k\delta_{ik}$ and the notation $x_{rs} \equiv x_r - x_s$ are used).

The functions (3.2) are distributions over $S(R^{2(n+m)})$. The parameter $\mu > 0$ is completely arbitrary. In order to study this arbitrariness we shall fix a unit and count μ with respect to it. The quantities in which μ is replaced by $z\mu$, $z > 0$ will be denoted by a lower index (z) in brackets. Bearing in mind that

$$D^+_{(z)}(x) = D^+(x) - \frac{\ln z}{2\pi},$$

$$\tilde{D}^+_{(z)}(x) = \tilde{D}^+(x) \tag{3.4}$$

and defining u_k as in ref. /1/

$$|u_k| = \frac{1}{\sqrt{2}}\mu^{(\alpha^2+\beta^2)/4\pi}, \quad k = 1,2 \tag{3.5}$$

one finds from the explicit form of the $(n+m)$-point functions (3.2), (3.3) that

$$w^{(n,m)}_{(z)j_1,\dots,j_{n+m}}(x_1,\dots,x_{n+m}) = z^N w^{(n,m)}_{j_1,\dots,j_{n+m}}(x_1,\dots,x_{n+m}), \quad (3.6)$$

where

$$N = (n+m)\frac{\alpha^2+\beta^2}{4\pi} + \Big\{ \sum_{r=1}^{n-1} \sum_{s=r+1}^{n} \frac{\alpha^2+\beta^2(-1)^{j_r+j_s}}{2\pi} - \sum_{r=1}^{n} \sum_{s=n+1}^{n+m} \frac{\alpha^2-\beta^2(-1)^{j_r+j_s}}{2\pi} + \sum_{r=n+1}^{n+m-1} \sum_{s=r+1}^{n+m} \frac{\alpha^2+\beta^2(-1)^{j_r+j_s}}{2\pi} \Big\}. \quad (3.7)$$

Let us calculate the sum in the braces. It equals to

$$\frac{\alpha^2}{2\pi}\Big\{\sum_{r=1}^{n-1}(n-r) - mn + \sum_{r=n+1}^{n+m-1}(n+m-r)\Big\} + \frac{\beta^2}{2\pi}\Big\{\sum_{r=1}^{n-1}(-1)^{j_r}[(-1)^{j_{r+1}}+\dots+(-1)^{j_n}] +$$

$$+\sum_{r=1}^{n}(-1)^{j_r}\sum_{s=n+1}^{n+m}(-1)^{j_s} + \sum_{r=n+1}^{n+m}(-1)^{j_r}[(-1)^{j_{r+1}}+\dots+(-1)^{j_{n+m}}]\Big\} = \quad (3.8)$$

$$= \frac{\alpha^2}{4\pi}[(n-m)^2 - (n+m)] + \frac{\beta^2}{4\pi}\Big\{\Big[\sum_{k=1}^{n+m}(-1)^{j_k}\Big]^2 - (n+m)\Big\}.$$

If we denote the number of indices j_k, $k=1,2,\dots,n+m$, which are equal to 1, by n_1, and of those which are equal to 2- by n_2, we finally obtain

$$N = N(n,m;n_1,n_2) = \frac{\alpha^2}{4\pi}(n-m)^2 + \frac{\beta^2}{4\pi}(n_1-n_2)^2 \geq 0. \quad (3.9)$$

It is clear that in the limit $z \to 0$ only those Wightman functions survive which do not depend on μ. The necessary and sufficient condition for that is $n=m$ and $n_1=n_2$. When both these conditions are fulfilled the product $\prod_{k=1}^{n} u_{j_k} \prod_{\ell=n+1}^{n+m} \bar{u}_{j_\ell}$ from (3.2) is equal to $|u_1|^n |u_2|^n$. The condition (3.5) is imposed by Hadjiivanov et al. /1/ in order to express the current J_ρ conversely in terms of the fields ψ, $\bar\psi$. It is clear, in any case, that the Wightman functions for $n=m$ and $n_1=n_2$ have the homogeneity degree in μ which depends on the total number $(n+m)$ of the fields ψ, $\bar\psi$ only. So, by a suitable renormalization of the form $\psi \to \mu^d \psi$ one can make the gauge invariant functions independent of μ whereas the homogeneity degree of all other functions will become positive. Thus, in the limit $\mu \to 0$ the symmetry of the Wightman functions both under the ordinary global gauge transformations (2.2) and the γ^5-gauge transformations (2.3) is restored. The spontaneous breaking of these gauge symmetries was pointed out by Aneva et al. [12]

4. EXPLICIT FORM OF THE GAUGE INVARIANT WIGHTMAN FUNCTIONS AND COMPARISON WITH THOSE FOR KLAIBER'S SOLUTION

Let us consider the gauge-invariant functions of the form

$$W_n^{\ell}(x_1,\dots,x_n,\, x_{n+1},\dots,x_{2n}) = \langle \prod_1^{\ell} \psi_1(x_j) \prod_{\ell+1}^{n} \psi_2(x_j) \prod_{n+1}^{n+\ell} \bar\psi_2(x_j) \prod_{n+\ell+1}^{2n} \bar\psi_1(x_j) \rangle_0$$

According to (3.2)

$$W_n^{\ell}(x_1,\dots,x_n,x_{n+1},\dots,x_{2n}) = \frac{(\mu^2)^{n\frac{\alpha^2+\beta^2}{4\pi}}}{(2\pi)^n}\, e^{F_n^{\ell}(x_1,\dots,x_n,x_{n+1},\dots,x_{2n})}, \quad (4.2)$$

(4.1)

where the functions $F_n^{\ell}(x_1,\dots,x_n,x_{n+1},\dots,x_{2n})$ are sums of logarithms. Using the decomposition

$$D^{+}(x) = -\frac{1}{4\pi}\{i\pi + \ln(\mu x_- - i0) + \ln(\mu x_+ - i0)\}$$

$$\tilde{D}^{+}(x) = \frac{1}{4\pi}\{\ln(\mu x_- - i0) - \ln(\mu x_+ - i0)\} \quad (4.3)$$

$$x_{\pm} = x^{\circ} \pm x^{1}$$

one can get the functions (4.1) as products of powers of $u_{rs} = \mu x_{rs_+} - i0$ and $v_{rs} = \mu x_{rs_-} - i0$. In particular, when $\ell = n$

$$W_n^{n}(x_1,\dots,x_n,x_{n+1},\dots,x_{2n}) = \langle \prod_1^{n} \psi_1(x_j) \prod_{n+1}^{2n} \bar\psi_2(x_j) \rangle_0 =$$

$$= \frac{(\mu^2)^{n\frac{\alpha^2+\beta^2}{4\pi}}}{(2\pi)^n}\, e^{-in\frac{\alpha^2+\beta^2}{4}} \prod_{r=1}^{n-1} \prod_{s=r+1}^{n} u_{rs}^{\delta_1} v_{rs}^{\delta_2} \times \quad (4.4)$$

$$\times \prod_{r=1}^{n} \prod_{s=n+1}^{2n} u_{rs}^{-\delta_1} v_{rs}^{-\delta_2} \prod_{r=n+1}^{2n-1} \prod_{s=r+1}^{2n} u_{rs}^{\delta_1} v_{rs}^{\delta_2}, \quad \delta_1 = \frac{(\alpha+\beta)^2}{4\pi} \quad \delta_2 = \frac{(\alpha-\beta)^2}{4\pi}.$$

The functions

$$W_n^{\circ}(x_1,\dots,x_n,x_{n+1},\dots,x_{2n}) = \langle \prod_1^n \psi_2(x_j) \prod_{n+1}^{2n} \bar{\psi}_1(x_j) \rangle_0 \qquad (4.5)$$

follow from Eq. (4.4) by the change $u_{rs} \leftrightarrow v_{rs}$.

For the mixed functions we get

$$W_n^{\ell}(x_1,\dots,x_\ell,x_{\ell+1},\dots,x_n,x_{n+1},\dots,x_{n+\ell},x_{n+\ell+1},\dots,x_{2n}) =$$

$$= \prod_{r=1}^{\ell} \prod_{s=\ell+1}^{n} u_{rs}^{\delta} v_{rs}^{\delta} \prod_{r=1}^{\ell} \prod_{s=n+\ell+1}^{2n} u_{rs}^{-\delta} v_{rs}^{-\delta} \prod_{r=\ell+1}^{n} \prod_{s=n+1}^{n+\ell} u_{rs}^{-\delta} v_{rs}^{-\delta} \prod_{r=n+1}^{n+\ell} \prod_{s=n+\ell+1}^{2n} u_{rs}^{\delta} v_{rs}^{\delta} \times,$$

$$\times W_\ell^{\ell}(x_1,\dots,x_\ell,x_{n+1},\dots,x_{n+\ell})\, W_{n-\ell}^{\circ}(x_{\ell+1},\dots,x_n,x_{n+\ell+1},\dots,x_{2n})$$

$$\delta = \frac{\alpha^2-\beta^2}{4\pi}.$$

One can compare Eqs. (4.4)-(4.6) with the corresponding formulae of Klaiber /4/, (p.162-165). (The signs before α and β are coordinated). These formulae prove to coincide in essence. Certain different signs are due to the conventional character of the spinors in the two-dimensional space.

5. LORENTZ TRANSFORMATIONS AND DILATATIONS

It is interesting to investigate the covariance properties of the functions (3.2) (they are obviously translationally invariant) under Lorentz and dilatation transformations, for as it is shown by Aneva et al. /12/ one cannot attribute definite "spin" and scale dimension to the field ψ. One essentially makes use of the results of Hadjiivanov et al. /1/:

$$\psi_j(x) \xrightarrow{\Lambda\chi} \psi_j^{(\chi)}(x) = e^{\chi L^-(j)} \psi_j(\Lambda_\chi x)\, e^{-\chi L^+(j)}, \qquad (5.1)$$

where

$$L^{\pm}(j) = \frac{\beta(-1)^j q^{\pm} - \alpha \tilde{q}^{\pm}}{2\pi}, \quad \Lambda_\chi = \begin{pmatrix} \operatorname{ch}\chi & \operatorname{sh}\chi \\ \operatorname{sh}\chi & \operatorname{ch}\chi \end{pmatrix}$$

and

$$\psi_j(x) \xrightarrow{\lambda} \psi_j^{(\lambda)}(x) = e^{Q^-(j)\ln\lambda}\ \psi_j(\lambda x) e^{-Q^+(j)\ln\lambda},$$

$$Q^{\pm}(j) = \frac{\beta(-1)^j \bar{q}^{\pm} - \alpha q^{\pm}}{2\pi}, \quad \lambda > 0. \tag{5.2}$$

Supposing that the vacuum state $|0>$ is Lorentz- and dilatation-invariant the Wightman functions can be shown to have the following covariance laws under the Lorentz transformations.

$$w^{(n,m)}_{j_1,\dots,j_{n+m}}(x_1,\dots,x_{n+m}) = e^{\frac{\alpha\beta}{4\pi}\chi(P^{(+)}_{nm}+P^{(-)}_{nm})} w^{(n,m)}_{j_1,\dots,j_{n+m}}(\Lambda x_1,\dots,\Lambda x_{n+m}), \tag{5.3}$$

where

$$P^{(+)}_{nm} = (n-1)\sum_{k=1}^{n}(-1)^{j_k} - m\sum_{k=1}^{n}(-1)^{j_k} + n\sum_{k=n+1}^{n+m}(-1)^{j_k} - (m-1)\sum_{k=n+1}^{n+m}(-1)^{j_k} =$$

$$= (n-m-1)\sum_{k=1}^{n}(-1)^{j_k} + (n-m+1)\sum_{k=n+1}^{n+m}(-1)^{j_k} = P^{(-)}_{nm} \tag{5.4}$$

and under the scale transformations

$$w^{(n,m)}_{j_1,\dots,j_{n+m}}(x_1,\dots,x_{n+m}) =$$

$$= e^{[(n+m)\frac{\alpha^2+\beta^2}{4\pi} - \frac{\alpha^2(n-m)^2+\beta^2(n_1-n_2)^2}{4\pi}]\ln\lambda} w^{(n,m)}_{j_1,\dots,j_{n+m}}(\lambda x_1,\dots,\lambda x_{n+m}). \tag{5.5}$$

The functions (3.2) satisfy the last two equations identically. Therefore, there is no spontaneous breaking of the Lorentz and scale symmetries.

The covariance properties of the $(n+m)$-point functions are obtained in accordance with the fact that the field ψ has no fixed spin and dimension.

The gauge-invariant functions however do have the usual properties:

$$<\psi(x_1)\dots\psi(x_n)\bar{\psi}(x_{n+1})\dots\bar{\psi}(x_{2n})>_0 =$$

$$= e^{-\frac{\alpha\beta}{2\pi}\chi\sum_{k=1}^{n}\gamma^5_{(k)}} <\psi(\Lambda x_1)\dots\psi(\Lambda x_n)\bar{\psi}(\Lambda x_{n+1})\dots\bar{\psi}(\Lambda x_{2n})>_0 e^{\frac{\alpha\beta}{2\pi}\chi\sum_{\ell=n+1}^{2n}\gamma^5_{(\ell)}} \tag{5.6}$$

$$\langle\psi(x_1)\dots\psi(x_n)\bar{\psi}(x_{n+1})\dots\bar{\psi}(x_{2n})\rangle_0 =$$
$$= \lambda^{2n\frac{\alpha^2+\beta^2}{4\pi}} \langle\psi(\lambda x_1)\dots\psi(\lambda x_n)\bar{\psi}(\lambda x_{n+1})\dots\psi(\lambda x_{2n})\rangle_0 . \quad (5.7)$$

Equalities (5.6) and (5.7) show us that if we consider the theory we get in the limit $\mu\to 0$ in the Wightman functions, besides the disappearance of the spontaneous breaking of the gauge symmetries, another remarkable feature is obtained: the field ψ acquires the fixed spin

$$s = \frac{|\alpha\beta|}{2\pi} \quad (5.8)$$

and scale dimension

$$d = \frac{\alpha^2+\beta^2}{4\pi} . \quad (5.9)$$

Canonical dimension is the lowest possible dimension, the spin being kept fixed (see Kupsch et al. [13]). Obviously $d \geq s$ so that

$$d_{can} = s. \quad (5.10)$$

Note that $d=d_{can}=s$ is equivalent to $g=0$. (Evidently $\alpha=-\beta$ leads to trivial solution, since then $\alpha=\beta=0$. In general, if we restrict $|g|<2\pi$, then α and β have the same signs).

6. LOCAL COMMUTATIVITY

From the explicit form of the solution (2.8) and the commutation relations (2.11) and (2.12) it follows that

$$\psi_{j_1}(x_1)\psi_{j_2}(x_2) = e^{iA_{j_1j_2}(x_1-x_2)}\psi_{j_2}(x_2)\psi_{j_1}(x_1), \quad (6.1)$$

$$\psi_{j_1}(x_1)\psi^*_{j_2}(x_2) = e^{-iA_{j_1j_2}(x_1-x_2)}\psi^*_{j_2}(x_2)\psi_{j_1}(x_1), \quad (6.2)$$

where

$$A_{j_1j_2}(x) = -[\alpha^2+\beta^2(-1)^{j_1+j_2}]D(x) + \alpha\beta[(-1)^{j_1}+(-1)^{j_2}]\tilde{D}(x) \quad (6.3)$$

(i.e., expressions which depend on μ do not appear).

Further, for $(x_1-x_2)^2<0$ (when $D(x_1-x_2)=0$) one obtains:

$$[\psi_{j_1}(x_1), \psi_{j_2}(x_2)] = 0 = [\psi_{j_1}(x_1), \psi^*_{j_2}(x_2)], \quad j_1 \neq j_2 . \qquad (6.4)$$

For $s = N,\ N = 0, \pm 1, \pm 2, \ldots$

$$[\psi_j(x_1), \psi_j(x_2)] = 0 = [\psi_j(x_1), \psi^*_j(x_2)] \qquad (6.5)$$

and for $s = N + \frac{1}{2}$

$$\{\psi_j(x_1), \psi_j(x_2)\} = 0 = \{\psi_j(x_1), \psi^*_j(x_2)\}. \qquad (6.6)$$

The commutation relations obtained do not fulfill the requirements of the relation between spin and statistics. However, as it was pointed out by Stoyanov [11], the correct spin-statistics relation can be achieved by a simple field renormalization. The fields $\tilde{\phi}(x)$ used in this paper up to this point have to be replaced by new fields

$$\tilde{\phi}^{\pm}_s(x) = \tilde{\phi}^{\pm}(x) + \frac{i}{8}\sqrt{2\pi}(-a^+(0) + a^-(0)), \qquad (6.7)$$

where $a^{\pm}(0)$ are related to the operators $q^{\pm}$ in Eq. (2.14) as follows:

$$q^{\pm} = \mp i\sqrt{\frac{\pi}{2}}\, a^{\pm}(0) \qquad (6.8)$$

(Unfortunately, there was a misprint in the first of formulae [11]; the correct formula is Eq. (6.7)).Owing to this field redefinition the Thirring fields change by a constant phase factor

$$\Psi_j(x) = e^{\frac{i}{2}\beta q(-1)^j}\, \psi_j(x) \qquad (6.9)$$

with $q = q^+ + q^-$. Then, using Eqs. (2.16), (2.17) and (6.4)-(6.6) one can easily find that for $(x_1 - x_2)^2 < 0$

$$\{\Psi_{j_1}(x_1), \Psi_{j_2}(x_2)\} = 0 = \{\Psi_{j_1}(x_1), \Psi^*_{j_2}(x_2)\} \qquad (6.10)$$

if the spin s of the fields is half-integer

$$s = \frac{\alpha\beta}{2\pi} = N + \frac{1}{2}$$

and

$$[\Psi_{j_1}(x_1), \Psi_{j_2}(x_2)] = 0 = [\Psi_{j_1}(x_1), \Psi^*_{j_2}(x_2)] \qquad (6.11)$$

if s is integer

$$s = \frac{\alpha\beta}{2\pi} = N.$$

So, we have the correct relation between spin and statistics for the fields (6.9).

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REFERENCES

1. Hadjiivanov L., Mikhov S., Stoyanov D. Journal of Phys. A., 1979, 12, p.119.
2. Hadjiivanov L., Stoyanov D. JINR, E2-10950, Dubna, 1977.
3. Johnson K. Nuovo Cimento, 1961, 20, p.773.
4. Klaiber B. Boulder Lectures in Theoretical Physics. Gordon and Breach, N.Y., 1968, vol.X-A.
5. Dell'Antonio G.F., Frishman Y., Zwanziger D. Phys. Rev., 1972, D6, p.988.
6. Wightman A. Lectures at the Summer School of Theoretical Physics, Cargese, 1964.
7. Nakanishi N. Progress of Theor.Phys., 1977, 57, pp.269, 580, 1025 and 58, p.1007.
8. Skyrme T.H.R. Proc.Roy.Soc., 1961, A262, p.237.
9. Streater R.F., Wilde I.F. Nucl.Phys., 1970, B24, p.561.
10. Streater R.F. Charges and Currents in the Thirring Model. In: Physical Reality and Mathematical Description, 1974, p.375. Enz/Mehra (eds.), D.Reidel Publishing Company, Dordrecht-Holland.
11. Stoyanov D.T. JINR, E2-11884, Dubna, 1978.
12. Aneva B., Mikhov S., Stoyanov D. JINR, E2-11885, Dubna, 1978.
13. Kupsch J., Rühl W., Yunn B.C. Ann. of Phys., 1975, 89, p.115.

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