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# SECONDARY PARTICLES AVERAGE MULTIPLICITY IN HADRON-NUCLEUS INTERACTIONS

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Средняя множественность вторичных частиц
в адрон-ядерных взаимодействиях

Рассматривается средняя множественность вторичных частиц в адрон-ядерных взаимодействиях в рамках модели многократного рассеяния.

Показано, что предсказания модели каскада лидирующей частицы с учетом неопределенности в значениях коэффициента неупругости в адрон-нуклонных взаимодействиях находятся в удовлетворительном согласии с существующими экспериментальными данными.

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Alaverdian G.B. et al. **E2 - 12799**

Secondary Particles Average Multiplicity
in Hadron-Nucleus Interactions

The mean multiplicity of the secondary particles in hadron-nucleus interactions is considered in the framework of the multiple scattering theory.

It is shown that predictions of the cascade model of leading particles are in satisfactory agreement with the existing experimental data (according to the uncertainty in inelasticity coefficient in hadron-nucleon interactions).

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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## 1. INTRODUCTION

Most of the presently available models, which claim to describe the inelastic hadron-nucleus (hA) experimental data, can be conventionally divided into two classes.

Different kinds of cascade models[1,2] belong to the first class. The second class includes "collective" type models[3,4], when the incident hadron interacts with the whole group of nucleons (in particular, with all together or with one) simultaneously. As a rule, it is single-step type model. In favour of the "collective" type models and against the cascade ones the following arguments are usually listed: the experimentally found weak A-dependence of the secondary relativistic particle average multiplicity, the inelasticity coefficient, the average transverse momentum of the leading particles and also the so-called "hadron-like" behaviour of some characteristics of the hA -interactions, for example, the coincidence of the KNO-functions, describing the multiplicity distributions of secondary particles produced in hN-and hA-interactions. However these objections can be hardly considered well justified. As a rule they are either purely speculative or are the result of rough enough and not always correct numerical estimation[2]. The well-known divergence of the naive cascade model predictions with the experimental data plays not the last role in these objections. But as far as this model can be used for small energies the above-mentioned divergence is not a satisfactory reason for its rejection. It is rather an indication to modify it. One of such possibilities is the model in which the cascading of a leading particle only is taken into account, further called the cascade model of the leading particle (CMLP). The argument in favour of this model is the successful description of the leading particle spectra in the reactions like $hA \to hX$ in the framework of the multiple scattering theory/5/.

Further we show that the existing experimental data on the multiple production of particles in hA-interaction in no way contradict the picture of multiple collisions of an incident hadron with nucleons inside the nucleus. Therefore these data can be described in the framework of the multiple scattering model.

In the present paper the mean multiplicity of the secondary relativistic particles is calculated according to such an approach and its dependence on energy, nuclear-target atomic number, and type of the incident particles is analysed.

## 2. MEAN MULTIPLICITY OF THE SECONDARIES PRODUCED IN hA-COLLISION

Suppose that the initial (leading) hadron undergoes $n$ inelastic collisions in the process of its interaction with nucleus A, and in each such collision it produces on the average $\bar{n}_{hN}(E_k)$ secondaries. Then the total number of detected hadrons will be

$$\sum_{k=1}^{n} \bar{n}_{hN}(E_k). \qquad (1)$$

Multiplying expression (1) by the $n$-fold collision probability

$$W_n(\sigma, A) = N_n(\sigma, A)/N(0, \sigma) \qquad (2)$$

and summing over all $n$, one finds the resulting multiplicity in the hA-interaction

$$\bar{n}_{hA}(E) = \sum_{n=1}^{A} W_n(\sigma, A) \sum_{k=1}^{n} \bar{n}_{hN}(E_k). \qquad (3)$$

Here

$$N_n(\sigma, A) = \frac{1}{\sigma n!} \int d^2 b [\sigma T(\vec{b})]^n e^{-\sigma T(\vec{b})},$$

$$N(0,\sigma) = \sum_{n=1}^{A} N_n(\sigma, A) = \frac{1}{\sigma} \int d^2 b [1 - e^{-\sigma T(\vec{b})}]$$

are effective numbers, and $T(\vec{b})$ is thickness of nucleus, defined as $T(\vec{b}) = \int_{-\infty}^{+\infty} \rho(\vec{b}, z) dz$, $\rho(\vec{b}, z)$ is density of the nucleon distribution in the nucleus ($\int \rho(r) d^3 r = A$), $E_k$ is the leading particle energy before $k$-th collision. First suppose that the leading particle loses its energy by discrete portions. (This energy should be calculated generally with the fluctuations of the energy loss in each collision taken into account). Let $f$ be a mean part of the energy carried by the leading particle after inelastic hN-collision. Then $E_k = E f^{k-1}$, where $E$ is the initial beam energy. There are different predictions for the energy dependence $\bar{n}_{hN}(E)$ of the mean multiplicity of the secondaries produced in the hN-collisions. According to the hydrodynamical model/6/ the $\bar{n}_{hN}(E)$ value should grow slowly with energy

$$\bar{n}_{hN}(E) = c\left(\frac{E}{E_0}\right)^{a}, \qquad (4)$$

where $E_0$ is the scale parameter. The logarithmic dependence appears in the multiperipheral model/7/

$$\bar{n}_{hN}(E) = a \ln(E/E_0), \qquad (5a)$$

$$\bar{n}_{hN}(E) = b + a \ln^2(E/E_0). \qquad (5b)$$

In the first case one can obtain for the mean multiplicity on the nucleus

$$\bar{n}_{hA}(E) = \bar{n}_{hN}(E) \frac{N(0, \sigma_1)}{N(0, \sigma)}, \qquad (6)$$

where $\sigma_1 = \sigma(1 - f^{a})$. In the case of the logarithmic dependence the result is somewhat more complicated

$$\bar{n}_{hA}(E) = \bar{n}_{hN}(E) \frac{A}{N(0,\sigma)} + \frac{aD \ln f}{N(0,\sigma)},$$

$$\bar{n}_{hA}(E) = \bar{n}_{hN}(E) \frac{A}{N(0,\sigma)} + \frac{aD \ln f}{N(0,\sigma)} * \qquad (7)$$

$$* \frac{2\ln(E/E_0) + \ln f}{\bar{n}_{hN}(E)} + \frac{aG \ln^2 f}{N(0,\sigma)\, \bar{n}_{hN}(E)},$$

where $D = \frac{\sigma}{2} \int T^2(\vec{b}) d^2 b$, $G = \frac{\sigma^2}{3} \int T^3(\vec{b}) d^2 b$.

The normalized multiplicity (multiplication coefficient) is usually considered in the analysis of experimental data:

$$R_A = \bar{n}_{hA} / \bar{n}_{hN}. \quad (8)$$

We note that both power and logarithmic energy dependence of the secondary particle mean multiplicity does not contradict experimental data in the wide energy range.

## 3. $R_A$ ENERGY DEPENDENCE

Expressions (6) and (7) show that cascade model of leading particle (CMLP) leads to non energy dependence of $R_A$ in the case of the power-like growth of $\bar{n}_{hN}(E)$ and that $R_A$ value weakly increases with energy with a constant limit at $E \to \infty$ in the case where $\bar{n}_{hN}(E)$ depends logarithmically on energy

$$R_A = N(0, \sigma_1) / N(0, \sigma) = \mathrm{const}(E), \quad (9a)$$

$$R_A = \bar{\nu}\left[1 + \frac{D \ln f}{a \ln(E/E_0)}\right], \quad (9b)$$

$$R_A = \bar{\nu}\left[1 + \frac{a D \ln f}{A} \cdot \frac{2\ln(E/E_0) + \ln f}{\bar{n}_{hN}(E)} + \frac{a G \ln^2 f}{A\, \bar{n}_{hN}(E)}\right]. \quad (9c)$$

Here $\bar{\nu}$ is the mean number of inelastic collisions of the incident hadron with nucleons inside nucleus.

$$\bar{\nu} = \sum_{n=1}^{\infty} n N_n(\sigma, A) / N(0, \sigma) = A / N(0, \sigma). \quad (10)$$

Therefore the asymptotical behaviour of $R_A$ is different in the case of power and logarithmic energy dependence of $\bar{n}_{hN}(E)$, both being compatible with experimental data in the accessible accelerator energy region. Both cases result in the numerically close predictions for $R_A$ at finite energies, therefore at the energies achieved these two possibilities cannot be distinguished. For example, values of $R_A$ are calculated with the use of the Fermi distribution of the nuclear density $\rho(\vec{r})$ for all three parametrizations of $\bar{n}_{hN}(E)$ and for different values of the inelasticity coefficient in hN-collisions ($K_N = 1 - f$) and the results are presented in Table 1.

The parameters for $\bar{n}_{hN}(E)$ are taken from refs./8/. It is obvious that the value of $R_A$ with the energy variation from $10^2$ to $10^4$GeV varies within (3÷5)% for light nucleus and (9–18)% for intermediate and heavy ones depending on different values of the inelasticity coefficient.

Before passing to the comparison of the predictions of the model with experimental data we consider effects occurring due to the energy loss fluctuation of the leading particle.

Represent eq.(3) in the form

$$\bar{n}_{hA}(E) = \sum_{n=1}^{A} \tilde{W}_n \bar{n}_{hN}(E_n), \quad (11)$$

where $\tilde{W}_n = \sum_{k=n}^{A} W_k$ is the probability for the leading particle to undergo more than $n$ inelastic collisions inside the nucleus.

Taking into account the energy loss fluctuations one replaces $\bar{n}_{hN}(E_n)$ by the expression

$$\frac{1}{\sigma} \int \left(\frac{d\sigma}{dx}\right)^{(n-1)} \bar{n}_{hN}(EX)\, dx, \quad (12)$$

where $\frac{1}{\sigma}\left(\frac{d\sigma}{dx}\right)^{(n)}$ is the leading particle energy spectrum after $n$-fold collision.

According to the results of ref./5/ it is defined as

$$\frac{1}{\sigma}\left(\frac{d\sigma}{dx}\right)^{(n)} = \int_0^1 \prod_{i=1}^{n} \left[\frac{1}{\sigma} \frac{d\sigma}{dx_i} dx_i\right] \delta\left(x - \prod_{i=1}^{n} x_i\right). \quad (13)$$

Here $\frac{1}{\sigma}\frac{d\sigma}{dx_i}$ is the spectrum in hN-interactions.

Assuming that

$$\frac{1}{\sigma}\frac{d\sigma}{dx_i} = (1+\beta) x_i^{\beta}, \quad (14)$$

where

$$\beta = (2f-1)/(1-f), \quad f = \frac{1}{\sigma} \int x \frac{d\sigma}{dx} dx.$$

Table 1

| A | E GeV | $R_A$ ($\bar{n}_{hN}=a+b\ln s+c\ln^2 s$) $K_N=0,4$ | $=0,5$ | $=0,6$ | ($\bar{n}_{hN}=a+b\ln s$) $=0,4$ | $=0,5$ | $=0,6$ | ($\bar{n}_{hN}=as^{0.25}$) $=0,4$ | $=0,5$ | $=0,6$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | I00 | I,462 | I,425 | I,383 | I,448 | I,400 | I,343 | | | |
| | 300 | I,476 | I,442 | I,404 | I,480 | I,445 | I,40I | | | |
| | 500 | I,48I | I,449 | I,4I2 | I,490 | I,458 | I,4I9 | | | |
| | I000 | I,488 | I,458 | I,423 | I,50I | I,473 | I,439 | | | |
| | I500 | I,49I | I,462 | I,428 | I,507 | I,480 | I,448 | | | |
| | I800 | I,493 | I,464 | I,43I | I,509 | I,483 | I,452 | | | |
| I2 | 2000 | I,494 | I,465 | I,432 | I,5I0 | I,485 | I,454 | I,48 | I.43 | I,38 |
| | 2500 | I,496 | I,468 | I,435 | I,5I2 | I,488 | I,459 | | | |
| | 3000 | I,497 | I,469 | I,437 | I,5I4 | I,49I | I,462 | | | |
| | 4000 | I,499 | I,472 | I,44I | I,5I7 | I,494 | I,467 | | | |
| | I0000 | I,505 | I,480 | I,450 | I,524 | I,504 | I,480 | | | |
| | I00 | 2,372 | 2,I97 | 2,02I | 2,235 | I,959 | I,820 | | | |
| | 300 | 2,439 | 2,274 | 2,I0I | 2,426 | 2,2I7 | I,962 | | | |
| | 500 | 2,466 | 2,307 | 2,I36 | 2,486 | 2,299 | 2,070 | | | |
| | I000 | 2,500 | 2,347 | 2,I82 | 2,550 | 2,385 | 2,I84 | | | |
| | I500 | 2,5I8 | 2,369 | 2,207 | 2,580 | 2,427 | 2,240 | | | |
| | I800 | 2,526 | 2,379 | 2,2I8 | 2,593 | 2,444 | 2,262 | | | |
| 94 | 2000 | 2,530 | 2,384 | 2,224 | 2,600 | 2,453 | 2,274 | 2,40 | 2,2I | 2,I0 |
| | 2500 | 2,539 | 2,395 | 2,237 | 2,6I4 | 2,472 | 2,299 | | | |
| | 3000 | 2,546 | 2,404 | 2,247 | 2,624 | 2,487 | 2,3I8 | | | |
| | 4000 | 2,557 | 2,4I8 | 2,263 | 2,640 | 2,508 | 2,347 | | | |
| | I0000 | 2,589 | 2,457 | 2,3I0 | 2,683 | 2,566 | 2,423 | | | |
| | I00 | 2,87I | 2,588 | 2,334 | 2,553 | 2,030 | I,989 | | | |
| | 300 | 2,982 | 2,707 | 2,438 | 2,9I4 | 2,5I9 | 2,036 | | | |
| | 500 | 3,029 | 2,759 | 2,489 | 3,027 | 2,673 | 2,240 | | | |
| | I000 | 3,087 | 2,826 | 2,558 | 3,I49 | 2,838 | 2,457 | | | |
| | I500 | 3,II8 | 2,863 | 2,597 | 3,207 | 2,9I7 | 2,56I | | | |
| | I800 | 3,I32 | 2,878 | 2,6I4 | 3,23I | 2,949 | 2,604 | | | |
| 208 | 2000 | 3,I40 | 2,888 | 2,623 | 3,244 | 2,966 | 2,627 | 2,93 | 2,62 | 2,22 |
| | 2500 | 3,I56 | 2,906 | 2,644 | 3,270 | 3,002 | 2,674 | | | |
| | 3000 | 3,I68 | 2,92I | 2,660 | 3,290 | 3,030 | 2,7II | | | |
| | 4000 | 3,I87 | 2,944 | 2,686 | 3,320 | 3,070 | 2,764 | | | |
| | I0000 | 3,244 | 3,0I2 | 2,762 | 3,400 | 3,I79 | 2,908 | | | |



one obtaines

$$\frac{1}{\sigma}\left(\frac{d\sigma}{dx}\right)^{(n)} = \frac{(1+\beta)^n}{(n-1)!}\left(\ln\frac{1}{x}\right)^{n-1} x^{\beta}, \quad \frac{1}{\sigma}\left(\frac{d\sigma}{dx}\right)^{(0)} = \delta(1-x). \qquad (15)$$

It is easy to find, using eq.(15), that the accounting of the energy loss fluctuations results in the replacement of the value $f$ in (6) by $f_{eff}$

$$f_{eff} = \left(\frac{1+\beta}{1+\alpha+\beta}\right)^{1/\alpha} = \left(\frac{f}{f+(1-f)\alpha}\right)^{1/\alpha}. \qquad (16)$$

Since at the variation of $f$ within $0.4\leq f\leq 0.6$, $f_{eff}$ varies within the interval $0.25\leq f_{eff}\leq 0.55$ then the effective discrete energy loss in each collision is larger than in free hN-interactions. It leads to the decrease of the value of $R_A$.

$$R_A = N\left(0, \sigma\frac{\alpha}{1+\alpha+\beta}\right)/N(0,\sigma). \qquad (17)$$

If the multiplicity in hN-collisions does not depend on projectile energy quantity, $R_A$ must coincide with the mean collision number of the leading particles $\bar{\nu}$. This situation corresponds to the limit $\alpha\to 0$, i.e., $R_A = A\,N(0,\sigma) = \bar{\nu}$ in the case of power parametrization.

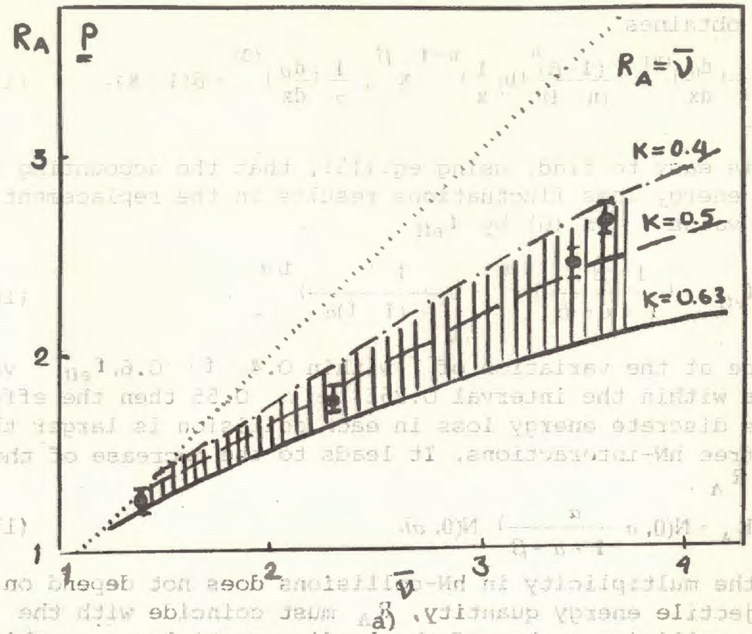
However, even comparatively weak energy dependence of the mean multiplicity in hN-collisions (experimentally $\alpha\sim 0.25$ which agrees with the prediction of hydrodynamical model) diminishes this ratio sharply (see fig. 1).

The sensitivity of $R_A$ to the inelasticity coefficients is shown in fig. 1 (a,b,c) where the $R_A$ value strip is presented in comparison with experimental data for three types of incident particles ($\pi$, K, p). The inelasticity coefficient in hN-collisions is estimated now with large uncertainty. The comparison of our results with experimental data speaks in favour of value $K_N = 0.5$.

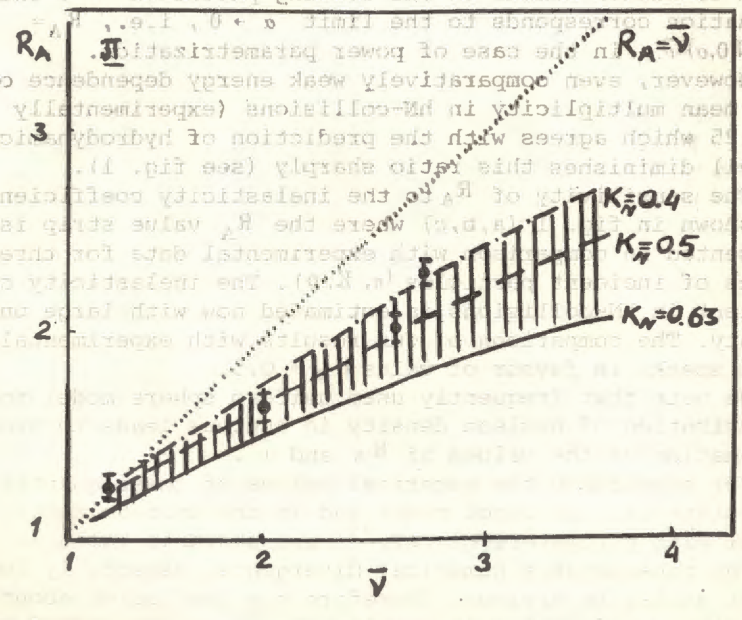
We note that frequently used uniform sphere model for distribution of nucleon density in nucleus leads to overestimation of the values of $R_A$ and $\bar{\nu}$.

For comparison the numerical values of these quantities calculated in the fermi model and in the uniform sphere model with parameters of ref./2/ are shown in Table 2.

The considerable numerical divergence, especially for light nuclei is obvious. Therefore the conclusion about the failure of CMLP proposed in ref./2/ is not indeed well justified result which is proved by the comparison of the

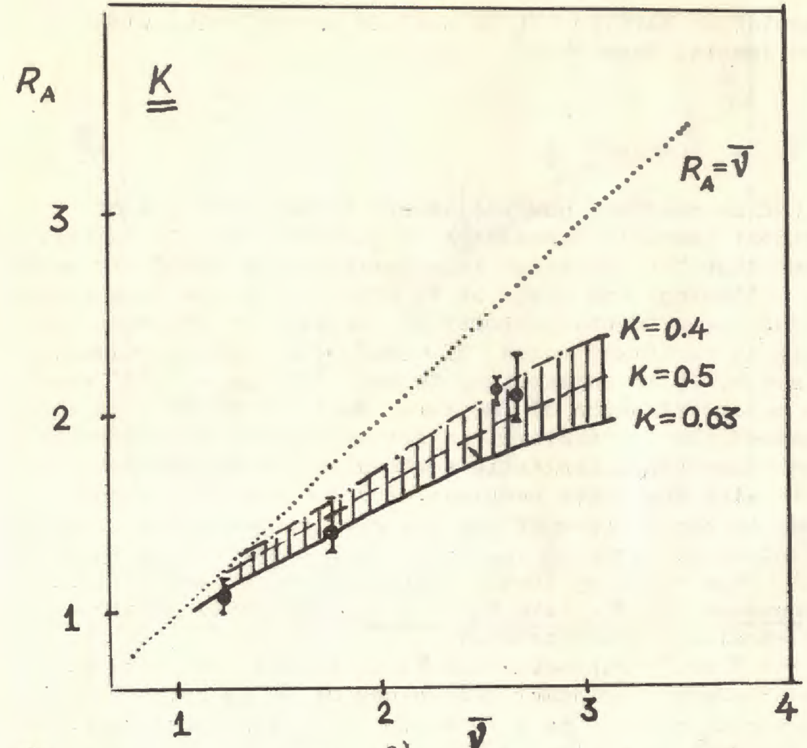



a)




b)






c)

Fig. 1. The values of $R_A$ in comparison with experimental data /9/. The curves are the results of calculation at the different values of inelasticity coefficient for proton (a), for $\pi$-meson (b) and for K-meson (c).

Table 2

| A | $\nu$ Fermi model | $\nu$ Uniform sphere model | $R_A$ Fermi model | $R_A$ Uniform sphere model |
|---|---|---|---|---|
| 12 | 1,62 | 2.16 | 1.25 | 1.87 |
| 27 | 2.09 | 2.60 | 1.67 | 2.12 |
| 64 | 2.77 | 3.24 | 2.09 | 2.57 |
| 108 | 3.18 | 3.66 | 2.31 | 2.79 |
| 207 | 4.04 | 4.54 | 2.64 | 3.10 |

calculation carried out in uniform sphere model with experimental data.

4. "$\bar{\nu}$ - SCALING"

Let us consider now the atomic number and type of incident particle dependence of quantity $R_A$. It is well known that "$\bar{\nu}$ -scaling" is experimentally found. It means the following: the value of $R_A$ turns out to be independent (within experimental errors) of the kind of incident particle at the fixed value of $\bar{\nu}$ and with correspondingly chosen A. It is interesting to note that in ref./9/ where the parametrization of the form $R_A = 0.47 \pm 0.61\bar{\nu}$ is $\bar{\nu}$ obtained the "$\bar{\nu}$ -scaling" is interpreted as a manifestation of multiple inelastic scattering of the incident particle with the cross sections of secondary collisions equal to the section of the incident hadron interaction with free nucleons, i.e., in evident connection with CMLP. This model by itself does not lead to any definite expression for $R_A$ like $R_A = a + b\bar{\nu}$, that would manifest "$\bar{\nu}$ -scaling" transparently.

The A and $\bar{\nu}$ dependence of $R_A$ is rather complicated. Nevertheless, the numerical values of $R_A$ calculated with the use of eq.(17) as a function of $\bar{\nu}$ for three types of incident particles $(\pi, K, p)$ and for $K_N = 0.5$ turn out to be rather close and within experimental errors (which are about 15% in this case) can be considered to be equal (see fig. 2).

Therefore CMLP reproduces this experimentally found regularity.

Figure 3 shows the same experimental data for $\pi, K, p$ but now they are shown as a function of the value $A^{1/3}$ together with the results calculated for $K_N = 0.5$. It is evident that the geometrical scaling ("$A^{1/3}$ -scaling") is absent in this case.

Experimental data are not fitting also with the assumption idea that the cross sections of secondary collisions of the cascading particle supposed to differ from that of the incident hadron interacting with free nucleons.

Similar assumptions are made for the models where the multiplication of secondary particles depends on the cascading of the secondary (produced) particles. The mean collision number in such approach is $\bar{\nu}' = 1 + (\bar{\nu} - 1)\sigma_{\pi p}/\sigma_{hp}$.



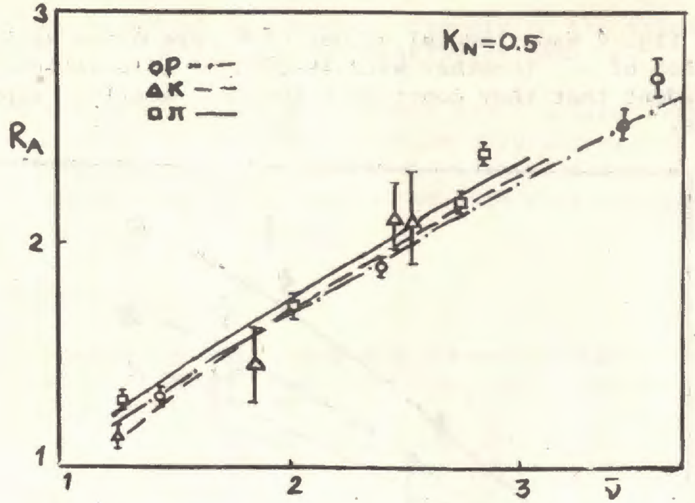



Fig. 2. $\bar{\nu}$ -dependence of $R_A$ at $K_N = 0.5$ for three particles $p, \pi$ and K. The experimental points are taken from ref./9/.

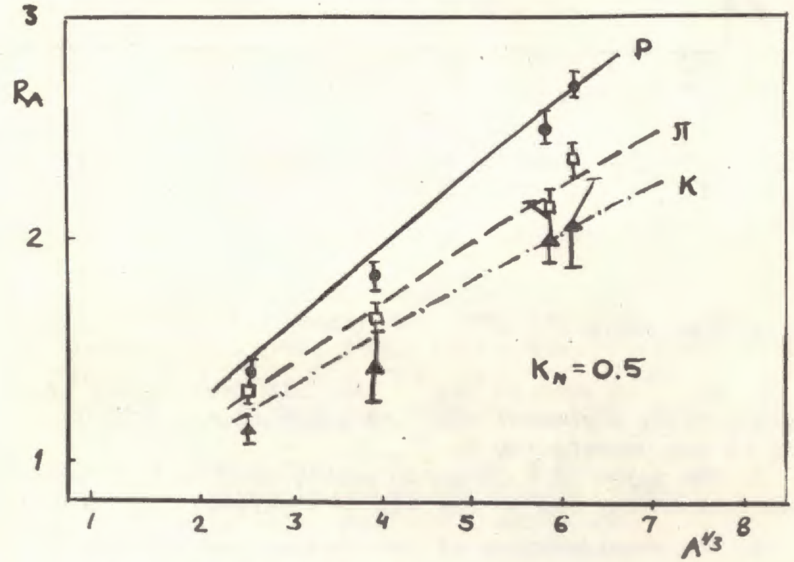



Fig. 3. A-dependence of $R_A$. The curves are results of calculation at $K_N = 0.5$.

In fig. 4 experimental values of $R_A$ are drawn as a function of $\bar{\nu}'$ together with results of calculation. It is evident that they contradict the "$\bar{\nu}'$ -scaling" hypothesis.

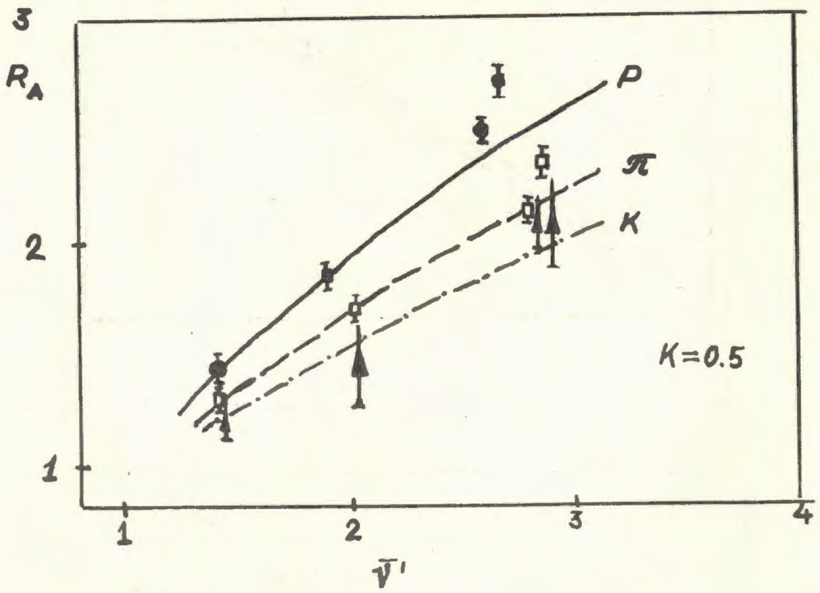



Fig. 4. $\bar{\nu}'$ -dependence of $R_A$. Experimental points are taken from ref. /9/. The curves are results of calculation at $K_N$ = 0.5.

5. CONCLUSION

1. As is seen on fig. 1 the CMLP predictions are in satisfactory agreement with the experimental data (according to the uncertainty in $K_N$ ).

2. The value of $R_A$ depends weakly both on the incident particle energy and on the $\bar{n}_{hN}(E)$ ) parametrization kind.

3. The consideration of the leading particle energy loss fluctuations leads to decrease of the value of $R_A$.

4. CMLP does not give a strict " $\bar{\nu}$ - scaling however numerical values of $R_A$ for different kinds of incident particles and the same value of $\bar{\nu}$ (i.e., at appropriate choice of A) happen to coincide.



5. The value of $R_A$ depends strongly on the value of inelasticity coefficient in hN-interactions.

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