ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2 - 12708

J.Antoš, Yu.A.Budagov, V.S.Rumyantsev

RELATION BETWEEN TWO-DIMENSIONAL SPECTRA OF UNSTABLE PARTICLES AND THEIR DECAY PRODUCTS

1979

E2 - 12708

J.Antoš, Yu.A.Budagov, V.S.Rumyantsev*

RELATION BETWEEN TWO-DIMENSIONAL SPECTRA OF UNSTABLE PARTICLES AND THEIR DECAY PRODUCTS

Submitted to "Nuclear Physics B"

* Institute of Physics, Minsk, USSR

Антош Я., Будагов Ю.А., Румянцев В.С. E2 - 12708

О связи между двумерными спектрами нестабильных частиц и продуктов их распада

Найдены ядро и границы интегрирования для интегрального уравнения, устанавливающего связь между инвариантными структурными функциями нестабильных частиц и одного из продуктов их изотропного распада на две массивные частицы. Распад $\pi^{0}\to 2\gamma$ является частным случаем полученных соотношений.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1979

Antoš J., Budagov Yu.A., Rumyantsev V.S. E2 - 12708

Relation between Two-Dimensional Spectra of Unstable Particles and Their Decay Products

The integral equation which connects invariant structure functions of unstable particles and one of the products of their isotropic two-particle decay is considered. The explicit form of kernel and integration limits are derived. The decay $\pi^{0}\to 2\gamma$ is a special case of obtained relations.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1979

The task of reconstruction of the unstable-particle spectrum from the spectrum of their decay products reduces to finding the explicit form of kernel of an integral equation

$$n(q) = \int G(q,Q)N(Q)dQ, \tag{1}$$

the integration limits and solving this equation. Here $n(q)$ and $N(Q)$ are densities of the q and Q distributions of secondary and primary particles, respectively. The kernel $G(q,Q)$ determines the probability of finding decay product with kinematical characteristic q if a decaying particle has kinematical characteristic Q . When each of symbols q and Q represents a couple of kinematical variables, e.g., P_L, E or P_L, P_T (P_L, P_T are the longitudinal and the transversal components of the momentum, E is the energy/, then eq. (1) establishes a connection between two-dimensional spectra.

For the case of one-dimensional spectra as a function of energy, longitudinal and transversal momentum the equation (1) was analyzed by several authors[1-3]. Results obtained by these authors are widely used for many years in reconstruction of π° -meson spectra [4-10] produced in inclusive reaction

$$\begin{aligned} a + b \to \pi^{\circ} &+ X \\ &\to \gamma + \gamma \end{aligned} \tag{2}$$

under conditions of the poor γ -detection efficiency. In paper[11] dedicated to the study of bremsstrahlung contribution to the photon distribution from the decay $\pi^{\circ} \to 2\gamma$ the integral equation connecting two-dimensional π°- and γ - spectra was obtained. However, this equation is valid only in the c.m. system of reaction (2) and under the condition when masses of incident particles are equal.

In this paper we present an explicit form of kernel $G(q,Q)$ and integration limits for an integral equation of kind (1), which connects two-dimensional spectra of unstable particles and one of the products of their isotropic decay into two massive particles. Relations were obtained for invariant structure functions and are valid in an arbitrary reference frame. Decay $\pi^0 \to 2\gamma$ is a special case of obtained relations.

Consider an isotropic decay $0 \to 1 + 2$. The primary particle 0 has mass M, energy E and momentum $\underline{P}$ in the lab.system. The same quantities for the secondary particle 1 are m, ω and $\underline{q}$, respectively.

To describe the two-dimensional spectra, we take as independent variables energy and projection of momentum on an arbitrary axis. Let this axis be directed along the unit vector $\underline{i}$. The density of the E, P_i distribution $N(E, P_i)$ is connected with invariant structure function $E\, d^3\sigma/d^3P$ by the simple relation

$$N(E, P_i) \equiv \frac{1}{N} \frac{d^2N}{dE\,dP_i} = \frac{2\pi}{\sigma}\left(E \frac{d^3\sigma}{d^3P}\right).$$

This relation shows the invariance of the distribution $N(E, P_i)$. According to eq. (1) we can write an expression for the density of the ω, q_i distribution

$$n(\omega, q_i) = \int_{E_-}^{E_+} dE \int_{P_-}^{P_+} dP_i\, G(\omega, q_i, E, P_i) N(E, P_i). \qquad (3)$$

To find an explicit form of function G and integration limits in the eq. (3), we consider the decay of particle 0 in its rest system. In this reference frame the energy and momentum of particle 1 are constant and equal to

$$\omega^* = \frac{M^2 + m^2 - m_2^2}{2M}, \qquad q^* = \frac{\sqrt{(M^2 - m^2 - m_2^2)^2 - 4m^2m_2^2}}{2M},$$

where m_2 is the mass of particle 2. Let the particle 1 be emitted along the direction determined by the solid and azimuthal angles θ^* and ϕ^*. Then the density of the ϕ^* and $\cos\theta^*$ distribution has the form

$$\frac{d^2g}{d\phi^*\, d(\cos\theta^*)} \equiv g(\phi^*, \cos\theta^*) = \frac{1}{4\pi},$$

where ϕ^* and $\cos\theta^*$ vary within $0 \le \phi^* \le 2\pi$ and $-1 \le \cos\theta^* \le 1$ limits. Now we can write

$$G(\omega, q_i, E, P_i) = \frac{\partial(\cos\phi^*, \cos\theta^*)}{\partial(\omega, q_i)} \frac{\partial\phi^*}{\partial(\cos\theta^*)} g(\phi^*, \cos\theta^*), \qquad (4)$$

where $\partial(\cos\phi^*, \cos\theta^*)/\partial(\omega, q_i)$ is the Jacobian of the transformation from variables $\cos\phi^*$ and $\cos\theta^*$ to ω and q_i. Let us establish the relations connecting angles θ^* and ϕ^* with characteristics of particles 0 and 1 in the lab. system. From the relation connecting ω^* with ω we find

$$\cos\theta^* = \frac{\omega M - \omega^* E}{q^* P}. \qquad (5)$$

In order to obtain the similar expression for $\cos\phi^*$ it is convenient to introduce the system of orthogonal unit vectors $\underset{\sim}{e}_1$, $\underset{\sim}{e}_2$ and $\underset{\sim}{e}_3$. We direct the vector $\underset{\sim}{e}_1$ along the vector $\underset{\sim}{P}$. Then vectors $\underset{\sim}{e}_2$ and $\underset{\sim}{e}_3$ will lie in the plane perpendicular to the vector $\underset{\sim}{P}$. In this plane all possible directions of vector $\underset{\sim}{e}_3$ are equivalent. Therefore without loss of generality we can fix a direction of $\underset{\sim}{e}_3$ by the requirement for $\underset{\sim}{e}_3$ to be perpendicular to the $\underset{\sim}{i}$, $\underset{\sim}{P}$ plane. According to these conditions we can write

$$\underset{\sim}{e}_3 = \frac{\underset{\sim}{P} \times \underset{\sim}{i}}{|P \times i|} = \frac{P(\underset{\sim}{e}_3 \times i)}{P \sin\theta} = \frac{P}{P_T}(\underset{\sim}{e}_3 \times \underset{\sim}{i}) \quad \text{and} \quad \underset{\sim}{e}_2 = \underset{\sim}{e}_3 \times \underset{\sim}{e}_1,$$

where θ is the solid angle of particle 0 in the lab. system, $P_T = \sqrt{P^2 - P_i^2}$ and $P = \sqrt{E^2 - M^2}$. It follows from this that

$$\underset{\sim}{e}_1 \cdot \underset{\sim}{i} = \cos\theta = \frac{P_i}{P}, \quad \underset{\sim}{e}_2 \cdot \underset{\sim}{i} = \sin\theta = \frac{P_T}{P}, \quad \underset{\sim}{e}_3 \cdot \underset{\sim}{i} = 0. \qquad (6)$$

In the orthogonal base of vectors $\underset{\sim}{e}_1$, $\underset{\sim}{e}_2$, and $\underset{\sim}{e}_3$ the momentum of particle 1 can be expressed in the form

$$\underset{\sim}{q} = q'_L \underset{\sim}{e}_1 + q'_T(\underset{\sim}{e}_2 \cos\phi^* + \underset{\sim}{e}_3 \sin\phi^*), \qquad (7)$$

where

$$q'_L = \frac{E}{M} q^* \cos\theta^* + \frac{P}{M}\omega^* = \frac{\omega E - \omega^* M}{P},$$

$$q'_T = q^* \sin\theta^* = \sqrt{q^2 - q'^2_L} \quad \text{and} \quad q = \sqrt{\omega^2 - m^2}.$$

From (6) and (7) we have

$$q_i = \underset{\sim}{q} \cdot \underset{\sim}{i} = \frac{1}{P}(q'_L P_i + q'_T P_T \cos\phi^*),$$

then

$$\cos\phi^* = \frac{q_i P - q'_L P}{q'_T P_T}. \quad (8)$$

With the help of the formulas (5) and (8) one can calculate the transformation Jacobian in the formula (4). Then we obtain for the function G the following expression

$$G(\omega, q_i, E, P_i) = \frac{M}{2\pi q^* \sqrt{(q'_T P_T)^2 - (q_i P - q'_L P_i)^2}}.$$

The integration limits in the relation (3) are derived from the equation of the second order for E and P_i obtained from (5) and (8) under the conditions that $|\cos\theta^*| \leq 1$ and $|\cos\phi^*| \leq 1$. As a solution, we have

$$E_\pm = \frac{M}{m^2}(\omega\omega^* \pm q q^*), \quad \text{for} \quad m \neq 0,$$

$$P_\pm = \frac{P}{q^2}(q_i q'_L \pm q_T q'_T),$$

where $q_T = \sqrt{q^2 - q_i^2}$.

In the case of m = 0 the equation for E reduces to the linear equation and we obtain

$$E_- = \frac{M}{2}\left(\frac{\omega}{\omega^*} + \frac{\omega^*}{\omega}\right), \quad E_+ = \infty, \qquad \text{for} \quad m = 0.$$

In the remaining formulas it is sufficient to take m = 0 and

$$\omega^* = q^* = \frac{M^2 - m_2^2}{2M}.$$

For m = 0 and $m_2 = 0$ one has $\omega^* = q^* = M/2$ and a new expression for the integration limits

$$E_- = \omega + M^2/4\omega, \quad E_+ = \infty, \qquad \text{for} \quad m = 0, \quad m_2 = 0.$$

The case of $m = m_2 = 0$ can be used for the description of $\pi^0 \to 2\gamma$ decay.

Let us show that integral equations for one-dimensional spectra obtained in papers [2,3] are special cases of our results. For example, the density of the ω distribution of particle 1 is determined by the integral

$$n(\omega) = \int_{-q}^{q} dq_i n(\omega, q_i).$$

Substitute here $n(\omega, q_i)$ from the expression (3) and change the order of integration. As a result we obtain

$$n(\omega) = \int_{E_-}^{E_+} dE \int_{-P}^{P} dP_i N(E, P_i) \int_{q_-}^{q_+} dq_i G(\omega, q_i, E, P_i),$$

where the integration limits $q_\pm$ are as follows

$$q_\pm = \frac{1}{P}(q'_L P_i \pm q'_T P_T).$$

It is easy to check that the integral of G over q_i within limits $q_\pm$ equals to $M/2q^*P$. Using the definition

$$N(E) = \int_{-P}^{P} dP_i N(E, P_i),$$

we have

$$n(\omega) = \frac{M}{2q^*} \int_{E_-}^{E_+} \frac{dE}{P} N(E).$$

This expression coincides fully with the integral equation from paper [3].

Using the similar arguments it is easy to show that the integral of $n(\omega, q_i)$ over ω leads to a well-known equation connecting the one-dimensional q_i and P_i distributions [3].

In order to find the spectrum $N(E, P_i)$ from eq.(3) it is possible to employ numerical methods used earlier for reconstruction of π°-spectra [4-10,12]. Among them note a statistical regularization method [13] and a method based on the parametrization of spectrum $N(E, P_i)$ by a function dependent on free parameters.

REFERENCES

1. Sternheimer R., Phys.Rev., 1955, 99, p.277.
2. Glasser R.G. Phys.Rev., 1972, 6D, p.1993.
3. Kopylov G.I. Nucl.Phys., 1973, B52, p.126.
4. Angelov N.S., Grishin V.G., Kopylov G.I. JINR P1-7546, Dubna, 1973.
5. Abdurakhimov A.U. et al., Yad.fiz., 1974, 20, p.384.
6. Biswas N.N. et al., Phys.Rev., 1974, 10D, p.3579.
7. Elliott J.R. et al., Phys.Rev., 1978, 17D, p.83.
8. Boratov M. et al., Nucl.Phys., 1976, B111, p.529.

9. Barluto R. et al., Yad.fiz., 1976, 24, p.580.
10. Amaglobeli N.S. et al., Yad.fiz., 1978, 27, p.995.
Amaglobeli N.S. et al., Yad.fiz., 1978, 28, 1511.
11. Cahn R.N., Phys.Rev., 1973, 7D, p.247.
12. Azhinenko I.V. et al., IHEP 78-58, 1978.
13. Turchin V.F., Kozlov V.P., Malkewitch M.S., Uspekhi fiz.nauk (Sov. Phys.) 1970, 102, p.345.

Received by Publishing Department
on July 31 1979.