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Об импульсных распределениях дейтронов от квазиупругого d - d рассеяния при высоких энергиях

В рамках модели многократного нуклон-нуклонного рассеяния получены выражения для описания импульсных спектров релятивистских дейтронов, испытавших квазиупругое /с развалом мишени/ рассеяние на дейтронах. Приводятся результаты расчетов для дейтронов с начальным импульсом 8,9 ГэВ/с и углов рассеяния 100-160 мрад, демонстрирующие эволюцию структуры высокоимпульсных частей спектров дейтронов, обусловленную вкладом N-N соударений различной кратности.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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On the Momentum Distributions of Deuterons from Quasi-Elastic d-d Scattering at High Energies

The expressions for the momentum spectra of relativistic deuteron experienced a quasi-elastic (with target deuteron break-up) scattering on deuterons are obtained in the framework of the multiple nucleon-nucleon scattering model. The results of calculations, exhibiting the development of the structure of the high-momentum parts of the deuteron spectra due to the contributions of the various multiple N-N scatterings, are given for an initial deuteron momentum of 8.9 GeV/c and scattering angles of 100-160 mr.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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The problem of calculating the differential cross section for the high-energy deuteron-deuteron scattering has already been considered within the framework of the Glauber multiple scattering model both in the case of elastic [1-3] and quasi--elastic scattering occurring with target deuteron break-up but without production of new particles [4]. Present experimental data on the high-energy elastic d-d scattering are reasonably described in terms of the multiple nucleon-nucleon scattering model [5,6]. Recently, the momentum spectra of deuterons with initial momenta of 4.3, 6.3 and 8.9 GeV/c experienced elastic and quasi-elastic scattering at an angle of 103 mr on deuterons [6] and nuclei [7] have been measured. The momentum spectra of deuterons from quasi-elastic d-d scattering show a structure which is new and potentially informative object to examine the predictions of the multiple scattering model. In this work we obtain the expressions to describe the momentum distribution of secondary deuterons emitted in the reaction

$$d + d \rightarrow d + p + n, \tag{1}$$

where the incident deuteron stays in the ground state $|d\rangle$, while the target deuteron is disintegrated, and it goes from the ground state to the state of two unbound nucleons $|p,n\rangle$. On the basis of the expressions obtained the momentum spectra of secondary deuterons from reaction (1) are calculated for an initial deuteron momentum of 8.9 GeV/c and scattering angles in the interval from 103 to 160 mr (lab).

In the multiple nucleon-nucleon scattering model, the amplitude for reaction (1) is given by

$$\mathfrak{F}(\vec{q}) = \langle p,n;d\,|\,\hat{\mathfrak{F}}\,|\,d;d\rangle = \langle p,n|\,\tilde{\mathfrak{F}}\,|\,d\rangle, \tag{2}$$

where the multiple scattering operator $\hat{\mathfrak{F}}$ has the expression

$$\hat{\mathfrak{F}}(\vec{q}) = \frac{ip_0}{2\pi} \int e^{i\vec{q}\vec{b}} \{1 - \prod_{j=1}^{2} \prod_{\ell=1}^{2} [1 - \Gamma_{j\ell}(\vec{b} - \vec{s}_j + \vec{x}_\ell)]\} d^2 b . \qquad (3)$$

Here p_0 is the incident deuteron momentum, $\vec{q}$ is the momentum transferred to the target deuteron, $\vec{b}$ is the impact parameter between the incident deuteron and the target one, $\vec{s}_j$ and $\vec{x}_\ell$ are the projections of the coordinate-vectors of nucleons of the incident and target deuterons, respectively, on the impact parameter plane. The profile function $\Gamma_{j\ell}$ for the interaction of the nucleon j with the nucleon ℓ is given by

$$\Gamma_{j\ell}(\vec{b}) = \frac{1}{2\pi i p'} \int e^{-i\vec{q}'\vec{b}} f(\vec{q}') d^2 q' , \qquad (4)$$

where $p' = \frac{1}{2} p_0$ is the momentum of the incident nucleon, and $f(\vec{q}')$ is the amplitude of the elastic nucleon-nucleon scattering. The scattering angles in reaction (1) are assumed to be small enough in order that vector $\vec{q}$ may be regarded as lying in a plane perpendicular to the incident beam direction. However, the multiple scattering model turns out to be successful in describing experimental data as well in the cases when there are deviations from this condition /8/.

The multiple scattering operator $\hat{\mathfrak{F}}$ can be represented as

$$\hat{\mathfrak{F}} = \hat{\mathfrak{F}}_1 - \hat{\mathfrak{F}}_2 + \hat{\mathfrak{F}}_3 - \hat{\mathfrak{F}}_4 , \qquad (5)$$

where $\hat{\mathfrak{F}}_k$ is the operator corresponding to the multiplicity k of nucleon-nucleon scattering. The various kinds of multiple N-N collisions resulting in the quasi-elastic d-d scattering are schematically represented in fig. 2 of ref. /6/.

By carrying out the integration over the space coordinates of the incident deuteron nucleons, we obtain the expressions for the matrix elements

$$\tilde{\mathfrak{F}}_k(\vec{q}) = \langle d | \hat{\mathfrak{F}}_k | d \rangle \qquad (6)$$

entering into eq.(2) for the amplitude of reaction (1):

$$\tilde{\mathfrak{F}}_1 = K_1 f(\vec{q}) S(\frac{\vec{q}}{2}) \{ e^{-i\vec{u}\frac{\vec{q}}{2}} + e^{i\vec{u}\frac{\vec{q}}{2}} \} ,$$

$$\tilde{\mathcal{F}}_2 = \tilde{\mathcal{F}}_{2(1)} + \tilde{\mathcal{F}}_{2(2)} + \tilde{\mathcal{F}}_{2(3)} = 2K_2 S(\frac{\vec{q}}{2}) \int D(\vec{q},\vec{q}_1) e^{i\vec{u}\vec{q}_1} d^2 q_1 +$$
$$+ K_2 \{ e^{-i\vec{u}\frac{\vec{q}}{2}} + e^{i\vec{u}\frac{\vec{q}}{2}} \} \int D(\vec{q},\vec{q}_1) S(\vec{q}_1) d^2 q_1 +$$
$$+ 2K_2 \int D(\vec{q},\vec{q}_1) S(\vec{q}_1) e^{i\vec{u}\vec{q}_1} d^2 q_1 , \qquad (7)$$
$$\tilde{\mathcal{F}}_3 = K_3 \int T(\vec{q},\vec{q}_1,\vec{q}_2) S(\vec{q}_1) \{ e^{-i\vec{u}\vec{q}_2} + e^{i\vec{u}\vec{q}_2} \} d^2 q_1 d^2 q_2 ,$$
$$\tilde{\mathcal{F}}_4 = K_4 \int Q(\vec{q},\vec{q}_1,\vec{q}_2,\vec{q}_3) S(\vec{q}_1) e^{i\vec{u}\vec{q}_2} d^2 q_1 d^2 q_2 d^2 q_3 .$$

Here we use the following notations:

$$S(\vec{q}) = \int |\phi(\vec{r})|^2 e^{-i\vec{q}\vec{r}} d\vec{r}, \qquad (8)$$

$S(\vec{q})$ being the deuteron form factor, $\phi(\vec{r})$ is the deuteron wave function in the coordinate space,

$$D(\vec{q},\vec{q}_1) = f(\frac{\vec{q}}{2} + \vec{q}_1) f(\frac{\vec{q}}{2} - \vec{q}_1),$$
$$T(\vec{q},\vec{q}_1,\vec{q}_2) = f(\frac{\vec{q}}{2} - \vec{q}_1) f(\frac{\vec{q}}{2} - \vec{q}_2) f(\vec{q}_1 + \vec{q}_2), \qquad (9)$$
$$Q(\vec{q},\vec{q}_1,\vec{q}_2,\vec{q}_3) = f(\vec{q}_3) f(\frac{\vec{q}}{2} - \vec{q}_1 - \vec{q}_3) f(\frac{\vec{q}}{2} - \vec{q}_2 - \vec{q}_3) f(\vec{q}_1 + \vec{q}_2 + \vec{q}_3),$$

and the factors K_k are

$$K_1 = 4, \quad K_2 = -\frac{4i}{2\pi p_0}, \quad K_3 = -\frac{16}{(2\pi p_0)^2}, \quad K_4 = \frac{16i}{(2\pi p_0)^3}. \qquad (10)$$

In formulas (7) the expression for $\tilde{\mathcal{F}}_2$ is split into three terms each corresponding to a different type of the double N-N scattering: $\tilde{\mathcal{F}}_{2(1)}$ represents the collisions when one of the nucleons of the incident deuteron scatters consecutively on both nucleons of the target deuteron, $\tilde{\mathcal{F}}_{2(2)}$ corresponds to the case when both the projectiles scatter on one of the nucleons of the target deuteron, and, at last,

$\tilde{\mathcal{F}}_{2(3)}$ corresponds to the simultaneous scatterings of both the incident nucleons each on one of the two target nucleons.

In order to describe the state of two unbound nucleons which appear as a result of reaction (1) we make use of modified plane waves which are orthogonal to the wave function of the deuteron ground state $|d\rangle$ [9]:

$$|p,n\rangle = \frac{1}{(2\pi)^{3/2}} e^{i\vec{k}\vec{u}} - \Phi(\vec{k})\phi(\vec{u}), \tag{11}$$

where $\Phi(\vec{k})$ is the deuteron wave function in the momentum space. Such a choice of the function $|p,n\rangle$ permits to a certain extent the final state interaction in the p-n system to be taken into account and a more reliable description of the deuteron momentum spectra in the regions near the elastic d-d scattering peaks to be obtained.

Putting expressions (7) and (11) into (2) and carrying out the integration over the space coordinates of the target deuteron nucleons, one obtains the expression for the amplitude of reaction (1) in the form

$$\mathcal{F}(\vec{q}) = \mathcal{F}_1 - \mathcal{F}_2 + \mathcal{F}_3 - \mathcal{F}_4 ,$$

$$\mathcal{F}_1 = K_1 f(\vec{q}) S(\frac{\vec{q}}{2}) \{ \Phi(\vec{k} + \frac{\vec{q}}{2}) + \Phi(\vec{k} - \frac{\vec{q}}{2}) - 2\Phi(\vec{k}) S(\frac{\vec{q}}{2}) \},$$

$$\mathcal{F}_2 = \mathcal{F}_{2(1)} + \mathcal{F}_{2(2)} + \mathcal{F}_{2(3)} = 2K_2 S(\frac{\vec{q}}{2}) \{ \int D\Phi(\vec{k} - \vec{q}_1) d^2q_1 - \Phi(\vec{k}) \int DS(\vec{q}_1) d^2q_1 \} + K_2 \{ \Phi(\vec{k} + \frac{\vec{q}}{2}) + \Phi(\vec{k} - \frac{\vec{q}}{2}) - 2\Phi(\vec{k}) S(\frac{\vec{q}}{2}) \} \times \int DS(\vec{q}_1) d^2q_1 + 2K_2 \{ \int DS(\vec{q}_1) \Phi(\vec{k} - \vec{q}_1) d^2q_1 - \Phi(\vec{k}) \int DS^2(\vec{q}_1) d^2q_1 \}, \tag{12}$$

$$\mathcal{F}_3 = K_3 \{ \int TS(\vec{q}_1) [\Phi(\vec{k} + \vec{q}_2) + \Phi(\vec{k} - \vec{q}_2)] d^2q_1 d^2q_2 - 2\Phi(\vec{k}) \int TS(\vec{q}_1) S(\vec{q}_2) d^2q_1 d^2q_2 \},$$

$$\mathcal{F}_4 = K_4 \{ \int QS(\vec{q}_1) \Phi(\vec{k} - \vec{q}_2) d^2q_1 d^2q_2 d^2q_3 - \Phi(\vec{k}) \int QS(\vec{q}_1) S(\vec{q}_2) d^2q_1 d^2q_2 d^2q_3 \}.$$

The differential cross section for reaction (1) is given by

$$\frac{d\sigma}{d\Omega} = |\mathcal{F}(\vec{q})|^2. \tag{13}$$

In order to obtain the expressions describing the momentum distributions of deuterons emitted in reaction (1) we follow by ref.[10] where the momentum spectra of protons from the quasi-elastic p-d scattering have been investigated. To this end for the kinematical description of the final state we use the following variables in the rest frame of the target deuteron[11]:

$$\vec{q} = \vec{p} - \vec{p}_0 = -(\vec{k}_p + \vec{k}_n), \quad \vec{k} = \frac{1}{2}(\vec{k}_p - \vec{k}_n),$$
$$s = (\hat{k}_p + \hat{k}_n)^2, \quad t = (\hat{p}_0 - \hat{p})^2, \tag{14}$$

where $\vec{p}$ is the momentum of the scattered deuteron, $\vec{k}_{p(n)}$ is the recoiling proton (neutron) momentum, s is the square of the invariant mass of two target nucleons, and t is the square of the four-momentum transfer. The invariant mass spectrum of the p-n pair is given by

$$\frac{d^2\sigma}{d\Omega\, ds} = \int |\mathcal{F}(\vec{q})|^2 \delta\{ s + q^2 - [\sqrt{m^2 + (\vec{k} + \frac{\vec{q}}{2})^2} + \sqrt{m^2 + (\vec{k} - \frac{\vec{q}}{2})^2}]^2 \} d^3k, \tag{15}$$

and the momentum spectrum of the scattered deuterons is related to it by

$$\frac{d^2\sigma}{d\Omega\, dp} = \frac{2}{E}[(E_0 + M)p - E p_0 \cos\theta] \frac{d^2\sigma}{d\Omega\, ds} \simeq 2M \frac{d^2\sigma}{d\Omega\, ds}. \tag{16}$$

Here m and M are the nucleon and deuteron masses, respectively, E_0 and E are the energies of a deuteron with the momenta p_0 and p, and θ is the scattering angle.

The deuteron wave function in the momentum space has been written as[6] :

$$\Phi(\vec{k}) = \sum_{i=1}^{2} N_i \exp(-a_i k^2) \tag{17}$$

with the parameter values

$$N_1 = 29.4 \ (\mathrm{GeV/c})^{-3/2}, \qquad a_1 = 450 \ (\mathrm{GeV/c})^{-2},$$
$$N_2 = 9.8 \ (\mathrm{GeV/c})^{-3/2}, \qquad a_2 = 50 \ (\mathrm{GeV/c})^{-2}.$$

Parametrization (17) of the wave function gives rise to the following expression for the deuteron form factor:

$$S(\vec{q}) = \sum_{m=1}^{3} C_m \exp(-\gamma_m q^2), \qquad (18)$$

where

$C_1 = 0.178,\qquad \gamma_1 = 225\ (\mathrm{GeV}/c)^{-2},$
$C_2 = 0.287,\qquad \gamma_2 = 45\ (\mathrm{GeV}/c)^{-2},$
$C_3 = 0.535,\qquad \gamma_3 = 25\ (\mathrm{GeV}/c)^{-2}.$

In the present analysis we neglect the spin and isospin dependences of the elastic nucleon-nucleon scattering amplitude. In deriving the formulae this amplitude has been assumed as usual to have the form

$$f(\vec{q}) = \frac{p'\sigma_t}{4\pi}(i+\rho)\exp(-\frac{1}{2}\beta q^2), \qquad (19)$$

where σ_t is the total N-N cross section, $\rho = \mathrm{Re}\, f(0)/\mathrm{Im}\, f(0)$, and β is the slope parameter for the differential cross section of the elastic N-N scattering. However, the experimental data on the elastic N-N scattering in the range of not very small momentum transfers are better described with an exponential dependence on t than on q^2. On the other hand, the numerical value of $\sqrt{-t}$ is closer than $|\vec{q}|$ to the value of the projection of $\vec{q}$ onto the plane perpendicular to the beam direction, over that plane the integration in eqs. (12) is carried out. Therefore, in the final expressions for $d^2\sigma/d\Omega ds$ value of q^2 has been interpreted as $-t$. In our calculations at $p_0 = 8.9$ GeV/c the parameters for N-N scattering amplitude are taken to be $\sigma_t = 42.4$ mb, $\beta = 6.3\ (\mathrm{GeV}/c)^{-2}$, and $\rho = -0.43$.

By putting expressions (17-19) into (12) and then into (15), and by integrating in (15) over the relative momentum $\vec{k}$ of the p-n system with a glance to δ function which provides for the energy-momentum conservation, after fairly cumbersome calculations we obtain the expression for the invariant mass spectrum in the form

$$\frac{d^2\sigma}{d\Omega\, ds} = g_{11} - g_{1,2(1)} + g_{2(1),2(1)} - g_{1,2(2)} + g_{2(1),2(2)} + g_{2(2),2(2)} -$$
$$- g_{1,2(3)} + g_{2(1),2(3)} + g_{2(2),2(3)} + g_{2(3),2(3)} + g_{13} - g_{2(1),3} - \qquad (20)$$
$$- g_{2(2),3} - g_{2(3),3} + g_{33} - g_{14} + g_{2(1),4} + g_{2(2),4} + g_{2(3),4} -$$
$$- g_{34} + g_{44},$$

where $g_{k\ell}$ represent contributions of amplitudes $\mathcal{F}_k$ and their interference to the differential cross section. Indices k and ℓ note a multiplicity of N-N scattering, and index 2(n), n=1,2,3, corresponds to the definite type of the above-mentioned double N-N scatterings. The expressions for $g_{k\ell}$ are given in the Appendix.

The expressions which are similar to those obtained in this work have been used for the description of the momentum spectra of deuterons emitted in reaction (1) at an angle of 103 mr and initial deuteron momenta of 4.3, 6.3 and 8.9 GeV/c /6/. Here we present in fig.1 the calculation results for an initial deuteron momentum of 8.9 GeV/c and scattering angles of 103, 120, 140 and 160 mr, with taking account of the experimental resolution attained in ref./6/. At 8.9 GeV/c the momentum resolution function may be approximated by a Gaussian distribution with a standard deviation of 23.2 MeV/c. The dashed curves in fig. 1 show the contributions from the elastic d-d scattering calculated according to ref./2/.

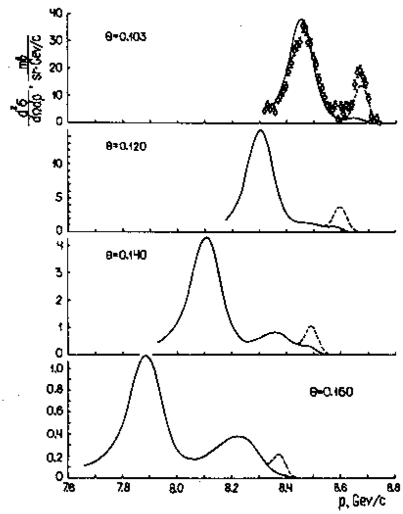

Fig. 1. Momentum spectra of deuterons emitted in reaction (1) at angles of 103, 120, 140 and 160 mr and an initial deuteron momentum of 8.9 GeV/c (solid curves). The dashed curves show the contributions from the elastic scattering calculated according to ref./2/. The experimental data for an angle of 103 mr are taken from ref./6/.

The distributions presented in fig. 1 demonstrate the development of the structure of the high-momentum parts of the deuteron spectra from the quasielastic d-d scattering with increasing a scattering angle. It is seen that in the spectra for 140 and 160 mr the maximum at $p \simeq p_0 - 5(p_0\theta)^2/16m$

is exhibited clearly. This maximum is due to the contribution of tripple N-N collisions. It is worth noting that at momentum transfers under consideration $(0.8 \lesssim |t| \lesssim 1.9\ (GeV/c)^2)$ the contributions of amplitudes $\mathfrak{F}_1$ and $\mathfrak{F}_{2(1)}$ as well as the contributions of terms representing their interference and the interference of those amplitudes with other ones to the differential cross section $d^2\sigma/d\Omega ds$ are small, and those can be neglected without appreciable altering final results.

Figure 2 shows the modification of the deuteron momentum spectrum at p_0 = 8.9 GeV/c, θ = 140 mr as N-N collisions of ever increasing multiplicity are successively taken into account. The final shape of the spectrum is strongly dependent on the contribution of the terms representing the interference of amplitudes which describe the collisions with different multiplicities.

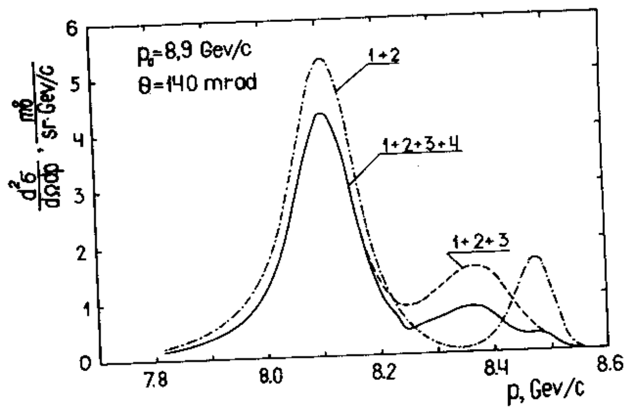

Fig. 2.The modification of the momentum spectrum of secondary deuterons from reaction (1) for p_0 =8.9 GeV/c, θ = 140 mr, as N-N collisions of ever increasing multiplicity are successively taken into account. The dash-dotted curve represents the contributions of the single and the double scatterings only, the dashed curve incorporates the triple scattering too, and the solid curve corresponds to the full amplitude up to the quadruple scattering.

The peaks at $p \simeq p_0 - (p_0\theta)^2/2m$ in the deuteron spectra are due, in the main, to such double N-N collisions where both the incident nucleons scatter on one of the two target nucleons. The widths of these peaks are not related directly to the internal momentum distribution of nucleons in the deuteron because their shape is also affected by the interference of the amplitudes corresponding to the double and triple N-N collisions with the result that these peaks grow narrow. (The lower momentum peak in fig.2 has 140 MeV/c FWHM for the (1+2) -curve, and 132 MeV/c FWHM for the (1+2+3) -curve).

The contributions of the quasielastic d-d scattering to the spectrum regions corresponding kinematically to the elastic d-d scattering are small; as it can be seen from fig.1, these amount to only 15-30%. This situation is qualitatively different from that for p-d scattering: at momentum transfers in the range from o.6 to 1.8 $(GeV/c)^2$ the contributions of the quasielastic p-d scattering to the regions of the peaks corresponding to the elastic p-d scattering amount to 60-70% (see, for example, ref. [12]). Thus, though the deuteron is a very loose system, under certain conditions the elastic scattering in bombarding deuterons by deuterons proves to be relatively more probable than in bombarding deuterons by protons. This is due, of course, to the occurrence of such double collisions in the case of d-d scattering when both the incident nucleons scatter simultaneously each on one of the two target nucleons. The existence of multiple collisions of that kind has to lead to an enrichment of the upper parts of the deuteron spectra in the case of the deuteron-nucleus scattering as well through the deuteron scattering on nucleon groups inside the nucleus. These groups may be emitted as fragments of the target nucleus in accordance with the kinematics close to that for the elastic d-fragment scattering. The recent experimental data on the nuclear scattering of relativistic deuterons [7] do not contradict to this interpretation.

The experimental test of predictions obtained in the present work on the basis of the multiple scattering model seems may be employed to clarify the validity of some assumptions used in deriving expressions for the description of the momentum spectra of deuterons emitted in reaction (1). Furthermore, certain features of multiple interactions can be explored only when both colliding particles are composite, due to the occurrence of new types of multiple collisions. Investigations in this field seem to be of interest as well in connection with the possibility of interpreting high-energy hadron-hadron scattering as a result of multiple scattering effects of hypothetical constituents of those hadrons [13].

APPENDIX

We report here the expressions of the functions appearing in the differential cross section (20).

$$g_{11} = 16 r^2 (1+\rho^2) S^2(\tfrac{q}{2}) e^{\beta t} \{ \sum_{ik} N_i N_k G_1(\alpha_i+\alpha_k, \alpha_i+\alpha_k, \alpha_i+\alpha_k) +$$

$$+ 2 \sum_{ik} N_i N_k G_2(\alpha_i+\alpha_k, \alpha_i-\alpha_k) - 4S(\tfrac{q}{2}) H_1 + 4S^2(\tfrac{q}{2}) H_0 \},$$

$$g_{1,2(1)} = \frac{32}{p_0} r^3 (1+\rho^2) S^2(\tfrac{q}{2}) e^{\frac{3}{4}\beta t} \{ \sum_{ik} \frac{N_i N_k}{\beta+\alpha_i} G_3(\alpha_i+\alpha_k, \frac{\beta\alpha_i}{\beta+\alpha_i}+\alpha_k, \alpha_k, \alpha_k) -$$

$$- S_1 H_1 - 2S(\tfrac{q}{2}) H_{2(1)} + 2S(\tfrac{q}{2}) S_1 H_0 \},$$

$$g_{2(1),2(1)} = \frac{16}{p_0^2} r^4 (1+\rho^2) S^2(\tfrac{q}{2}) e^{\frac{1}{2}\beta t} \{ \sum_{ik} \frac{N_i N_k}{(\beta+\alpha_i)(\beta+\alpha_k)} G_4(\alpha_i+\alpha_k,$$

$$\frac{\beta\alpha_i}{\beta+\alpha_i} + \frac{\beta\alpha_k}{\beta+\alpha_k}) - 2S_1 H_{2(1)} + S_1^2 H_0 \}, \quad g_{1,2(2)} = -2K g_{11},$$

$$g_{2(1),2(2)} = -K(1+\rho^2) g_{1,2(1)}, \quad g_{2(2),2(2)} = K^2 (1+\rho^2) g_{11},$$

$$g_{1,2(3)} = \frac{32}{p_0} r^3 (1+\rho^2) S(\tfrac{q}{2}) e^{\frac{3}{4}\beta t} \{ \sum_{ikm} \frac{N_i N_k C_m}{\beta+\alpha_i+\gamma_m} G_3(\alpha_i+\alpha_k,$$

$$\frac{\alpha_i(\beta+\gamma_m)}{\beta+\alpha_i+\gamma_m} + \alpha_k, \alpha_k, \alpha_k) - S_2 H_1 - 2S(\tfrac{q}{2}) H_{2(3)} + 2S(\tfrac{q}{2}) S_2 H_0 \},$$

$$g_{2(1),2(3)} = \frac{32}{p_0^2} r^4 (1+\rho^2)^2 S(\frac{q}{2}) e^{\frac{1}{2}\beta t} \{ \sum_{ikm} \frac{N_i N_k C_m}{(\beta+\alpha_i)(\beta+\alpha_k+\gamma_m)} G_4(\alpha_i+\alpha_k,$$

$$\frac{\beta\alpha_i}{\beta+\alpha_i} + \frac{\alpha_k(\beta+\gamma_m)}{\beta+\alpha_k+\gamma_m}) - S_2 H_{2(1)} - S_1 H_{2(3)} + S_1 S_2 H_0 \},$$

$$g_{2(2),2(3)} = -K(1+\rho^2) g_{1,2(3)},$$

$$g_{2(3),2(3)} = \frac{16}{p_0^2} r^4 (1+\rho^2)^2 e^{\frac{1}{2}\beta t} \{ \sum_{ikmn} \frac{N_i N_k C_m C_n}{(\beta+\alpha_i+\gamma_m)(\beta+\alpha_k+\gamma_n)} G_4(\alpha_i+\alpha_k,$$

$$\frac{\alpha_i(\beta+\gamma_m)}{\beta+\alpha_i+\gamma_m} + \frac{\alpha_k(\beta+\gamma_n)}{\beta+\alpha_k+\gamma_n}) - 2S_2 H_{2(3)} + S_2^2 H_0 \},$$

$$g_{13} = \frac{32}{p_0^2} r^4 (1-\rho^4) S(\frac{q}{2}) e^{\frac{3}{4}\beta t} \{ \sum_{ikm} \frac{N_i N_k C_m}{(\beta+\alpha_i-\epsilon_m)(\beta+\gamma_m)} [G_3(\alpha_i+\alpha_k, \eta_{im}+\alpha_k,$$

$$\kappa_{im}+\alpha_k,\ \alpha_k-\zeta_{im}) + G_3(\alpha_i+\alpha_k, \eta_{im}+\alpha_k, \kappa_{im}-\alpha_k, \alpha_k-\zeta_{im})] -$$

$$-2S_3 H_1 - 2S(\frac{q}{2}) H_3 + 4S(\frac{q}{2}) S_3 H_0 \},$$

$$g_{2(1),3} = \frac{32}{p_0^3} r^5 (1+\rho^2)^2 S(\frac{q}{2}) e^{\frac{1}{2}\beta t} \{ \sum_{ikm} \frac{N_i N_k C_m}{(\beta+\alpha_i-\epsilon_m)(\beta+\alpha_k)(\beta+\gamma_m)} \times$$

$$\times G_3(\alpha_i+\alpha_k,\ \eta_{im} + \frac{\beta\alpha_k}{\beta+\alpha_k},\ \kappa_{im},\ -\zeta_{im}) - 2S_3 H_{2(1)} - S_1 H_3 + 2S_1 S_3 H_0 \},$$

$$g_{2(2),3} = -K \frac{1+\rho^2}{1-\rho^2} g_{13},$$

$$g_{2(3),3} = \frac{32}{p_0^3} r^5 (1+\rho^2)^2 e^{\frac{1}{2}\beta t} \Big\{ \sum_{ikmn} \frac{N_i N_k C_m C_n}{(\beta+\alpha_i-\epsilon_m)(\beta+\alpha_k+\gamma_n)(\beta+\gamma_m)} \times$$

$$\times G_3\left(\alpha_i+\alpha_k,\ \eta_{im}+\frac{\alpha_k(\beta+\gamma_n)}{\beta+\alpha_k+\gamma_n},\ \kappa_{im},\ -\zeta_{im}\right) - 2S_3 H_{2(3)} - S_2 H_3 +$$

$$+ 2S_2 S_3 H_0 \Big\},$$

$$g_{33} = \frac{16}{p_0^4} r^6 (1+\rho^2)^3 e^{\frac{1}{2}\beta t} \Big\{ \sum_{ikmn} \frac{N_i N_k C_m C_n}{(\beta+\alpha_i-\epsilon_m)(\beta+\alpha_k-\epsilon_n)(\beta+\gamma_m)(\beta+\gamma_n)} \times$$

$$\times [G_3(\alpha_i+\alpha_k,\ \eta_{im}+\eta_{kn},\ \kappa_{im}+\kappa_{kn},\ -\zeta_{im}-\zeta_{kn}) +$$

$$+ G_3(\alpha_i+\alpha_k,\ \eta_{im}+\eta_{kn},\ \kappa_{im}-\kappa_{kn},\ -\zeta_{im}-\zeta_{kn})] - 4S_3 H_3 + 4S_3^2 H_0 \Big\},$$

$$g_{14} = \frac{32}{\beta p_0^3} r^5 (1+\rho^2)(1-3\rho^2) S\left(\frac{q}{2}\right) e^{\frac{5}{8}\beta t} \Big\{ \sum_{ikm} \frac{N_i N_k C_m}{(\beta+2\alpha_i)(\beta+2\gamma_m)} \times$$

$$\times G_3(\alpha_i+\alpha_k,\ \chi_i+\alpha_k,\ \alpha_k,\ \alpha_k) - S_4 H_1 - 2S\left(\frac{q}{2}\right) H_4 + 2S\left(\frac{q}{2}\right) S_4 H_0 \Big\},$$

$$g_{2(1),4} = \frac{32}{\beta p_0^4} r^6 (1+\rho^2)(1-\rho^4) S\left(\frac{q}{2}\right) e^{\frac{3}{8}\beta t} \Big\{ \sum_{ikm} \frac{N_i N_k C_m}{(\beta+2\alpha_i)(\beta+\alpha_k)(\beta+2\gamma_m)} \times$$

$$\times G_4\left(\alpha_i+\alpha_k,\ \chi_i+\frac{\beta\alpha_k}{\beta+\alpha_k}\right) - S_4 H_{2(1)} - S_1 H_4 + S_1 S_4 H_0 \Big\},$$

$$g_{2(2),4} = -K \frac{1-\rho^4}{1-3\rho^2} g_{14},$$

$$g_{2(3),4} = \frac{32}{\beta p_0^4} r^6 (1+\rho^2)(1-\rho^4) e^{\frac{3}{8}\beta t} \{ \sum_{ikmn} \frac{N_i N_k C_m C_n}{(\beta+2\alpha_i)(\beta+\alpha_k+\gamma_n)(\beta+2\gamma_m)} \times$$

$$\times G_4(\alpha_i + \alpha_k, \chi_i + \frac{\alpha_k(\beta+\gamma_n)}{\beta+\alpha_k+\gamma_n}) - S_4 H_{2(3)} - S_2 H_4 + S_2 S_4 H_0 \},$$

$$g_{34} = \frac{32}{\beta p_0^5} r^7 (1+\rho^2)^3 e^{-\frac{3}{8}\beta t} \{ \sum_{ikmn} \frac{N_i N_k C_m C_n}{(\beta+2\alpha_i)(\beta+\alpha_k-\epsilon_n)(\beta+2\gamma_m)(\beta+\gamma_n)} \times$$

$$\times G_3(\alpha_i + \alpha_k, \chi_i + \eta_{kn}, \kappa_{kn}, -\zeta_{kn}) - S_4 H_3 - 2 S_3 H_4 + 2 S_3 S_4 H_0 \},$$

$$g_{44} = \frac{16}{\beta^2 p_0^6} r^8 (1+\rho^2)^4 e^{\frac{1}{4}\beta t} \{ \sum_{ikmn} \frac{N_i N_k C_m C_n}{(\beta+2\alpha_i)(\beta+2\alpha_k)(\beta+2\gamma_m)(\beta+2\gamma_n)} \times$$

$$\times G_4(\alpha_i + \alpha_k, \chi_i + \chi_k) - 2 S_4 H_4 + S_4^2 H_0 \}.$$

Here we use the following notations:

$$r = \mathrm{Im} f(0), \quad K = \frac{r}{2p_0} \frac{\exp(-\beta t/4)}{S(q/2)} S_1, \quad \epsilon_m = \frac{\beta^2}{4(\beta+\gamma_m)}, \; \eta_{im} = \alpha_i - \frac{\alpha_i^2}{\beta+\alpha_i-\epsilon_m},$$

$$\kappa_{im} = \frac{\alpha_i(\beta/2-\epsilon_m)}{\beta+\alpha_i-\epsilon_m}, \quad \zeta_{im} = \epsilon_m + \frac{(\beta/2-\epsilon_m)^2}{\beta+\alpha_i-\epsilon_m}, \; \xi_{mn} = \epsilon_m + \frac{(\beta/2-\epsilon_m)^2}{\beta+\gamma_n-\epsilon_m},$$

$$\chi_i = \frac{\beta\alpha_i}{\beta+\alpha_i}, \quad S_1 = \sum_m \frac{C_m}{\beta+\gamma_m}, \quad S_2 = \sum_{mn} \frac{C_m C_n}{\beta+\gamma_m+\gamma_n},$$

$$S_3 = \sum_{mn} \frac{C_m C_n}{(\beta+\gamma_m)(\beta+\gamma_n-\epsilon_m)} e^{-\frac{1}{4}\xi_{mn} t},$$

$$S_4 = \sum_{mn} \frac{C_m C_n}{(\beta+2\gamma_m)(\beta+2\gamma_n)}, \quad H_0 = \sum_{ik} N_i N_k G_1(\alpha_i+\alpha_k, 0, 0),$$

$$H_1 = \sum_{ik} N_i N_k G_1(\alpha_i+\alpha_k, \alpha_i, \alpha_i),$$

$$H_{2(1)} = \sum_{ik} \frac{N_i N_k}{\beta+\alpha_i} G_4(\alpha_i+\alpha_k, \frac{\beta\alpha_i}{\beta+\alpha_i}+\alpha_k),$$

$$H_{2(3)} = \sum_{ikm} \frac{N_i N_k C_m}{\beta+\alpha_i+\gamma_m} G_4(\alpha_i+\alpha_k, \frac{\alpha_i(\beta+\gamma_m)}{\beta+\alpha_i+\gamma_m}+\alpha_k),$$

$$H_3 = \sum_{ikm} \frac{N_i N_k C_m}{(\beta+\alpha_i-\epsilon_m)(\beta+\gamma_m)} G_3(\alpha_i+\alpha_k, \eta_{im}+\alpha_k, \kappa_{im}, -\zeta_{im}),$$

$$H_4 = \sum_{ikm} \frac{N_i N_k C_m}{(\beta+2\alpha_i)(\beta+2\gamma_m)} G_4(\alpha_i+\alpha_k, \chi_i+\alpha_k).$$

The functions $G_1(a,b,c)$ and $G_2(a,b)$ are expressed in terms of the error function

$$G_1(a,b,c) = \frac{\pi}{2\sigma q}\exp(-\frac{c}{4}q^2)\{\frac{b\sigma+a\mu q}{a^2}\exp(-\frac{a}{4}\sigma^2\mu^2+\frac{b}{2}\sigma\mu q)-$$

$$-\frac{b\sigma-a\mu q}{a^2}\exp(-\frac{a}{4}\sigma^2\mu^2-\frac{b}{2}\sigma\mu q)+\sqrt{\frac{\pi}{a}}[\frac{\sigma^2}{2}(1-\frac{b^2}{a^2})-\frac{1}{a}]\times$$

$$\times[\mathrm{erf}(\frac{b\sigma+a\mu q}{2\sqrt{a}})-\mathrm{erf}(\frac{b\sigma-a\mu q}{2\sqrt{a}})]\exp(-\frac{a}{4}s\mu^2+\frac{b^2}{4a}\sigma^2)\},$$

$$G_2(a,b) = \frac{\pi}{2\sigma q}\exp(-\frac{a}{4}q^2)\{\frac{b\sigma}{a^2}\exp(-\frac{a}{4}s\mu^2)-\frac{b\sigma-a\mu q}{a^2}\exp(-\frac{a}{4}\sigma^2\mu^2-$$

$$-\frac{b}{2}\sigma\mu q)+\sqrt{\frac{\pi}{a}}[\frac{\sigma^2}{2}(1-\frac{b^2}{a^2})-\frac{1}{a}][\mathrm{erf}(\frac{b\sigma+a\mu q}{2\sqrt{a}})-\mathrm{erf}(\frac{b\sigma}{2\sqrt{a}})]\times$$

$$\times\exp(-\frac{a}{4}s\mu^2+\frac{b^2}{4a}\sigma^2),$$

where $\sigma^2 = s + q^2$, $\mu^2 = 1 - 4m^2/s$, and to calculate the functions $G_3(a,b,c,d)$ and $G_4(a,b)$ numerical integrations have to be done:

$$G_3(a,b,c,d) = \frac{1}{2}\sigma\mu\, e^{-\frac{d}{4}q^2} \int_0^1 \{ \int_0^{\pi/2} F(z,u)\,du - \frac{\mu^2 q^2}{\sigma^2}(1-z^2) \times$$

$$\times \int_0^{\pi/2} F(z,u)\cos^2 u\,du \}\,dz ,$$

where

$$F(z,u) = \exp[-\frac{a}{4}s\mu^2 z^2 - \frac{b}{4}s\mu^2(1-z^2)(1+\frac{q^2}{s}\cos^2 u) - \frac{c}{2}\sigma\mu q\sqrt{1-z^2}\cos u] +$$

$$+ \exp[-\frac{a}{4}s\mu^2 z^2 - \frac{b}{4}s\mu^2(1-z^2)(1+\frac{q^2}{s}\cos^2 u) + \frac{c}{2}\sigma\mu q\sqrt{1-z^2}\cos u],$$

$$G_4(a,b) = \frac{\pi}{4}\sigma\mu \int_0^1 \exp\{ -\frac{s\mu^2}{4}[a z^2 + b(1+\frac{q^2}{2s})(1-z^2)] \} [I_0(\nu) -$$

$$- \frac{4\nu}{b\sigma^2}\{I_0(\nu) - I_1(\nu)\}]\,dz ,$$

where $\nu = \frac{b}{8}\mu^2 q^2 (1-z^2)$, and I_0, I_1 are the modified Bessel functions.

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