сообщения
объединенного
института
ядерных
исследований
дубна

E2 - 12652

A.D.Linkevich, N.B.Skachkov

NUCLEON DEEP-INELASTIC STRUCTURE FUNCTIONS IN A QUARK MODEL WITH FACTORIZABILITY ASSUMPTIONS

1979

E2 - 12652

A.D.Linkevich, N.B.Skachkov

NUCLEON DEEP-INELASTIC STRUCTURE FUNCTIONS IN A QUARK MODEL WITH FACTORIZABILITY ASSUMPTIONS

Линкевич А.Д., Скачков Н.Б. **E2 - 12652**

Структурные функции глубоконеупругого eN-рассеяния в кварковой модели с гипотезой факторизуемости

В работе находится явный вид структурных функций глубоконеупругого рассеяния электронов на нуклонах. Для этой цели используется динамическая модель факторизующихся кварковых амплитуд. Проведенное сравнение с экспериментальными данными показывает хорошее согласие модели с опытом.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1979

Linkevich A.D., Skachkov N.B. **E2 - 12652**

Nucleon Deep-Inelastic Structure Functions in a Quark Model with Factorizability Assumptions

Based on the quark model with the hypothesis of factorizability we derive formulae for structure functions of deep inelastic eN-scattering well describing the experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1979

I. Introduction

The investigation of the processes of deep inelastic scattering of leptons on hadrons is an important problem of high-energy physics. Since the first experiments on inelastic eN-scattering which have established an approximate Bjorken scaling[1], a number of approaches and models have been developed for describing lepton-hadron interactions.

In papers[2,3] the scale-invariant behaviour of hadron structure functions was studied on the basis of general principles of quantum field theory. Among field-theoretical models used to describe the deviation from scaling the most preferable is quantum chromodynamics (QCD)[4,5] which is expected to substitute for the naive quark-parton model[6]. However, QCD fails to directly determine the form of structure functions because of the divergence of the Fourier transform of the current-operator product (see, e.g.,[4]). It can reliably establish only the behaviour of moments of structure functions and only up to the unknown moments of gluon and quarks distribution functions. These values can be found either by using the wave functions of composite hadrons unknown in QCD, or experimental data[7], or some parametrizations for distribution functions of quark and gluons. However, the reconstruction of structure functions from their moments encounters mathematical difficulties and also requires the parametrization of structure functions (see, e.g.[9]).

The hadron structure functions can be determined within the field-theoretical formulations of the quark model by using the approaches describing composite systems as bound states of quarks. For instance, in paper[10] the structure functions were calculated on the basis of the Bethe-Salpeter equation, and in paper[11] the meson structure functions have been found on the basis of the covariant three-dimensional formulation of the two-body problem. The problem of description of three-particle composite systems within quantum field theory is rather complicated and yet far to be solved. In view of this, the description of the lepton-nucleon interaction and other hadron systems composed of more than two quarks should use the model representations. In this paper, the form of structure functions for deep inelastic eN scattering will be determined on the basis of the simple quark model with the hypothesis of factorizability, the dynamical model of factorizing quarks (DMFQ). In the next section we present the main formulae of DMFQ and in section III apply them to determine the structure functions.

2. Dynamical Model of Factorizing Quarks (DMFQ)

DMFQ[12,13,14] is an extension of the model of factorizing quarks (MFQ) proposed earlier[15] for the description of two-particle elastic and quasi-elastic processes. The DMFQ is based on the assumption that the description of elementary-particle interactions in the presently attained preasymptotical region of energies is possible if one uses the scale parameter, quark mass M_q, with which one compares the portion of the transfer momentum per one quark, $\sqrt{-t_q}$ *). So, in paper [12] the MFQ was extended with the dynamical hypothesis that at large-angle scattering the range of quark interaction is characterized by an effective size set to be equal to the quark Compton wavelength, M_q^{-1}.

The MFQ assumes that the hadron collision produces a certain effective field V_{eff} on which the quarks, constituting the hadrons, scatter as quasi-independent objects. As for independent

*) For simplicity it is assumed that each of quarks shares equal portion of the transfer momentum, so that $t_q \approx \frac{t}{n^2}$, with n, the quark number in hadron.

events the total probability is a product of individual probabilities, the amplitude $M_{AB\to AB}(\theta)$ of scattering in c.m.s. of hadrons A and B at an angle θ equals the product of the scattering amplitudes $g_q(\theta)$ of individual quarks on the potential V_{eff} :

$$M_{AB\to AB}(\theta) = \prod_{q_A}^{n_A} g_q(\theta) \cdot \prod_{q_B}^{n_B} g_q(\theta) . \tag{1}$$

The expansion over unitary irreducible representations of the Lorentz group allows one to derive the form of multiplicative amplitude $g_q(\theta)$ for which the corresponding interaction radius is known to equal the Compton wave length, M_q^{-1} [12]:

$$g_q(\theta) = \frac{\chi_q}{sh\chi_q} = \frac{2M_q^2 \ln\left[1 - \frac{t_q}{2M_q^2} + \frac{1}{2M_q^2}\sqrt{t_q(t_q - 4M_q^2)}\right]}{\sqrt{t_q(t_q - 4M_q^2)}} \tag{2}$$

Here $\chi_q = Ar\, ch\left(1 - \frac{t_q}{2M_q^2}\right)$ is the rapidity corresponding to $t_q \approx t/n^2$.

The Wu-Yang generalized formula[16]

$$\frac{d\sigma}{dt}(AB\to AB) \approx \frac{1}{s^2}\, G_A^2(t) \cdot G_B^2(t) \tag{3}$$

and (1), (2) give the following formula for the asymptotic behaviour of the form factor of hadron A composed of n valence quarks[13]

$$G_A(t) = b_A \mu_A \left(\frac{\chi_q}{sh\chi_q}\right)^n \approx \left(\frac{\ln |t| n^{-2} M_q^{-2}}{|t| n^{-2} M_q^{-2}}\right)^n . \tag{4}$$

Formula (4) may also be written in the power form

$$G_A(t) \approx \left(|t| n_A^{-2} M_q^{-2}\right)^{-n_{eff}(t)}$$

$$n_{eff}(t) \approx n - n\,\frac{\ln\left(\ln |t| n^{-2} M_q^{-2}\right)}{\ln |t| n^{-2} M_q^{-2}} . \tag{5}$$

The behaviour (4) can be found in the DMFQ by using the approach by Landshoff, Polkinghorne[17] (see ref.[14]), as well.

The DMFQ formulae are in good agreement with the data on proton and pion form factors[13,18,19] and on hadron-hadron interactions (see ref.[14]).

3. Determination of the Form of Structure Functions of eN -Scattering

Based on the DMFQ, in papers[18,19] the structure functions have been calculated for different hadrons near the elastic threshold. This calculation used the elastic form factor (4) and the Bloom-Gilman relation of local duality[20]

$$F_i^A(\omega_S^A) = \frac{t}{1-\omega_S^A} \cdot \frac{d}{dt}\left[G_i^A(t)\right]^2, \tag{6}$$

where $i = 1,2$; $A = p, \pi, d$, etc., $\left[G_i^A(t)\right]^2$ are quadratic combinations of elastic form factors of hadron A, or within an accuracy of δ-functions, are structure functions of elastic scattering. For instance, for nucleons N

$$G_1^N(t) = G_M^N(t)$$

$$G_2^N(t) = \frac{\left[G_E^N(t)\right]^2 - \tau\left[G_M^N(t)\right]^2}{1-\tau},$$

where $\tau = t/4M^2$; G_E^N and G_M^N are Sach electric and magnetic form factors. The Bloom-Gilman scaling variable in (6), ω_S^A, in terms of which the early scaling is observed experimentally, is given as follows

$$\omega_S^A = \omega - \frac{M_S^2}{t} \longrightarrow 1 - \frac{W_A^2}{t}, \tag{7}$$

where $W_A^2 = (W_{in}^A)^2 + M_S^2 - M_A^2$ and $W_{in}^A = M_A + m_\pi$ is the inelastic threshold of eA -scattering, M_S^2 is a fitting parameter. As is shown in paper[19], the results of[21] make it possible to introduce the Bloom-Gilman variable in the parton model. The structure functions near threshold obtained in[18,19] are of the form

$$F_1^N(\omega_S^N) = c_1\, \Phi(\chi_S^N, 3) \tag{8}$$

$$F_2^p(\omega_S^p) = c_2\, \frac{\mu_p^{-2} - \tau_p}{1-\tau_p}\, \Phi(\chi_S^p, 3) + c_2\, \frac{\lambda_p(\mu_p^{-2} - 1)}{4M^2(1-\tau_p)^2}\left[G(\chi_S^p, 3)\right]^2 \tag{9}$$

$$F_2^n(\omega_s^n) = c_2 \frac{\tau_n}{\tau_n - 1} \Phi(\chi_s^n, 3) - \frac{c_2 \lambda_n}{4M^2(1-\tau_n)^2} \left[G(\chi_s^n, 3)\right]^2, \tag{10}$$

where the function

$$\Phi(\chi_s^A, n) = 4 b_A^2 \mu_A^2 M_q^2 n \, W_A^{-2} (\chi_s^A \, ch \chi_s^A - sh \chi_s^A) \cdot \frac{(ch \chi_s^A - 1)^2}{sh^3 \chi_s^A} \cdot \left(\frac{\chi_s^A}{sh \chi_s^A}\right)^{2n-1} \tag{11}$$

is derived by differentiating the form factor G, and factors of $[G(\chi_s^A, 3)]^2$ by differentiating the factors $(\mu_p^{-2} \tau_p)/(1-\tau_p)$ or $\tau_n / \tau_n - 1$, and also

$$G(\chi_s^A, n) = b_A \mu_A \left(\frac{\chi_s^A}{sh \chi_s^A}\right)^n, \tag{12}$$

$$\tau_A = \frac{W_A^2}{4M^2(1-\omega_s^A)} \; ; \; \lambda_A = \frac{W_A^2}{(1-\omega_s^A)^2} . \tag{13}$$

The variable χ_s^A in the above-listed formulae is given by the expression

$$\chi_s^A \equiv \chi_s^A(\omega_s^A) = Ar \, ch \left(1 + \frac{W_A^2}{2n^2 M_q^2 (\omega_s^A - 1)}\right). \tag{14}$$

Formulae (8) to (14) contain three parameters: The quark mass M_q, proportionality coefficient c_i, and parameter M_s^2 characterizing the kinematical region, where the resonance contribution is importent. The analysis of experimental data on the proton structure functions near threshold $x \geqslant 0.75$ reveals good agreement of formulae (8) and (9) with experiment: χ^2 per one degree of freedom equals, resp., 1.04 and 1.08 [18,19]. The values of quark mass calculated by comparing (8) and (9) with experiment are consistent with the values determined earlier in fitting the data on proton [13,18] and pion [19] elastic form factors in the region of space-like transfer momenta (as well as on the pion form factor in the time-like region [19]). The average value of the quark mass is about 0.16 GeV.

In the range $x \leq 0.75$ far from the elastic threshold the

Table 1.

The results of fitting the data on the proton structure function $F_2(x)$ by formula (8).

Q^2_{min} (Gev^2)	Number of points	Number of dropped points	$\chi^2_{d.f.}$	M_q (GeV)	C_1	M_s^2 (GeV^2)
2	I74	3	I.I0	0.I2	6.73	I.56
2.5	I68	– 3	I.I0 0.93	0.I2	6.35	I.54
2	I63		0.96 0.84	0.I2	5.98	I.50

Table 2.

The results of fitting the data on the proton structure function $F_2(x)$ /23/ by formula (9).

x_{min}	Q^2_{min} (GeV^2)	Number of points	Number of dropped points	$\chi^2_{d.f.}$	M_q (GeV)	C_2	M_s^2 (GeV^2)
0.25		54	2	I.I6	0.II5	8.97	I.I8
0.3	–	47	I	0.84	0.II6	8.I6	I.I5
0.4	–	39	–	0.68	0.II6	8.47	I.I9
0.5	–	3I	–	0.75	0.II7	8.5I	I.20
–	3	5I	2	I.I2	0.II7	9.63	I.29
–	3.5	45	–	0.9I	0.II8	9.62	I.35
–	4	43	–	0.64	0.II8	9.29	I.3I
0.25	2	5I	–	I.I0	0.II5	9.I9	I.20
0.2	2.5	52	–	I.I7	0.II7	9.65	I.28

value of the quark mass may no longer be fixed to equal the value obtained from fitting the data on the elastic form factor. Then, formula (8) can well describe all the data[22,23] on the proton structure function, F_2. The results of our analysis of data[22,23] listed in Table I show excellent agreement of formula (8) with experiment. (Note the stability of the values of parameters calculated in the analysis of different groups of data, taken with the cut off over Q^2 by Q^2_{min}, i.e., data with $Q^2 \geqslant Q^2_{min}$). Table 2 shows the result of fitting the data on structure function F_2. As is seen, formula (9) also well describes the experimental data; the values of parameters for different choices of Q^2_{min} and x_{min} are stable again. So, formulae (8) and (9) well describe all the experimental data[22,23] for $Q^2 \geqslant Q^2_{min} = 2 \div 3\,(GeV)^2$ with the quark mass $M_q \sim 0.12\,GeV$. It should be noted that formulae (8) and (9) give the decrease of structure functions with increasing Q^2 at large x and (small) growth with increasing Q^2 at small x (see Fig.1).

Still better agreement with experiment can be achieved if in formulae (8), (9) one takes into account the Regge behaviour of structure functions as $x \to 0$. The theory of Regge poles allows the following expression for structure functions[24]

$$F_2(Q^2,\nu) \underset{x \to 0}{\sim} \sum_i A_i(Q^2)\,\nu^{\alpha_i - 1}, \tag{15}$$

where α_i is the intercept of the i-Regge trajectory, $A_i(Q^2)$ is an unknown residue at a pole. To determine the scaling behaviour of the structure functions, it is assumed that

$$A_i(Q^2) = a_i \left(\frac{2M}{Q^2}\right)^{\alpha_i - 1}, \quad a_i = const, \tag{16}$$

so that

$$F_2(Q^2,\nu) \underset{x \to 0}{\sim} \sum_i a_i\, \omega^{\alpha_i - 1}.$$

To take account of the deviation from scaling observed in the preasymptotical energy region, we put, instead of (16) that

$$A_i(Q^2) = a_i\left[\frac{2M}{Q^2}(1 + b_i)\right]^{\alpha_i - 1}, \tag{17}$$

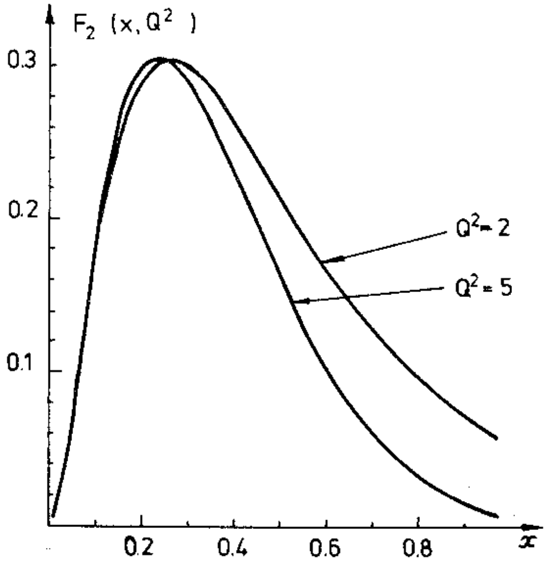


Fig. 1. The behaviour of proton structure function $F_2(x_s)$ defined by formula (9).

Table 3.

Results of fitting the data on proton $F_L(y)$ /22,23/ by formulae (20) and (8).

Q^2_{min} (GeV^2)	Number of points	Number of dropped points	$\chi^2_{d.f.}$	M_g (GeV)	C_L	M_s^2 (GeV^2)	α
–	I87	–	0.73	0.I7I	0.26	I.33	2.728
		I	0.64				
I.25	I83	–	0.69	0.I7	0.27	I.34	2.7I4
		I	0.6I				
2	I74	–	0.62	0.I69	0.30	I.37	2.63I
		I	0.54				
2.5	I68	–	0.6I	0.67	0.34	I.29	2.560
		I	0.53				

Table 4.

Results of fitting the data on proton structure function $F_2(x_s)$ /22,23/ by formulae (20) and (9).

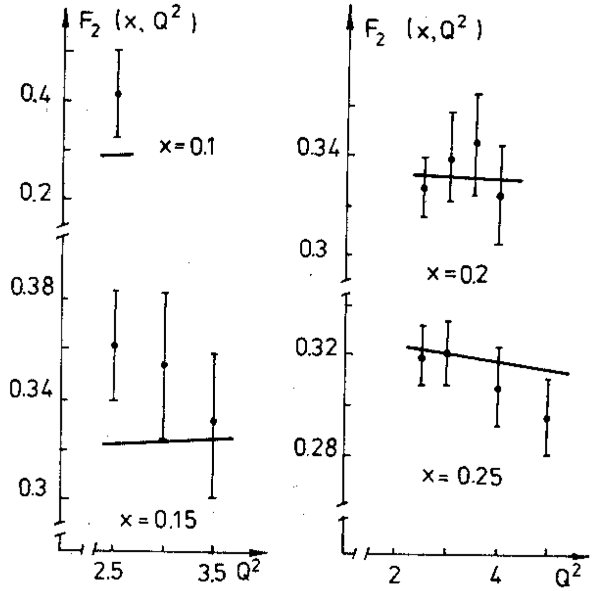
Q^2_{min} (GeV^2)	Number of points	Number of dropped points	$\chi^2_{d.f.}$	M_q (GeV)	C_2	M_s^2 (GeV^2)	α
I.25	7I	I	I.34	0.I5	0.7I	I.06	2.4
2	62	-	I.I7	0.I5	0.87	I.I7	2.32
2.5	56	-	I.00	0.I4	I.2I	I.20	2.I2
3	5I	-	0.9I	0.I4	I.54	I.20	I.98

Table 5.

Results of fitting the data on proton structure function $F_2(x_s)$/22,23/ by formulae (20) and (8) with the parameters M_q^2, M_s^2 and α fixed from the behaviour of F_2 (see table 4).

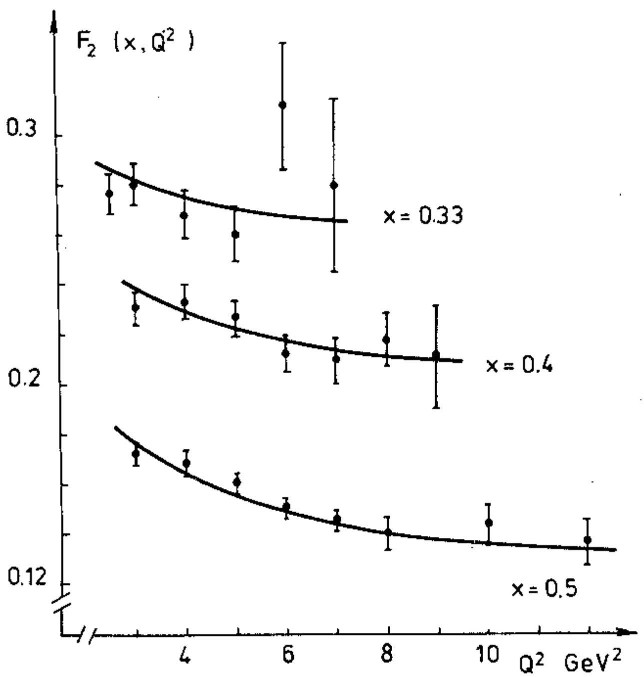
Q^2_{min} (GeV^2)	Number of points	$\chi^2_{d.f.}$	M_q (GeV)	C_2	M_s^2 (GeV^2)	α
2	I74	0.749	0.I5	0.52	I.I7	2.32
2.5	I68	0.772	0.I4	0.75	I.20	2.I2
3	I63	0.735	0.I4	0.95	I.20	I.98

a)

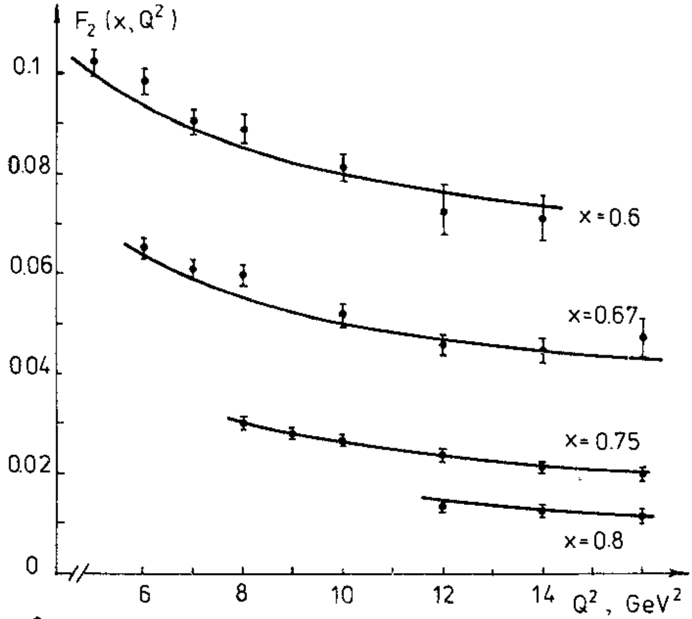


F_2 (x, Q^2)
0.4
0.2
x = 0.1
0.38
0.34
0.3
x = 0.15
2.5
3.5 Q^2
F_2 (x, Q^2)
0.34
0.3
x = 0.2
0.32
0.28
x = 0.25
2
4
Q^2

b)



F_2 (x, Q^2)
0.3
0.2
0.12
x = 0.33
x = 0.4
x = 0.5
4
6
8
10
Q^2 GeV^2



c)

Fig. 2. The behaviour of proton structure function $F_2(x_s)$. Data from /23/, the solid line corresponds to formulae (9) and (20). The fitted parameters are listed in table 4 (Q^2_{min} 2.5 GeV^2).

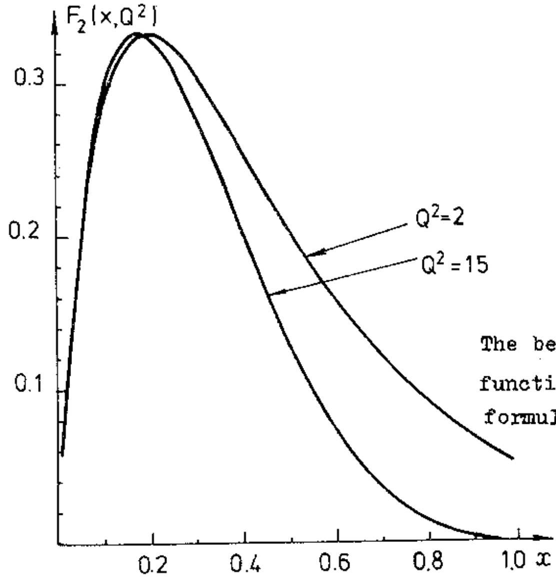


Fig. 3.

The behaviour of proton structure function $F_2(x_s)$ defined by formulae (9) and (20).

where b_i is a small additive correction to unity. Inserting (17) in (15) we get

$$F_2(Q^2,\nu) \underset{x\to 0}{\sim} \sum_i a_i \left[\omega + \frac{2Mb_i}{Q^2}\right]^{\alpha_i - 1}.$$

Considering $2M\nu b_i = M_i^2$ to be some phenomenological parameters we arrive at the description of structure functions in terms of the variable $\omega_i = \omega + M_i^2/Q^2$:

$$F_2(Q^2,\nu) \underset{x\to 0}{\sim} \sum_i a_i \omega_i^{\alpha_i - 1}. \tag{18}$$

As $x \to 0$ ($\omega_i \to \infty$) in formula (18) the term with the largest α_k dominates, and other terms may be neglected. Identifying M_k^2 with parameter M_s^2 entering into the Bloom-Gilman variable ω_s, we finally get

$$F_2(Q^2,\nu) \underset{x\to 0}{\sim} \omega_s^{\alpha - 1}. \tag{19}$$

Multiplying structure functions F_i^{thr} (8) to (10), calculated in the threshold region $x \approx 1$, by factor $\omega_s^{\alpha-1}$, we find

$$F_i(\omega_s) = F_i^{thr}(\omega_s)\cdot \omega_s^{\alpha - 1}. \tag{20}$$

We considered the leading α as a free parameter. The fit of experimental data /19,20/ on proton structure functions F_1 and F_2 by formula (20) with (8) to (10) taken into account is in good agreement with experiment (see Tables 3,4 and Fig.2). The value of parameters are also stable. Note that the average value of the quark mass is consistent with the average value $M_q \sim 0.16$ Gev obtained earlier in fitting F_1, F_2 in the threshold region and $G_p(t), G_n(t)$ /18,19/. The character of deviation from scaling remains unchanged (see Fig.2,3) after introducing the factor $\omega_s^{\alpha-1}$.

Formulae (20), (8) and (9) allow also good description for the data on both the proton structure functions with the same values of parameters M_q, M_s^2, and α. The results of comparison with experimental data on F_1 of formula (20)(with parameters M_q, M_s^2 and α fixed by the behaviour of function F_2 (Table 4)) are collected in Table 5.

Conclusion

Thus, on the basis of the simple quark model the formulae are found for structure functions of inelastic eN-scattering which well describe the experimental data with stable values of three or four parameters. The formulae obtained give the decrease of structure functions with increasing Q^2, transfer momentum squared, at large x and (small) growth with Q^2 at small x. In the region $x \leq 0.1$ a substantial contribution to the eN-scattering cross section should come also from the "sea" quark-antiquark pairs produced in the fragmentation of gluons. Within the DMFQ it is not difficult to take into account that contribution; this is a subject of a subsequent paper.

The authors are thankful to V.G.Kadyshevsky, I.A.Savin, V.A.Mescheryakov, V.G.Krivochisyn, S.P.Kuleshov and A.V.Sidorov for interest in the work and useful discussions.

References

1. Bjorken J.D. Phys.Rev., 1969, 179, p.1547.
2. Bogolubov N.N., Vladimirov V.S., Tavkhelidze A.N. Soviet Journal Theor.Math.Phys., 1972, 12, p.305.
3. Matveev V.A. et al. Sov.Journ.Theor.Math.Phys., 1973, 16, p.355.
   Zavialov B.I. Sov.Journ.Theor.Math.Phys., 1973, 17, p.179; 1974, 19, p.163.
   Kiselev A.V., Mestvirishvili M.A., Rochev V.E. Sov.Journ. Theor.Math.Phys., 1979, 39, p.35.
4. Gross D.J., Wilczek F. Phys.Rev., 1973, D8, p.3633; 1974, D9, p.980.
5. Georgi H., Politzer H.D. Phys.Rev., 1974, D9, p.416. Politzer H.D. Phys.Rev., 1974, 14C, p.129.
6. Feynman R. Photon Hadron Interactions, W.A.Benjamin Inc. New York, 1972;
   Bjorken J.D. Proc. of the Intern. School of Physics "Enrico Fermi", Course XLI, ed. by J.Steinberger. Academic Press Inc., New York, 1968.

7. Linkevich A.D., Skachkov N.B. JINR, E2-12213, Dubna, 1979.
Anderson H.L., Matis H.S., Myrianthopoulos L.C. Phys.Rev.Lett., 1978, 40, p.1061.
8. Yndurain F.J. Phys.Lett. 1978, 74, p. 68.
9. Gluck M., Reya E. Phys.Rev., 1976, D14, p. 3034.
Fox G.C. Nucl.Phys., 1977, B131, p. 108.
Buras A.J., Gaemers K.J.F. Nucl.Phys., 1978, B132, p. 249.
10. Hayashi R. Nagoya Univ. preprint DPNU - 29 -77, 1977.
11. Linkevich A.D., Skachkov N.B. JINR, E2-12893, Dubna, 1979.
12. Pashkov A.F., Skachkov N.B., Solovtsov I.L. JETP Letters 25 (1977) 452; JINR E2-10462, P2-10490, Dubna, 1977.
13. Skachkov N.B., Solovtsov I.L., JINR E2-10530, Dubna, 1977.
14. Pahkov A.F., Skachkov N,B., Solovtsov I.L. JINR P2-12003, Dubna, 1979.
15. Kawaguchi M., Sumi Y., Yokomi H. Progr.Theor.Phys., 1967, 38, p.1183; Phys.Rev., 1968, 168, p.1556;
Kawaguchi M., Yokomi H. Progr.Theor.Phys., 1977, 57, p.470.
16. Creutz M., Wang L. Preprint BNL, 1974.
17. Landshoff P.V., Polkinghorne I.C., Short R.D. Nucl.Phys., 1971, B28, p.225;
Landshoff P.V., Polkinghorne I.C. Nucl.Phys., 1973, B53, p.473;
Landshoff P.V. TH-2157-CERN, 1976.
18. Pashkov A.F., Skachkov N.B., Solovtsov I.L. JINR P2-11211, Dubna, 1978.
19. Linkevich A.D., Skachkov N.B. JINR, E2-12561, Dubna, 1979.
20. Bloom E., Gilman F. Phys.Rev.Lett., 1970, 25, p.1140;
Phys.Rev., 1971, D4, p.2001.
21. Hwa R.C., Matsuda S., Roberts R.G. Preprint CERN, TH 2456, 1978.
22. Atwood W.B. SLAC Report 185, 1975.
Atwood W.B. et al. SLAC-PUB-1758, 1976.
23. Riordan E.M. et al. SLAC-PUB-1634, 1975.
24. Abarfanel H.D.I., Goldberger M.L. Trieman S.B. Phys.Rev.Lett., 1969, 22, p.500.
Harari H. Phys.Rev.Lett., 1969, 22, p.1078.

Received by Publishing Department
on July 10 1979.