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Об однозначности ренормгрупповых вычислений в квантовой хромодинамике

Проведен подробный анализ сокращения неоднозначностей, обусловленных произволом в выборе ренормировочной процедуры, при ренормгрупповых вычислениях физических эффектов в квантовой хромодинамике.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Unambiguity of Renormalization Group Calculations in QCD

A detailed analysis of the cancellations of renormalization-scheme dependences is presented for the renormalization group calculations of physical quantities in QCD.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1979

1. An application of renormalization group (RG) methods to asymptotically free models such as QCD results in an "improved" perturbation theory for Green's functions and physical amplitudes. This new expansion is justified at large momenta, where its parameter, the effective charge $\bar{g}$, appears to be small. Up to now the two-loop calculations in QCD have been carried out for several deep-inelastic processes/1-3/. These results are consistent with the scaling violations presently observed/2,4/.

It is well known that at the two-loop level a renormalization -prescription dependence appears in some RG quantities. However, it must vanish in resulting expressions for observables. The machinery of these cancellations has been concerned in papers/1,2, 5,6/. Though refs./5,6/, when combined, seem to cover the whole subject, it may be worthwhile making an attempt to describe it in a more complete and compact way. This is a goal of the present paper which represents an extended English translation of §6 from our review written with D.V.Shirkov and submitted to Usp. Fiz. Nauk.

2. First we recall some basic formulas concerning the renormalization-scheme dependence of Green's and RG functions. We shall consider a massless asymptotically free theory with one coupling constant g . Green's functions of different renormalization

schemes, Γ and $\widetilde{\Gamma}$, are related by

$$\Gamma\left(\frac{Q^2}{\mu^2}, g\right) = Z\left(\frac{\tilde{\mu}^2}{\mu^2}, g\right)\widetilde{\Gamma}\left(\frac{Q^2}{\tilde{\mu}^2}, \tilde{g}\left(\frac{\tilde{\mu}^2}{\mu^2}, g\right)\right). \qquad (1)$$

Let $\tilde{\mu}$ be equal to μ. Then, introducing the notation

$$\frac{Q^2}{\mu^2} = t, \quad \tilde{g}(1, g) = q(g), \quad Z(1, g) = \rho(g),$$

we arrive at

$$\Gamma(t, g) = \rho(g)\widetilde{\Gamma}(t, q(g)). \qquad (2)$$

Differentiating (2) with respect to t and using the RG equation

$$\left(t\frac{\partial}{\partial t} - \beta(g)\frac{\partial}{\partial g} + \gamma(g)\right)\Gamma(t, g) = 0 \qquad (3)$$

and an analogous one for $\widetilde{\Gamma}$ (with β and γ replaced by $\tilde{\beta}$ and $\tilde{\gamma}$) results in the relations between the RG functions of different schemes /7,8,5/:

$$\tilde{\beta}(q(g)) = \beta(g)\frac{dq(g)}{dg}, \qquad (4)$$

$$\tilde{\gamma}(q(g)) = \gamma(g) - \beta(g)\frac{d\ln\rho(g)}{dg}. \qquad (5)$$

Let us also define an auxiliary function $\Psi(g)$ as an indefinite integral

$$\Psi(g) = \int^{g}\frac{du}{\beta(u)}, \qquad (6)$$

i.e.,within an arbitrary additive constant. From (4) one obtains

$$\tilde{\Psi}(q(g)) = \Psi(g) + const. \qquad (7)$$

3. All physical quantities are known to be renormalization-scheme independent. This holds for Green's functions as well up to the overall factor $\rho(g)$, that in the case of observables is cancelled by means of additional renormalization of external lines. Actually, with $\rho(g)$ omitted we see from (2), that Γ differs from $\tilde{\Gamma}$ by the change $g \to q(g)$ only. But this is just a change in coupling constant which has to accompany a transition from one scheme to another according to (1). As a result, the momentum dependence both of Γ and $\tilde{\Gamma}$ proves to be the same.

Let us be more specific. Write down $\Gamma(t,g)$ as

$$\Gamma(t,g) = \Gamma(1, \bar{g}(t,g)) \exp\left[-\int_g^{\bar{g}(t,g)} du \, \frac{\gamma(u)}{\beta(u)}\right], \quad (8)$$

with $\bar{g}(t,g)$ being an effective charge, normalized by the condition $\bar{g}(1,g) = g$. $\bar{g}(t,g)$ can be found from

$$\ln t = \int_g^{\bar{g}(t,g)} \frac{du}{\beta(u)} = \Psi(\bar{g}(t,g)) - \Psi(g). \quad (9)$$

We shall use the quantity

$$L = \ln t + \Psi(g) = \Psi(\bar{g}(t,g)) \quad (10)$$

as a new expansion parameter. The relationship (10) determines L only within a constant. To fix this constant, one must determine $\Psi(g)$ unambiguously.

To emphasize a logarithmic momentum dependence of L , another notation for it is often used, $L = \ln Q^2/\Lambda^2$. When doing this, the quantity $\Lambda = \Lambda(\mu^2, g)$, absorbing the whole dependence on the renormalization parameter μ and the coupling constant g , remains the only free parameter of the theory. An above-mentioned additive arbitrariness in L means in terms of Λ an arbitrariness in fixing the momentum scale.

With $\Psi(g)$ fixed somehow and $\bar{g}$ expressed in L with the use of (10) one can insert $\bar{g}(L)$ into (8) and obtain

$$\Gamma(t,g) = f(g)\,\Phi(L), \quad (11)$$

with $f(g)$ being a contribution of the lower limit of integral in (8), and $\Phi(L)$ allowing an expansion

$$\Phi(L) = L^{-\alpha}\left(1 + \sum_{k=1}^{\infty}\sum_{m=0}^{k} \alpha_{km} \frac{\ln^m L}{L^k}\right). \quad (12)$$

Most of the coefficients α_{km} depend on the way we have fixed a definition of $\Psi(g)$. So, they depend on the choice of L and vary when L is shifted by a constant in eq. (12), $L \to L +$ const.

Let us now change the renormalization scheme. According to (2) and (7),

$$\Phi(L) = \Phi(\ln t + \Psi(g)) = \tilde{\Phi}(\ln t + \tilde{\Psi}(q(g))) = \tilde{\Phi}(L + const) \quad (13)$$

The only result of changing a scheme is the replacement $\alpha_{km} \to \tilde{\alpha}_{km}$ which is exactly equal to the shift of L. This does not exceed the arbitrariness in α_{km} inherent in any given renormalization scheme due to an ambiguity in the definition of L. To co-ordinate the calculations of α_{km} in different schemes, one must shift L by a constant in one of them. The value of this constant can be found explicitly, provided the definitions of $\Psi(g)$ and $\tilde{\Psi}(g)$ are specified and function $q(g)$ is known in the one-loop approximation.

4. To illustrate the general relations given above, let us expand all relevant functions in power series in g to two- loop order (subscripts denote a number of loops):

$$\beta(g) = -\beta_1 g^2 + \beta_2 g^3 + O(g^4), \quad \gamma(g) = \gamma_1 g + \gamma_2 g^2 + O(g^3),$$
$$q(g) = g + q_1 g^2 + O(g^3), \quad (14)$$
$$\rho(g) = 1 + \rho_1 g + O(g^2), \quad \Gamma(1, g) = 1 + A_1 g + O(g^2).$$

As it follows from (2), (4) and (5), β_1, β_2, and γ_1 are scheme-independent,

$$\tilde{\beta}_1 = \beta_1 , \quad \tilde{\beta}_2 = \beta_2 , \quad \tilde{\gamma}_1 = \gamma_1 .$$

In contrast,

$$\tilde{\gamma}_2 = \gamma_2 + \beta_1 p_1 - \gamma_1 q_1 , \tag{15}$$

$$\tilde{A}_1 = A_1 - p_1 . \tag{16}$$

It should be noted that in papers[1,2] the $\gamma_1 q_1$ term in (15) has not been taken into account. We write down also an expansion for $\Psi(g)$,

$$\Psi(g) = \frac{1}{\beta_1 g} - \frac{\beta_2}{\beta_1^2} \ln g + \delta + O(g) , \tag{17}$$

where δ is an arbitrary constant. A specific choice for δ means fixing $\Psi(g)$ unambiguously. Now one can find from (7) and (17) the value of Δ , which describes the transition from one renormalization scheme to another according to (13), $\Phi(L) = \tilde{\Phi}(L+\Delta)$:

$$\Delta = \tilde{\delta} - \delta - \frac{q_1}{\beta_1} . \tag{18}$$

If $\tilde{\delta} = \delta$ we obtain[6] $\Delta = -\frac{q_1}{\beta_1}$. Using (10) and (17) gives

$$\bar{g}(L) = \frac{1}{\beta_1 L} + \frac{\beta_2}{\beta_1^3} \frac{\ln L}{L^2} + \frac{\beta_2 \ln \beta_1 + \beta_1^2 \delta}{\beta_1^3 L^2} + O\left(\frac{\ln^2 L}{L^3}\right). \tag{19}$$

Usually one excludes the $1/L^2$ term from (19) by setting $\delta = -\beta_2 \ln \beta_1 / \beta_1^2$. It is worth mentioning that such a choice fixes a definition of L completely, this fixation being different in different schemes.

However, we let δ be arbitrary and write down an expansion for Γ ,

$$\Gamma(t,g)=f(g)L^{-\alpha}\left(1+\frac{\alpha_{11}\ln L}{L}+\frac{\alpha_{10}}{L}+O\left(\frac{\ln^2 L}{L^2}\right)\right), \qquad (20)$$

where $\alpha=\frac{\gamma_1}{\beta_1}$ and $\alpha_{11}=\frac{\gamma_1\beta_2}{\beta_1^3}$ are scheme-independent quantities, whereas

$$\alpha_{10}=\frac{\beta_2\gamma_1(1+\ln\beta_1)}{\beta_1^3}+\frac{\gamma_2}{\beta_1^2}+\frac{A_1}{\beta_1}+\frac{\gamma_1\delta}{\beta_1} \qquad (21)$$

does depend upon the choice of scheme and varies as follows:

$$\tilde{\alpha}_{10}=\alpha_{10}+\frac{\gamma_1}{\beta_1}\Delta . \qquad (22)$$

This can be entirely cancelled by the shift $L\to L+\Delta$ with Δ given by (18).

We stress once more that the coefficient α_{10} is a nonunique quantity even within any given renormalization scheme, due to a freedom in defining δ. Therefore eq. (22) seems to show not a scheme dependence of α_{10} or some other coefficients but merely an arbitrariness in choosing the momentum scale.

One can completely eliminate all ambiguities in α_{KM} from Green's functions and amplitudes, fixing α_{10} for one of them, for instance, rendering α_{10} equal to zero in the expression for certain amplitude by the shift $L\to L+\Delta$. With one α_{10} fixed in such a way, all coefficients of the expansions in $1/L$ and $\ln L$ for any amplitude become scheme-independent. The corresponding values of Δ are to be found from (18) or (22).

5. The truncation of the series (12) at some finite order destroys an exact equivalence between the change $\alpha_{KM}\to\tilde{\alpha}_{KM}$ and the shift $L\to L-\Delta$. The truncated series obeys eq. (13) only approximately. In the case (20) this means that different values of α_{10} lead to different L-dependences of Γ, and this difference cannot be entirely absorbed in the additive redefinition of L. In other words, various numerical values of α_{10} appear to be nonequivalent for comparison of theoretical predictions with the data. Truncating the series (12) we obtain,

in addition to Λ , new free parameter α_{10}. There have been some attempts of adjusting this parameter in order to diminish as much as possible the discrepancies between theory and experiment[9]. It should be noted in this respect that α_{10} must be varied in a consistent manner in all amplitudes under consideration. Also, the arbitrariness in α_{10} being not relevant to that in renormalization prescriptions, the advantages of "physical" subtraction scheme over "unphysical" dimensional one, discussed in[6], seem to be no more than a fortuity.

6. In conclusion, several "rules" for RG calculations in QCD are to be formulated.

1). All contributions to an amplitude of a given process must be calculated by the same renormalization scheme.

2). The whole scheme dependence of amplitudes reduces to the redefinition $L \to L + \Delta$ $\left(L = \ln Q^2/\Lambda^2\right)$. Hence, computing other effects in other schemes must be accompanied by such a shift of L . The specific value of Δ is to be found from (18).

3). In practical calculations a truncation of power series in $1/L$ and $\ln L$ is unavoidable. Therefore the coefficient α_{10} becomes an extra free parameter, which may be simultaneously (and consistently) varied in the expressions for all amplitudes. One can put $\alpha_{10} = 0$ for a given process, or try to diminish the contributions from the α_{10}/L terms in all amplitudes at hand, striving for optimum position to compare theory and experiment.

4). The points 1)-3) present the necessary conditions for comparing with each other the values of Λ obtained from different experiments. It should be noted that various choices of α_{10} lead to various procedures of obtaining Λ from a fit to the data and must result in various numerical values of $\Lambda = \Lambda(\alpha_{10})$.

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