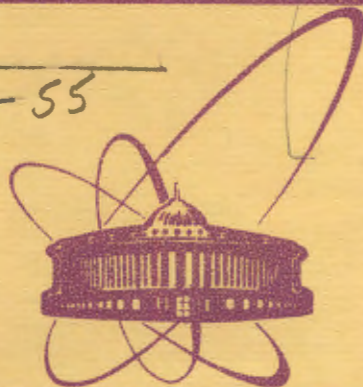


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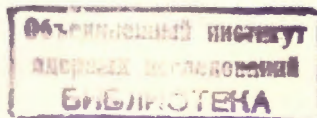
ELEMENTARY PARTICLE TREATMENT  
OF THE RADIATIVE MUON CAPTURE  
II. BOSON-LIKE TARGETS  $^{12}\text{C}$  AND  $^{16}\text{O}$

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**ELEMENTARY PARTICLE TREATMENT  
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Гмитро М., Овчинникова А.А.

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Радиационный  $\mu^-$ -захват в подходе "ядро как элементарная частица". II. Мишени с целым спином  $^{12}\text{C}$  и  $^{16}\text{O}$ .

Построены амплитуды радиационного  $\mu^-$ -захвата для процессов  $^{12}\text{C}(\mu, \gamma)^{12}\text{B}$ ,  $^{16}\text{O}(\mu, \gamma)^{16}\text{N}$ . Показано, что в рамках подхода к ядру как элементарной частице амплитуда процесса, построенная по теории возмущений с минимальной заменой, может не удовлетворять условиям СВС и РСАС, и даже сохранению электромагнитного тока. Предложенный метод позволяет избежать этих недостатков. Полученные амплитуды процессов оказываются, как и следовало ожидать, калибровочно-инвариантными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединяного института ядерных исследований. Дубна 1979

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Elementary Particle Treatment of the Radiative Muon Capture. II. Boson-Like Targets  $^{12}\text{C}$  and  $^{16}\text{O}$

Radiative-muon-capture amplitudes have been constructed for the  $^{12}\text{C}(0^+) \rightarrow ^{12}\text{B}(1^+)$  and  $^{16}\text{O}(0^+) \rightarrow ^{16}\text{N}(2^-)$  transitions using the assumptions about conservation of the electromagnetic and weak hadronic currents supplemented by a dynamical hypothesis.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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## 1. Introduction

In the present paper we shall construct the radiative-muon-capture (RMC) amplitudes for the  $^{12}\text{C}(\mu, \gamma)^{12}\text{B}(1^+)$  and  $^{16}\text{O}(\mu, \gamma)^{16}\text{N}(2^-)$  transitions. These are the first applications of the procedure suggested in our earlier paper /1/ (referred to as I). Relying fully on the notation introduced in I we give in Sec. 2 a condensed account of the procedure to be used in Sec. 3 and 4, where we present with necessary details the derivation for the two above-mentioned transitions. A short summary is given in Sec. 5.

## 2. Method

The hadron-radiating part  $T^{(h)}$  of the RMC amplitude contains the tensors  $V_{\mu\lambda}(k, q, Q)$  and  $A_{\mu\lambda}(k, q, Q)$  connected with the vector and axial-vector weak currents, respectively. The tensors  $V_{\mu\lambda}$  and  $A_{\mu\lambda}$  should be expanded in terms of the independent Lorentz covariants. Then, imposing the current-conservation conditions (CEC-charge, CVC, PCAC), one obtains the systems of constraint equations for the radiative form factors accompanying the kinematical covariants in the expansions for  $V_{\mu\lambda}$  and  $A_{\mu\lambda}$ . Now assuming a plausible explicit form (see Sec. 3 of I) of these solutions the equations may be solved if a few form factors are taken from the perturbation theory (PT). In this way the radiative-muon-capture amplitude is expressed in terms of (nonradiative) weak form factors.



### 3. The $^{12}C_{g.v.}(\mu^- \gamma \nu) ^{12}B_{g.v.}(1^+)$ reaction

In our treatment we have adopted the "elementary-particle" approach to the nuclear states in question. They are for our purpose fully characterized by the structure of the relativistically invariant equations corresponding to their spins and parities. First we consider the radiative transition  $O^+(^{12}C_{g.v.}) \rightarrow 1^+(^{12}B_{g.v.})$ . The weak currents needed in analysis of this process can be written following the construction due to Delorme /2/ as

$$\langle ^{12}B(p', \xi) | V_\lambda(0) | ^{12}C(p, i) \rangle = -\sqrt{2} \epsilon_{\lambda\alpha\beta\gamma} \xi_\alpha^* \frac{q_\beta}{2m_p} \frac{Q_\gamma}{2M} F_M(q^2), \quad (1)$$

$$\langle ^{12}B(p', \xi) | A_\lambda(0) | ^{12}C(p, i) \rangle = \sqrt{2} \left[ \xi_\lambda^* F_A(q^2) + \frac{q_\lambda q_\mu \xi_\mu^*}{m_p^2} F_P(q^2) - \frac{Q_\lambda}{2M} \frac{q_\mu \xi_\mu^*}{2m_p} F_F(q^2) \right], \quad (2)$$

where  $\xi_\alpha = (\vec{\xi}, i\xi_0)$  is the polarization 4-vector of the spin-one nuclear final state ( $^{12}B$ ) and  $\epsilon_{\alpha\beta\gamma\delta}$  is fully antisymmetric unit tensor ( $\epsilon_{0123} = +1$ ). The nuclear form factors  $F_M, F_A, F_P$  and  $F_F$  should be determined from the other experiments.

#### 3.1. Vector part of the RMC amplitude

A straightforward application of the rules discussed in Sec. 5 of I gives

$$V_{\mu\lambda} = \epsilon_{\mu\nu\rho\gamma} \xi_\alpha^* \frac{k_\rho Q_\nu}{m_p^2} (v_{11}^\alpha k_\lambda + v_{12}^\alpha Q_\lambda) + \epsilon_{\lambda\alpha\beta\gamma} \xi_\alpha^* \left[ \frac{Q_\beta q_\gamma}{m_p^2} (v_{11}^\beta k_\mu + v_{12}^\beta Q_\mu + v_{13}^\beta q_\mu) + \frac{q_\beta k_\gamma}{m_p^2} (v_{21}^\beta k_\mu + v_{22}^\beta Q_\mu + v_{23}^\beta q_\mu) + \frac{k_\rho Q_\nu}{m_p^2} (v_{31}^\beta k_\mu + v_{32}^\beta Q_\mu + v_{33}^\beta q_\mu) \right] \quad (3)$$

$$+ \epsilon_{\mu\alpha\beta\gamma} \frac{k_\rho Q_\nu q_\nu}{m_p^2} \left[ v_{00}^c \xi_\lambda^* + \frac{k_\mu \xi_\mu^*}{m_p^2} (v_{11}^c Q_\lambda + v_{12}^c q_\lambda) + \frac{q_\mu \xi_\mu^*}{m_p^2} (v_{21}^c Q_\lambda + v_{22}^c q_\lambda) \right]$$

$$+ \epsilon_{\lambda\alpha\beta\gamma} \frac{k_\rho Q_\nu q_\nu}{m_p^2} \cdot \frac{k_\mu}{m_p^2} (v_{11}^d k_\lambda^* \xi_\mu^* + v_{12}^d q_\lambda^* \xi_\mu^*) +$$

$$+ \epsilon_{\mu\lambda\alpha\beta} \left[ \frac{\xi_\alpha^*}{m_p} (v_{01}^e k_\beta + v_{02}^e Q_\beta + v_{03}^e q_\beta) + \frac{Q_\alpha q_\beta}{m_p^2} (v_{11}^e k_\lambda^* \xi_\mu^* + v_{12}^e q_\lambda^* \xi_\mu^*) \right]$$

$$+ \epsilon_{\alpha\beta\gamma\delta} \xi_\alpha^* \frac{k_\rho q_\nu Q_\delta}{m_p^2} \cdot \frac{k_\mu}{m_p^2} (v_{11}^f k_\lambda + v_{12}^f Q_\lambda).$$

As suggested above, we should substitute the general form (3) into constraint eqs. (12) and (14) of I to obtain the system of equations for the form factors  $v_{ij}^x$ . It can be seen that the system splits into two slightly connected subsystems. For the further reference we call them group A and B, respectively.

A:

$$v_{12}^e k Q + v_{13}^e k q = m_p^4 \cdot \frac{1}{2M} \frac{1}{2m_p} \sqrt{2} F_M(q^2) \quad (4)$$

$$v_{32}^e k Q + v_{33}^e k q + m_p^2 v_{02}^e = 0 \quad (5)$$

$$(v_{12}^e + v_{32}^e) k Q + (v_{13}^e + v_{33}^e) k q + m_p^2 v_{02}^e = m_p^4 \cdot \frac{1}{2M} \frac{1}{2m_p} \sqrt{2} F_M(q^2) \quad (6)$$

$$v_{12}^a k Q + v_{11}^a k q + (v_{11}^e + v_{31}^e) k q + (v_{13}^e + v_{33}^e) q^2 + (v_{11}^f k q + v_{12}^f k Q) \cdot k q - m_p^2 v_{02}^e = 0 \quad (7)$$

B:

$$v_{22}^b k Q + v_{23}^b k q - m_p^2 v_{03}^e = 0 \quad (8)$$

$$(v_{11}^e + v_{31}^e) k Q + (v_{12}^e + v_{32}^e) Q^2 + (v_{11}^f k q + v_{12}^f k Q) k Q + m_p^2 (v_{03}^e - v_{01}^e) = 0 \quad (9)$$

$$v_{11}^c k Q + v_{12}^c (k+q) q + m_p^2 (v_{11}^e + v_{00}^e) - m_p^2 (v_{11}^e + v_{31}^e) = 0 \quad (10)$$

$$v_{21}^c k Q + v_{22}^c (k+q) q + m_p^2 (v_{12}^e + v_{00}^e) - m_p^2 (v_{12}^e + v_{32}^e - v_{13}^e - v_{33}^e) = 0 \quad (11)$$

$$v_{11}^e = 0 \quad (12)$$

$$v_{12}^e = 0 \quad (13)$$

In Appendix A we describe the way of solution of these systems and display the final expressions for the radiative form factors  $v_{ij}^x$ . Substituting them into eq. (3) one obtains

$$\begin{aligned}
& \frac{1}{m_p} \epsilon_\mu^* V_{\mu\lambda} = \\
& = \sqrt{2} \left[ -\epsilon_{\lambda\alpha\beta\gamma} \xi_\alpha^* \frac{(q+k)_\beta}{2m_p} \frac{(Q-k)_\gamma}{2M} F_M(q^2) e_i \frac{P_i \epsilon^*}{P^i k} \right. \\
& \quad + \epsilon_{\lambda\alpha\beta\gamma} \xi_\alpha^* \frac{(q+k)_\beta}{2m_p} \frac{(Q+k)_\gamma}{2M} F_M(q^2) e_f \frac{P_f \epsilon^*}{P^f k} \\
& \quad + \epsilon_{\mu\alpha\beta\gamma} \epsilon_\mu^* \xi_\alpha^* \frac{k_\beta}{2m_p} \frac{Q_\gamma}{2M} F_M(q^2) \left( \frac{e_i(Q_\lambda - k_\lambda)}{2P^i k} - \frac{e_f(Q_\lambda + k_\lambda)}{2P^f k} \right) \\
& \quad + \epsilon_{\mu\lambda\alpha\beta} \epsilon_\mu^* \xi_\alpha^* \frac{Q_\beta}{2M} \frac{1}{2m_p} F_M(q^2) \\
& \quad - \epsilon_{\mu\lambda\alpha\beta} \epsilon_\mu^* \xi_\alpha^* \frac{k_\beta}{2m_p} \frac{1}{2M} F_M(q^2) (e_i + e_f) \\
& \quad \left. + \epsilon_{\mu\lambda\alpha\beta} \epsilon_\mu^* \xi_\alpha^* \frac{q_\beta}{2m_p} \frac{1}{2M} F_M(q^2) (e_i + e_f) \right] \quad (14)
\end{aligned}$$

In I we have mentioned that several equivalent sets of algebraically independent kinematical covariants may be constructed to fix the general form of the tensor  $V_{\mu\lambda}$  ( $A_{\mu\lambda}$ ) in unnatural (natural) parity transitions on integer-spin targets. The practical procedure to find the univocal choice is connected with the analysis of the corresponding systems of constraint equations. From the pragmatic point of view which is summarized in the "rules" given in Sec. 5 of I one always can avoid the ambiguities. In the present case the expansion (3) of  $V_{\mu\lambda}$  is the only one which provides the explicit solution which is unique within the framework of LH and minimal use of the perturbation theory.

### 3.2. Axial-vector part of the RMC amplitude

The construction of tensor  $A_{\mu\lambda}$  starts from the general form which is given by eq. (10) of Hwang and Primakoff <sup>13)</sup> (referred to as HP2 in what follows). We repeat here neither this expression nor the algebraic constraint systems derived in HP2 from the CEC and PCAC conditions. As for the solution, however, our procedure is described in Appendix A; it is based on the linearity hypothesis (LH) for the form factors entering into the expression for  $A_{\mu\lambda}$  only. LH must not be applied to the form factors of  $D_{\mu\nu}$ . We have

$$\begin{aligned}
& \frac{1}{m_p} \epsilon_\mu^* A_{\mu\lambda} = \\
& = \sqrt{2} \left[ \xi_\lambda^* F_A(q^2) + (q+k)_\lambda \frac{(q+k)_\lambda^*}{m_\pi^2} F_P(q^2) - \frac{(Q-k)_\lambda}{2M} \frac{(q+k)_\lambda^*}{2m_p} F_E(q^2) \right] e_i \frac{P_i \epsilon^*}{P^i k} \\
& - \sqrt{2} \left[ \xi_\lambda^* F_A(q^2) + (q+k)_\lambda \frac{(q+k)_\lambda^*}{m_\pi^2} F_P(q^2) - \frac{(Q+k)_\lambda}{2M} \frac{(q+k)_\lambda^*}{2m_p} F_E(q^2) \right] e_f \frac{P_f \epsilon^*}{P^f k} \quad (15) \\
& - \sqrt{2} \epsilon_\lambda^* \left[ \frac{q_\lambda^*}{m_\pi^2} F_P(q^2) + \frac{q_\lambda^*}{2m_p} \frac{1}{2M} F_E(q^2) (e_i + e_f) \right] \\
& - \sqrt{2} \epsilon_\lambda^* \xi^* \left[ \frac{(q_\lambda + k_\lambda)}{m_\pi^2} F_P(q^2) + \frac{k_\lambda}{2m_p} \frac{1}{2M} F_E(q^2) (e_i + e_f) - \frac{Q_\lambda}{2M} \frac{1}{2m_p} F_E(q^2) \right]
\end{aligned}$$

The form of the solution for  $A_{\mu\lambda}$  which we have obtained is the same as one would expect from the "perturbation theory + minimal electromagnetic coupling" treatment. The form factors are taken, however, at momentum  $q^2$  rather than at  $(q+k)^2$ . It should be interpreted <sup>13,4)</sup> as an effect of higher-order processes. One can easily check by substituting  $k_\mu$  for  $\epsilon_\mu^*$  that the radiative amplitude as given by eqs. (14), (15) and eq. (4) of HP2 is gauge invariant, this last property simply follows from the CEC constraint.

### 4. The $^{16}O_{g.s.}(\mu^-, \nu) ^{16}N_{g.s.}(2^-)$ reaction

The weak currents corresponding to this  $\Delta J=2$  parity-changing transition can be again constructed by the method of Delorme <sup>12)</sup>. They are

$$\langle ^{16}N(p', \xi_{\alpha p}^{(m)}) | V_A(0) | ^{16}O(p^i) \rangle = -\epsilon_{\lambda\alpha\beta\gamma} \xi_{\alpha p}^{(m)*} \frac{Q_\beta}{2M} \frac{q_\gamma}{2m_p} \frac{q_\gamma}{2m_p} F_M(q^2), \quad (16)$$

$$\begin{aligned}
\langle ^{16}N(p', \xi_{\alpha p}^{(m)}) | A_\lambda(0) | ^{16}O(p^i) \rangle &= \xi_{\alpha p}^{(m)*} \left[ \frac{Q_\lambda F_T(q^2)}{2M \cdot 2m_p} - \frac{q_\lambda F_P(q^2)}{m_\pi^2} \right] \frac{q_\alpha q_\beta}{2m_p} \\
&\quad - \xi_{\lambda\alpha}^{(m)*} \frac{q_\alpha}{2m_p} F_A(q^2), \quad (17)
\end{aligned}$$

where  $\xi_{\alpha p}^{(m)}$  is a spin-two-type polarization 4-tensor representable in terms of spin-one-type polarization 4-vector  $\xi_\mu$ , as,

$$\xi_{\alpha p}^{(m)} = \sum [ \begin{smallmatrix} 1 & 1 & 2 \\ m_1 & m_2 & m \end{smallmatrix} ] \xi_{\alpha}^{(m_1)} \xi_{p}^{(m_2)}$$



with the symbol  $[\dots]$  for  $SU(2)$  Clebsch-Gordan coefficients. The spin-two wave functions  $\{\psi_{\alpha\beta}^{(m)}\}$  ( $m = \pm 2, \pm 1, 0$ ) obey the equation

$$(p^2 + M^2) \{\psi_{\alpha\beta}^{(m)}(p) = 0$$

and supplementary conditions of Barita and Schwinger

$$\{\psi_{\mu\alpha}^{(m)} p_\alpha = 0, \{\psi_{\alpha\beta}^{(m)} p_\beta = 0, \{\psi_{\alpha\beta}^{(m)} = \{\psi_{\beta\alpha}^{(m)}, \{\psi_{\alpha\alpha}^{(m)} = 0.$$

A straightforward application of the ad hoc rules suggested in Sec. 5 of I allows one again to fix the general form of the tensors  $V_{\mu\lambda}, A_{\mu\lambda}$  and  $D_{\mu}$ .

$$V_{\mu\lambda} =$$

$$\begin{aligned} &= \epsilon_{\lambda\alpha\beta\gamma} \frac{\xi_{\alpha\beta}^*}{m_p^2} \left[ \frac{k_\mu k_\rho}{m_p^2} (v_{11}^a k_\rho q_\delta + v_{12}^a q_\rho q_\delta + v_{13}^a q_\rho k_\delta) \right. \\ &+ \delta_{\beta\mu} k_\gamma q_\rho \cdot v_{20}^a + \frac{k_\mu q_\rho}{m_p^2} (v_{21}^a k_\rho q_\delta + v_{22}^a q_\rho q_\delta + v_{23}^a q_\rho k_\delta) \\ &+ \frac{q_\mu k_\rho}{m_p^2} (v_{31}^a k_\rho q_\delta + v_{32}^a q_\rho q_\delta + v_{33}^a q_\rho k_\delta) \\ &+ \delta_{\beta\mu} q_\gamma q_\rho \cdot v_{40}^a + \frac{q_\mu q_\rho}{m_p^2} (v_{41}^a k_\rho q_\delta + v_{42}^a q_\rho q_\delta + v_{43}^a q_\rho k_\delta) \\ &+ \frac{q_\mu k_\rho}{m_p^2} (v_{51}^a k_\rho q_\delta + v_{52}^a q_\rho q_\delta + v_{53}^a q_\rho k_\delta) \\ &+ \delta_{\beta\mu} q_\gamma q_\rho \cdot v_{60}^a + \frac{q_\mu q_\rho}{m_p^2} (v_{61}^a k_\rho q_\delta + v_{62}^a q_\rho q_\delta + v_{63}^a q_\rho k_\delta) \\ &+ \delta_{\beta\mu} p (v_{01}^a k_\rho q_\delta + v_{02}^a q_\rho q_\delta + v_{03}^a q_\rho k_\delta) \left. \right] \\ &+ \epsilon_{\mu\lambda\beta\gamma} \xi_{\alpha\beta}^* \frac{k_\alpha k_\rho}{m_p^2} \frac{q_\rho k_\lambda}{m_p^2} v_{00}^c + \epsilon_{\mu\lambda\beta\gamma} \xi_{\alpha\beta}^* q_\beta \frac{q_\rho k_\lambda}{m_p^2} (v_{01}^c k_\lambda + v_{12}^c q_\lambda) \\ &+ \epsilon_{\mu\lambda\beta\gamma} \xi_{\alpha\beta}^* \frac{q_\mu k_\lambda}{m_p^2} (v_{01}^c k_\alpha k_\rho + v_{02}^c q_\alpha k_\rho + v_{03}^c q_\alpha q_\rho) \\ &+ \epsilon_{\lambda\beta\gamma\delta} \frac{k_\mu q_\rho q_\lambda}{m_p^2} \xi_{\alpha\beta}^* \left[ \delta_{\alpha\mu} (v_{01}^d k_\rho + v_{02}^d q_\rho) \right. \\ &\quad \left. + \frac{k_\alpha k_\rho}{m_p^2} (v_{11}^d k_\mu + v_{12}^d q_\mu + v_{13}^d q_\mu) \right] \end{aligned} \quad (18)$$

$$\begin{aligned} &+ \frac{q_\alpha k_\rho}{m_p^2} (v_{21}^d k_\mu + v_{22}^d q_\mu + v_{23}^d q_\mu) \\ &+ \frac{q_\alpha q_\rho}{m_p^2} (v_{31}^d k_\mu + v_{32}^d q_\mu + v_{33}^d q_\mu) \end{aligned}$$

$$+ \epsilon_{\alpha\beta\gamma\delta} \frac{k_\mu q_\rho q_\lambda}{m_p^2} \xi_{\alpha\beta}^* k_\mu \left[ \frac{k_\gamma}{m_p^2} (v_{11}^e k_\lambda + v_{12}^e q_\lambda + v_{13}^e q_\lambda) + \frac{q_\rho}{m_p^2} (v_{21}^e k_\lambda + v_{22}^e q_\lambda + v_{23}^e q_\lambda) \right]$$

$$A_{\mu\lambda} =$$

$$\begin{aligned} &= a_{00}^a \xi_{\mu\lambda}^* + \xi_{\mu\rho}^* \left[ \frac{q_\rho}{m_p^2} (a_{11}^a k_\lambda + a_{12}^a q_\lambda + a_{13}^a q_\lambda) + \frac{k_\rho}{m_p^2} (a_{21}^a k_\lambda + a_{22}^a q_\lambda + a_{23}^a q_\lambda) \right] \\ &+ \xi_{\lambda\rho}^* \left[ \frac{q_\rho}{m_p^2} (a_{31}^a k_\lambda + a_{32}^a q_\lambda + a_{33}^a q_\lambda) + \frac{k_\rho}{m_p^2} (a_{41}^a k_\lambda + a_{42}^a q_\lambda + a_{43}^a q_\lambda) \right] \\ &+ \xi_{\alpha\beta}^* \frac{k_\alpha k_\rho}{m_p^2} \left[ a_{00}^c \delta_{\mu\lambda} + \frac{k_\mu}{m_p^2} (a_{11}^c k_\lambda + a_{12}^c q_\lambda + a_{13}^c q_\lambda) \right. \\ &\quad \left. + \frac{q_\mu}{m_p^2} (a_{21}^c k_\lambda + a_{22}^c q_\lambda + a_{23}^c q_\lambda) + \frac{q_\mu}{m_p^2} (a_{31}^c k_\lambda + a_{32}^c q_\lambda + a_{33}^c q_\lambda) \right] \\ &+ \xi_{\alpha\beta}^* \frac{k_\alpha q_\rho}{m_p^2} \left[ a_{00}^d \delta_{\mu\lambda} + \frac{k_\mu}{m_p^2} (a_{11}^d k_\lambda + a_{12}^d q_\lambda + a_{13}^d q_\lambda) \right. \\ &\quad \left. + \frac{q_\mu}{m_p^2} (a_{21}^d k_\lambda + a_{22}^d q_\lambda + a_{23}^d q_\lambda) + \frac{q_\mu}{m_p^2} (a_{31}^d k_\lambda + a_{32}^d q_\lambda + a_{33}^d q_\lambda) \right] \\ &+ \xi_{\alpha\beta}^* \frac{q_\alpha q_\rho}{m_p^2} \left[ a_{00}^e \delta_{\mu\lambda} + \frac{k_\mu}{m_p^2} (a_{11}^e k_\lambda + a_{12}^e q_\lambda + a_{13}^e q_\lambda) \right. \\ &\quad \left. + \frac{q_\mu}{m_p^2} (a_{21}^e k_\lambda + a_{22}^e q_\lambda + a_{23}^e q_\lambda) + \frac{q_\mu}{m_p^2} (a_{31}^e k_\lambda + a_{32}^e q_\lambda + a_{33}^e q_\lambda) \right], \end{aligned} \quad (19)$$

$$D_\mu =$$

$$\begin{aligned} &= \xi_{\mu\rho}^* \frac{1}{m_p} (d_1^a q_\rho + d_2^a k_\rho) + \xi_{\alpha\beta}^* \frac{k_\alpha k_\rho}{m_p^2} (d_1^b k_\mu + d_2^b q_\mu + d_3^b q_\mu) \\ &+ \xi_{\alpha\beta}^* \frac{k_\alpha q_\rho}{m_p^2} (d_1^c k_\mu + d_2^c q_\mu + d_3^c q_\mu) + \xi_{\alpha\beta}^* \frac{q_\alpha q_\rho}{m_p^2} (d_1^d k_\mu + d_2^d q_\mu + d_3^d q_\mu) \end{aligned} \quad (20)$$

The constraint equations for these forms are much too lengthy to be displayed. In Appendix B we list only the resulting form factors  $v_{ij}^*$  and  $a_{ij}^*$ . Putting  $v_{ij}^*$  into (18) we obtain the explicit expression

$$\begin{aligned}
 \frac{1}{m_p} \epsilon_\mu^* V_{\mu\lambda} = & + \epsilon_{\lambda\alpha\beta\gamma} \frac{(q+k)_\alpha}{2m_p} \cdot \frac{(Q-k)_\beta}{2M} \cdot \xi_{\alpha\beta}^* \frac{(q+k)_\gamma}{2m_p} F_M(q^2) \cdot e_i \frac{p^i \epsilon^*}{p^i k} \\
 & - \epsilon_{\lambda\alpha\beta\gamma} \frac{(q+k)_\alpha}{2m_p} \cdot \frac{(Q+k)_\beta}{2M} \cdot \xi_{\alpha\beta}^* \frac{(q+k)_\gamma}{2m_p} F_M(q^2) \cdot e_f \frac{p^f \epsilon^*}{p^f k} \\
 & - \epsilon_{\mu\alpha\beta\gamma} \epsilon_\mu^* \frac{k_\beta}{2m_p} \cdot \frac{Q_\alpha}{2M} \cdot \xi_{\alpha\beta}^* \frac{q_\gamma}{2m_p} F_M(q^2) \left( \frac{e_i (Q-k)_\lambda}{2p^i k} - \frac{e_f (Q+k)_\lambda}{2p^f k} \right) \\
 & - \epsilon_{\mu\lambda\beta\gamma} \epsilon_\mu^* \frac{Q_\beta}{2M} \cdot \xi_{\alpha\beta}^* \frac{q_\gamma}{2m_p} \cdot \frac{1}{2m_p} F_M(q^2) \\
 & - \epsilon_{\mu\lambda\beta\gamma} \epsilon_\mu^* \frac{q_\beta}{2m_p} \cdot \xi_{\alpha\beta}^* \frac{q_\gamma}{2m_p} \cdot \frac{1}{2M} F_M(q^2) (e_i + e_f) \\
 & - \epsilon_{\mu\lambda\beta\gamma} \epsilon_\mu^* \frac{k_\beta}{2m_p} \cdot \xi_{\alpha\beta}^* \frac{q_\gamma}{2m_p} \cdot \frac{1}{2M} F_M(q^2) (e_i + e_f) \\
 & - \epsilon_{\lambda\alpha\beta\gamma} \frac{(q+k)_\alpha}{2m_p} \cdot \frac{Q_\beta}{2M} \cdot \xi_{\alpha\beta}^* \epsilon_\gamma^* \cdot \frac{1}{2m_p} F_M(q^2) \\
 & + \epsilon_{\lambda\alpha\beta\gamma} \frac{q_\alpha}{2m_p} \cdot \frac{k_\beta}{2m_p} \cdot \xi_{\alpha\beta}^* \epsilon_\gamma^* \cdot \frac{1}{2M} F_M(q^2) (e_i + e_f)
 \end{aligned} \tag{21}$$

It should be noted that substantial easement in derivation of  $V_{\mu\lambda}$  has been achieved by the use of independent kinematical covariants in this case of spin-two nuclear final state. The task of setting the general form of  $V_{\mu\lambda}$  becomes almost impracticable without reference to the identities as is eq. (22) of I.

The final result for  $A_{\mu\lambda}$  is

$$\begin{aligned}
 \frac{1}{m_p} \epsilon_\mu^* A_{\mu\lambda} = & \xi_{\alpha\beta}^* \frac{(q+k)_\alpha}{2m_p} \cdot (q+k)_\beta \left[ \frac{(Q-k)_\lambda}{2M \cdot 2m_p} F_T(q^2) - \frac{(q+k)_\lambda}{m_\pi^2} F_P(q^2) \right] e_i \frac{p^i \epsilon^*}{p^i k} \\
 & - \xi_{\alpha\beta}^* \frac{(q+k)_\alpha}{2m_p} \cdot (q+k)_\beta \left[ \frac{(Q+k)_\lambda}{2M \cdot 2m_p} F_T(q^2) - \frac{(q+k)_\lambda}{m_\pi^2} F_P(q^2) \right] e_f \frac{p^f \epsilon^*}{p^f k} \\
 & - \xi_{\lambda\beta}^* \frac{(q+k)_\alpha}{2m_p} F_A(q^2) \left[ e_i \frac{p^i \epsilon^*}{p^i k} - e_f \frac{p^f \epsilon^*}{p^f k} \right] +
 \end{aligned} \tag{22}$$

$$\begin{aligned}
 & + \xi_{\alpha\beta}^* \frac{q_\alpha}{2m_p} q_\beta \cdot \epsilon_\lambda^* \left[ \frac{1}{2M \cdot 2m_p} F_T(q^2) (e_i + e_f) + \frac{1}{m_\pi^2} F_P(q^2) \right] \\
 & + \xi_{\mu\beta}^* \frac{(2q+k)_\mu}{2m_p} \left[ \frac{k_\lambda}{2m_p} \cdot \frac{1}{2M} F_T(q^2) (e_i + e_f) - \frac{Q_\lambda}{2M} \cdot \frac{1}{2m_p} F_T(q^2) + \frac{(q+k)_\lambda}{m_\pi^2} F_P(q^2) \right] \\
 & + \xi_{\lambda\beta}^* \epsilon_\beta^* \cdot F_A(q^2) \frac{1}{2m_p}
 \end{aligned}$$

The comments of Sec. 3 concerning the relation of the "diagrammatic" approach to our treatment and the gauge invariance are valid also for this case of RMC on  $^{16}\text{O}$ .

To proceed in calculations, one should estimate the numerical values for the nuclear form factors  $F_M, F_A, F_P$  and  $F_T$ . An example of such estimates can be found in ref. 15/.

## 5. Conclusions

The  $0^+ \rightarrow 1^+$  and  $0^+ \rightarrow 2^-$  radiative-muon-capture amplitudes are presented within the elementary-particle treatment of  $^{12}\text{C}, ^{12}\text{B}, ^{16}\text{O}$  and  $^{16}\text{N}$  nuclei. The appealing features of the method used are both the conceptual simplicity and a general correspondence of the results obtained to the ones to be expected from diagrammatic treatment. As a main difficulty we wish to point out that the interpretation of the dynamical assumptions ("linearity hypothesis") is not very transparent. Further, the PCAC condition, due to its peculiar structure, works rather ineffectively. A new dynamical hypothesis about the structure of the auxiliary vector  $D_\mu$  is needed.

It is indeed clear from general considerations that the amplitude of a radiative process cannot be uniquely determined in terms of non-radiative quantities using the CEC, CVC, and PCAC constraints only. The uniqueness of the solutions we obtain is intimately connected with the dynamical assumptions which were used to solve the constraint equations.

Now it will be interesting to obtain numerical results for the observable characteristics of this process and compare them with those calculated via impulse approximation. Indeed, most important will be the comparison with the new experimental data for RMC on light nuclei to appear soon.



Appendix A

Hadron-radiating amplitude for the  $O^+ \rightarrow I^+$  transition.

A1. Tensor  $V_{\mu\lambda}$ .

Here we outline the solution of systems A and B (eqs. (4)-(13)) of Sec. 3. The form factors entering into A can be obtained without any reference to PT. Actually, using LH (see Appendix A of I) one gets from eq. (4)

$$v_{12}^e = \frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_M(q^2) \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right), \quad (A.1)$$

$$v_{13}^e = -\frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_M(q^2) \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right). \quad (A.2)$$

Eq. (5) shows that  $v_{02}^e$  does not contain any term proportional to  $q^2 / [(Q \pm q)k]$ . Therefore, in eq. (7) we have  $v_{13}^e + v_{33}^e = 0$ , i.e.

$$v_{33}^e = \frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_M(q^2) \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right). \quad (A.3)$$

From eqs. (5) and (A.3) and using LH one obtains

$$v_{32}^e = -\frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_M(q^2) \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right), \quad (A.4)$$

$$v_{02}^e = \frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_M(q^2). \quad (A.5)$$

Now, comparing eqs. (5) and (7) we have

$$v_{12}^a = -v_{32}^b, \quad (A.6)$$

$$v_{14}^a = -v_{33}^b. \quad (A.7)$$

To obtain these last results we have put

$$v_{14}^f = v_{12}^f = v_{11}^b = v_{31}^b = 0 \quad (A.8)$$

since the corresponding covariants, being proportional to  $k_\mu$  will vanish in the expression  $\epsilon_{\mu\nu}^* V_{\mu\lambda}$ .

To solve the system B (eqs. (8) through (13)), we have to use the perturbation theory result\*

\*) A word of warning is in order concerning the consistent use of PT. Normally, (none through) several form factors are needed to supplement LH for determination of a whole group of  $v_{ij}^e$ . To be consistent one must take from PT the term which is nonzero there (e.g.  $v_{12}^e$  or  $v_{13}^e$  in the above case). If the solution starts from the form factor which does not appear in PT (e.g. starts with  $v_{03}^e = 0$  here), it can result in a clear contradiction: from LH we get then both  $v_{12}^e$  and  $v_{13}^e$  zero as well. Examples of such a "destructive" use of PT can be traced in HP2.

$$v_{22}^e = \frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_M(q^2) \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right). \quad (A.9)$$

Eqs. (8) and (A.9) together with LH lead to

$$v_{23}^e = -\frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_M(q^2) \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right), \quad (A.10)$$

$$v_{03}^e = \frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_M(q^2) (e_i + e_f). \quad (A.11)$$

Now, using (A.1) and (A.2) we have from eq. (9)

$$v_{01}^e = \frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_M(q^2) (e_i + e_f). \quad (A.12)$$

We rewrite eqs. (10) and (11) in the form

$$v_{00}^e + v_{21}^e kQ + v_{22}^e (k+q)q = 0, \quad (A.13)$$

$$v_{00}^e + v_{11}^e kQ + v_{12}^e (k+q)q = 0. \quad (A.14)$$

None of the form factors contained in eqs. (A.13) and (A.14) appears in PT, therefore an arbitrary one of them can be chosen zero, then all the others via LH assume zero value as well, a.g.

$$v_{11}^c = 0 \longrightarrow v_{00}^c = v_{11}^c = v_{12}^c = v_{12}^c = 0. \quad (A.15)$$

We have determined completely the form factors of tensor  $V_{\mu\lambda}$ . To achieve this, we used LH and two form factors fixed through PT. This is in contrast to HP2, where 14 form factors were built in via perturbation theory.

A2. Tensor  $A_{\mu\lambda}$ .

Constraint equations to be solved for the radiative form factors which determine  $A_{\mu\lambda}$  have been derived in HP2. In I we have mentioned already that the PCAC condition gives no additional constraint on the form of  $A_{\mu\lambda}$  since the dynamical structure of the "auxiliary" vector  $D_\mu$  is arbitrary. Therefore we consider only the equations which arise from the CEC condition. In notation of HP2 they are

$$G_{02}^e kQ + G_{03}^e kq = m_p^2 \sqrt{2} F_A(q^2) \quad (A.16)$$

$$G_{21}^a kQ + G_{31}^a kq = -m_p^2 (G_{01}^a + G_{00}^a) \quad (A.17)$$



$$G_{22}^a kQ + G_{32}^a kq = -m_p^2 G_{02}^a \quad (\text{A.18})$$

$$G_{23}^a kQ + G_{33}^a kq = -m_p^2 G_{03}^a \quad (\text{A.19})$$

$$G_{21}^e kQ + G_{31}^e kq = -m_p^2 G_{00}^e \quad (\text{A.20})$$

$$G_{22}^e kQ + G_{32}^e kq = -\frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_E(q^2) \quad (\text{A.21})$$

$$G_{23}^e kQ + G_{33}^e kq = \frac{m_p^4}{m_\pi^2} \sqrt{2} F_P(q^2). \quad (\text{A.22})$$

Eqs. (A.16), (A.21) and (A.22) along with LH lead to

$$G_{02}^e = m_p^2 \sqrt{2} F_A \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$G_{03}^e = -m_p^2 \sqrt{2} F_A \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$G_{22}^e = -\frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_E \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$G_{32}^e = \frac{m_p^3}{2} \cdot \frac{1}{2M} \sqrt{2} F_E \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$G_{23}^e = \frac{m_p^4}{m_\pi^2} \sqrt{2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$G_{33}^e = -\frac{m_p^4}{m_\pi^2} \sqrt{2} F_P \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

For the further solution we have to calculate several form factors via perturbation theory:

$$G_{21}^e = \frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right) + \frac{m_p^3}{2 \cdot 2M} F_E \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$G_{21}^a = \frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right) + \frac{m_p^3}{2 \cdot 2M} F_E \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$G_{21}^a = -\frac{m_p^3}{2 \cdot 2M} F_E \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$G_{23}^a = \frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right).$$

Using these expressions and LH we obtain from eqs. (A.17)-(A.20)

$$G_{31}^a = -\frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right) - \frac{m_p^3}{2 \cdot 2M} F_E \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$G_{01}^a + G_{00}^a = -\frac{m_p^2}{m_\pi^2} F_P - \frac{m_p^2}{2M \cdot 2m_p} F_E (e_i + e_f) \quad (\text{A.23})$$

$$G_{32}^a = \frac{m_p^3}{2 \cdot 2M} F_E \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$G_{02}^a = \frac{m_p}{2 \cdot 2M} F_E$$

$$G_{33}^a = -\frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$G_{03}^a = -\frac{m_p^4}{m_\pi^2} F_P$$

$$G_{31}^e = -\frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right) - \frac{m_p^3}{2 \cdot 2M} F_E \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$G_{00}^e = -\frac{m_p^2}{m_\pi^2} F_P - \frac{m_p}{2 \cdot 2M} F_E (e_i + e_f).$$

Now we should like to call attention to equation (A.23). Here we have an example of the possible ambiguity in solution. None of the form factors  $G_{00}^a, G_{01}^a$  appears in perturbation theory. Hence we can not solve this equation. When we consider the expression for  $(k+q)_\lambda A_{\mu\lambda}$ , the term with  $G_{00}^a$  will be proportional to  $k_\mu$ . It is not dangerous, it simply gives rise to additional terms proportional to  $k_\mu$  in  $D_\mu$ . Though it is not a convincing reason we shall put  $G_{00}^a$  to be zero.

Then

$$G_{01}^a = -\frac{m_p^1}{m_\pi^2} F_P - \frac{m_p^2}{2M \cdot 2m_p} F_E (e_i + e_f).$$

We shall not display the expression for  $D_\mu$  here. It can easily be obtained substituting  $G_{ij}^a$  into eqs. (15a)-(15g) of HP2.

This expression certainly differs from that in HP2. But we would like to stress that the form factor  $F_D$  which has a physical interpretation and can be explored experimentally, is the same as in HP2,

$$F_D = \left(1 + \frac{q^2}{m_x^2}\right) \left(F_A + \frac{q^2}{m_x^2} F_P\right) \sqrt{2}.$$

In other words, though we do not apply LH to determine  $D_\mu$  and make no assumption about the structure of its form factors, the physical form factor  $F_D$  has a correct dependence on dynamical variables.

#### Appendix B

Radiative form factors for the  $0^+ \rightarrow 2^-$  transition  
Terms calculated via perturbation theory are

$$\begin{aligned} v_{31}^a &= -\frac{m_p^5}{(2m_p)^2 \cdot 2M} F_M \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right) \\ v_{32}^a &= -\frac{m_p^5}{(2m_p)^2 \cdot 2M} F_M \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right) \\ v_{41}^a &= \frac{m_p^5}{(2m_p)^2 \cdot 2M} F_M \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right), \end{aligned}$$

besides, similarly to (A.15) we put

$$v_{04}^c = v_{12}^d = v_{22}^d = 0.$$

Solutions of the equations arising from the CEC and CVC constraints are as follows

$$\begin{aligned} v_{20}^a &= v_{10}^c \\ v_{31}^a &- \\ v_{32}^a &- \\ v_{33}^a &= v_{32}^c \\ v_{40}^a &= \frac{m_p}{8M} F_M (e_i + e_f) \\ v_{41}^a &- \\ v_{42}^a &= \frac{m_p^3}{8M} F_M \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right) \end{aligned}$$

$$v_{13}^a = -\frac{m_p^3}{8M} F_M \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$v_{11}^a = \frac{m_p^3}{8M} F_M \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$v_{52}^a = \frac{m_p^3}{8M} F_M \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$v_{53}^a = v_{52}^a$$

$$v_{60}^a = \frac{m_p}{8M} F_M$$

$$v_{61}^a = -\frac{m_p^3}{8M} F_M \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$v_{62}^a = -\frac{m_p^3}{8M} F_M \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$v_{63}^a = -v_{62}^a$$

$$v_{01}^a = -\frac{m_p}{8M} F_M$$

$$v_{02}^a = -\frac{m_p}{8M} F_M (e_i + e_f)$$

$$v_{03}^a = v_{02}^a$$

$$v_{11}^b = -v_{13}^a$$

$$v_{12}^b = -v_{63}^a$$

$$v_{00}^b = -v_{01}^c$$

$$v_{01}^e -$$

$$v_{02}^c = 0$$

$$v_{03}^c = 0$$

$$v_{01}^d = 0$$

$$v_{02}^d = 0$$

$$v_{12}^d -$$

$$v_{13}^d = 0$$

$$v_{22}^d -$$

$$v_{23}^d = 0$$

$$v_{32}^d = 0$$

$$v_{33}^d = 0$$

While solving the equations for  $v_{ij}^a$  we were not interested in  $v_{11}^a, v_{21}^a, v_{31}^a$  and  $v_{ij}^e$  ( $i, j = 1, 2, 3$ ) and took them zero, since the corresponding terms vanished in  $\epsilon_\mu^* V_{\mu\lambda}$ .

Solving the system arising from the CEC constraint in the same manner as earlier for  ${}^{12}C$  we can obtain the form factors  $a_{ij}^a$ . The form factors of the first group below were calculated via perturbation theory

$$a_{22}^b = -\frac{m_p^2}{2} F_A \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$



$$a_{21}^c = -\frac{m_p^3}{8M} F_T \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right) - \frac{m_p^4}{2m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$a_{22}^c = \frac{m_p^3}{8M} F_T \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$a_{23}^c = -\frac{m_p^4}{2m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$a_{21}^d = -\frac{m_p^3}{4M} F_T \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right) - \frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$a_{22}^d = \frac{m_p^3}{4M} F_T \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$a_{23}^d = -\frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$a_{21}^e = -\frac{m_p^3}{8M} F_T \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right) - \frac{m_p^4}{2m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right).$$

The form factors  $a_{11}^b, a_{21}^b, a_{ij}^c, a_{ij}^d, a_{ij}^e$  are considered to be zero, as the corresponding terms vanish in  $\epsilon_\mu^* A_{\mu\lambda}$ .  $a_{00}^c, a_{00}^d$  are taken to be zero also, for an analogous reason as  $G_{00}^a$  in  $A_{\mu\lambda}$  for  $^{12}\text{C}$ . (see App. A).

The other form factors calculated via LH are as follows:

$$a_{00}^a = \frac{1}{2} F_A$$

$$a_{11}^a = \frac{m_p}{4M} F_T (e_i + e_f) + \frac{m_p^2}{m_\pi^2} F_P$$

$$a_{12}^a = -\frac{m_p}{4M} F_T$$

$$a_{13}^a = \frac{m_p^2}{m_\pi^2} F_P$$

$$a_{21}^a = \frac{m_p}{8M} F_T (e_i + e_f) + \frac{m_p^2}{2m_\pi^2} F_P$$

$$a_{22}^a = -\frac{m_p}{8M} F_T$$

$$a_{23}^a = \frac{m_p^2}{2m_\pi^2} F_P$$

$$a_{12}^b = -\frac{m_p^2}{2} F_A \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$a_{13}^b = \frac{m_p^2}{2} F_A \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{23}^b = \frac{m_p^2}{2} F_A \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{31}^c = \frac{m_p^3}{8M} F_T \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right) + \frac{m_p^4}{2m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{32}^c = -\frac{m_p^3}{8M} F_T \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{33}^c = \frac{m_p^4}{2m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{31}^d = \frac{m_p^3}{4M} F_T \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right) + \frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{32}^d = -\frac{m_p^3}{4M} F_T \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{33}^d = \frac{m_p^4}{m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{00}^e = \frac{m_p}{8M} F_T (e_i + e_f) + \frac{m_p^2}{2m_\pi^2} F_P$$

$$a_{21}^e = \frac{m_p^3}{8M} F_T \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right) + \frac{m_p^4}{2m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{22}^e = -\frac{m_p^3}{8M} F_T \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{23}^e = \frac{m_p^4}{2m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$a_{22}^e = \frac{m_p^3}{8M} F_T \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$a_{23}^e = -\frac{m_p^4}{2m_\pi^2} F_P \left( \frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$F_D = -\left(1 + \frac{g^2}{m_\pi^2}\right) \left(F_A + \frac{g^2}{m_\pi^2} F_P\right) \cdot \frac{1}{2}$$

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