сообщения
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E2 - 12639

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ELEMENTARY PARTICLE TREATMENT OF THE RADIATIVE MUON CAPTURE.

1. METHOD

1979

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Радиационный μ^--захват в подходе "ядро как элементарная частица". 1. Метод

В работе излагается метод построения амплитуды радиационного μ-захвата ядрами с различным спином. Ядро рассматривается как элементарная частица и может быть полностью охарактеризовано электрическим зарядом e, магнитным моментом μ, спином J и четностью π. Амплитуду радиационного μ-захвата можно разбить на две части: первая $T^{(\ell)}$ соответствует случаю, когда γ-квант испускается мюоном, вторая часть $T^{(h)}$ учитывает все остальные возможности. Гипотезы сохранения электромагнитного, слабого векторного токов, а также гипотеза частичного сохранения аксиального тока позволяют связать радиационные формфакторы, входящие в $T^{(h)}$ с нерадиационными формфакторами, характеризующими соответствующий обычный μ-захват. Для однозначной связи радиационных и нерадиационных формфакторов приходится вводить дополнительную гипотезу о динамической структуре формфакторов. Рассмотрены недостатки метода, предложенного Хуангом и Примаковым и предложена свободная от этих недостатков конструкция амплитуды радиационного μ-захвата.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1979

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Elementary Particle Treatment of the Radiative Muon Capture. 1. Method

We outline the construction of the radiative-muon-capture amplitude on arbitrary-spin targets using the assumptions about conservation of the electromagnetic and weak hadronic currents and a simplifying dynamical hypothesis.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1979

The reaction of radiative muon capture (RMC) is already for more than two decades studied as a prospective source of information about the weak interaction features which are poorly known from other reactions. In earlier experiments the true signal of hard ($\lesssim$ 100 MeV) γ-quanta was always accompanied by a background of high-energy neutrons due to the much more frequent (factor of $\approx 10^4$) nonradiative muon capture. These data indeed fell down in comparison with any calculation. A new generation of the experimental data is being collected /1/ on the modern meson facilities, partly with the use of pair spectrometers: the γ-quanta are converted into e^+e^- pair which can be safely detected. Such a development requires indeed to polish the theoretical understanding of RMC as well.

The usual treatment /2/ is based on the simple "diagrammatic" description: it considers radiation due to the electric charges and anomalous magnetic momenta of the initial and final particles. To restore the gauge invariance, one should indeed add the terms corresponding to the minimal electromagnetic coupling which are connected with the "vertex radiation". Such an amplitude derived for the RMC on a free proton may be via impulse approximation used in the calculations for higher-Z targets: since RMC is a rare process, the experiments are presently feasible only on the nuclei carying charge of several units e .

The impulse approximation is connected with an assumption that the effects of (mesonic) exchange currents can be ignored. Their role is not yet assessed properly, therefore we prefer an alternative, the so-called elementary-particle treatment (EPT). Here the initial and final nuclear states are supposed to be objects fully characterized by the invariance properties corresponding to their spin-parities. The nuclear-structure information is parametrised in a few form factors which should provide a link with the other (weak, electromagnetic) reactions on the respective target. EPT was earlier applied to the radiative muon capture on ^{3}He by Beder /3/. The attempts to derive the EPT amplitude for RMC on boson-like objects within the above "perturbation theory + minimal electromagnetic coupling" approach fail: the amplitude may break not only the CVC and PCAC requirements (as in the $^{12}C(0^+) \to {}^{12}B(1^+)$ transition) but even leads to the electromagnetic-current nonconservation ($^{16}O(0^+) \to {}^{16}N(2^-)$ radiative capture). A more systematic approach is needed.

Hwang and Primakoff (HP1 /4/, HP2/5/), again on the basis of EPT, suggested a physically appealing program. Starting with the conditions of charge conservation (CEC), vector current conservation (CVC) and partial conservation of axial current (PCAC), they constructed the RMC amplitude entirely in terms of the (nonradiative) muon-capture form factors. As for the results, we should mention, however, several difficulties.

First, the use of algebraically independent set of Lorentz covariants in the construction of the general form of the RMC amplitude in EPT is indeed highly desirable. Second, the excessive and partly contradictory references to the perturbation theory (PT) should be avoided. An example of such an inconsistent use of the PT will be given in Part II of this series for RMC on the ^{12}C nucleus. Third, the additional assumptions concerning the structure of the radiative form factors introduced in HP1 (the so-called linearity hypothesis - LH) stem actually from the perturbation theory. Therefore they should not be applied in a wider context. In doing so (construction of the vector $D_{\mu\nu}$, see below), Hwang and Primakoff have obtained the $\frac{1}{2} \to \frac{1}{2}$ transition amplitude which differs (in its axial part) considerably from the standard results/2,3/.

Being inspired by the suggestion of Hwang and Primakoff, we present in this paper another method for the construction of RMC amplitude. Though fully based on the general framework of HP1, the new construction solves the above-mentioned difficulties.In particular, for RMC on the free proton our result can easily be connected with the earlier derivations. We consider this last feature particularly satisfying, since it provides an additional justification for the use of the method in derivation of the RMC amplitudes in the more complex cases, e.g.,in analysis of RMC on ^{12}C and ^{16}O.

The plan of the present paper is as follows. In Sec. 2 we summarize the formulation of Hwang and Primakoff for derivation of the RMC amplitude. In Sec. 3 we discuss the hypothesis about the dynamical structure of the form factors to be determined. Using these assumptions the RMC amplitude for the proton-neutron transition is obtained in Sec. 4. The construction of the general relativistically covariant form of RMC amplitudes for integer-spin nuclei is considered in Sec. 5. Examples of the $(\mu^-, \gamma\nu)$ reaction on such boson-like targets will be given in Part II of this series.

2. Formulation

Using our knowledge of the divergences of the electromagnetic hadronic current $J_\mu(x)$, vector current $V_\mu(x)$ and axial-vector current $A_\mu(x)$, namely

$$\partial_\mu J_\mu(x) = 0, \tag{1}$$

$$\partial_\mu V_\mu(x) = 0, \tag{2}$$

$$\partial_\mu A_\mu(x) = a_\pi m_\pi^3 \left(-\partial_\mu \partial_\mu + m_\pi^2\right)^{-1} J^\pi(x), \tag{3}$$

we can obtain the constraints on the RMC amplitude

$$T = T^{(l)} + T^{(h)}. \tag{4}$$

Here $J^\pi(x)$ is the pion-source current, $a_\pi (m_\pi)$ is the pion-decay constant (pion mass). Actually, the contribution $T^{(l)}$ corresponding to the muon radiation does not present any difficulty. In what follows we describe a method for construction of $T^{(h)}$ on the basis of constraints arising from eqs. (1)-(3).

The standard relativistic reduction technique [6] for the outgoing photon can be used to obtain the amplitude $T^{(h)}$ in the form

$$T^{(h)} = -\frac{G e}{\sqrt{2}}\, \bar{u}^{\nu}(p^{\nu}) \gamma_\lambda (1+\gamma_5) u^{\mu}(p^{\mu}) \frac{1}{m_p} \cdot \frac{\epsilon_\mu^*}{\sqrt{2k_0}} \left[V_{\mu\lambda}(k,q,Q) + A_{\mu\lambda}(k,q,Q) \right]_1 \quad (5)$$

where

$$V_{\mu\lambda}(k,q,Q) \equiv -i m_p \int d^4x \cdot e^{-ikx} \langle N_f(p^f) | T(J_\mu(x) V_\lambda(0)) | N_i(p^i) \rangle \quad (6)$$

$$A_{\mu\lambda}(k,q,Q) \equiv -i m_p \int d^4x \cdot e^{-ikx} \langle N_f(p^f) | T(J_\mu(x) A_\lambda(0)) | N_i(p^i) \rangle \quad (7)$$

with self-explaining notation (we follow closely refs. [4,5]) for initial (i,μ) and final (f,ν) state quantities. The photon 4-momentum k_λ, and centre-of-mass 4-momenta

$$q_\lambda = p^f_\lambda - p^i_\lambda, \qquad Q_\lambda = p^f_\lambda + p^i_\lambda \quad (8)$$

have been introduced. We use the Pauli metrics: $A_\lambda = (\vec{A}, iA_0)$.

We proceed now to apply the CEC, CVC, and PCAC constraints (1)-(3) to the tensors $V_{\mu\lambda}$ and $A_{\mu\lambda}$. Performing, when necessary, the shift

$$V_\lambda(x) = e^{-iPx} V_\lambda(0) e^{iPx}, \quad (9)$$

using the T product representation

$$T\left[J_\mu(x) K_\lambda(y)\right] = J_\mu(x) K_\lambda(y) + \theta(y_0 - x_0)\left[K_\lambda(y), J_\mu(x)\right]_{-} \quad (10)$$

and the $SU_2 \times SU_2$ commutation relations like

$$\left[V_\lambda(0), J_4(x)\right]_{x_0=0} = i V_\lambda(0)\, \delta^{(3)}(\vec{x}),$$
$$\left[V_4(-x), J_\mu(0)\right]_{x_0=0} = i V_\mu(0)\, \delta^{(3)}(-\vec{x}), \ldots \text{etc.} \quad (11)$$

the following conditions are obtained in a straightforward calculation:

CEC: $\quad k_\mu V_{\mu\lambda}(k,q,Q) = m_p \langle N_f(p^f) | V_\lambda(0) | N_i(p^i) \rangle \quad (12)$

$\quad k_\mu A_{\mu\lambda}(k,q,Q) = m_p \langle N_f(p^f) | A_\lambda(0) | N_i(p^i) \rangle \quad (13)$

CVC: $\quad (k+q)_\lambda V_{\mu\lambda}(k,q,Q) = m_p \langle N_f(p^f) | V_\mu(0) | N_i(p^i) \rangle \quad (14)$

PCAC: $\quad (k+q)_\lambda A_{\mu\lambda}(k,q,Q) = m_p \langle N_f(p^f) | A_\mu(0) | N_i(p^i) \rangle + m_p D_\mu(k,q,Q) \quad (15)$

where

$$D_\mu(k,q,Q) = \int d^4x \cdot e^{-ikx} \langle N_f(p^f) | T(J_\mu(x)\, \partial_\lambda A_\lambda(0)) | N_i(p^i) \rangle \quad (16)$$

and (again using the condition $\partial_\lambda J_\lambda(x) = 0$)

$$k_\mu D_\mu = i \langle N_f(p^f) | \partial_\lambda A_\lambda(0) | N_i(p^i) \rangle. \quad (17)$$

To complete the derivation of $T^{(h)}$ we should construct the Lorentz-covariant forms

$$V_{\mu\lambda}(k,q,Q) = \sum_{ijx} v^x_{ij}(k,q,Q) \cdot X^x_{ij,\mu\lambda}(k,q,Q), \quad (18)$$

and

$$A_{\mu\lambda}(k,q,Q) = \sum_{ijx} a^x_{ij}(k,q,Q) \cdot Y^x_{ij,\mu\lambda}(k,q,Q), \quad (19)$$

$$D_\mu(k,q,Q) = \sum_i d_i(k,q,Q) \cdot Z_{i,\mu}(k,q,Q) \quad (20)$$

for each particular transition characterized by the spins (J_i, J_f) and parities (π_i, π_f) of the initial and final states, respectively. Here the form factors v, a and d depend on the 4-momenta introduced above and X, Y and Z are kinematical covariants.

The substitution of the forms (18)-(20) into eqs. (12)-(15) gives us a system of equations to determine the radiative form factors $v^x_{ij}(k,q,Q)$ and $a^x_{ij}(k,q,Q)$ in terms of the nonradiative ones which enter through the vector current $V_\lambda(0)$ and axial-vector current $A_\lambda(0)$. By solving them we shall complete the derivation of $T^{(h)}$.

3. A Dynamical Hypothesis

To solve the equations for the radiative form factors v^x_{ij}, a^x_{ij} and d_i, we should assume a particular dynamical structure of these form factors. In HP1 it is the linearity hypothesis - LH. We refer the reader to HP1 for a detailed discussion. Our use of LH is less straightforward, therefore we wish to summarise here in few points the way in which LH is taken in our work:
(i) the nonradiative weak form factors $F_j(q^2, (p^i)^2, (p^f)^2)$ (index j stands for V, M, A, P, the vector, weak-magnetism, axial-vector or induced pseudo-scalar term) actually depend on q^2 only:

$$F_j\left((q+k)^2, (p^i-k)^2, (p^f)^2\right) \simeq F_j(q^2),$$

the electric charge (e_i, e_f) and anomalous-magnetic-momentum (μ_i, μ_f) form factors are assumed to be constants, e.g.,

$$e_i(k^2, (p^i)^2, (p^i-k)^2) \simeq e_i, \quad \ldots \qquad \text{, etc.}$$

(ii) The radiative form factors a^x_{ij}, v^x_{ij}, d_i (call them R) have definite dynamical structure, namely

$$R(q^2, Q\cdot k, q\cdot k) = \frac{R^+(q^2)}{(Q+q)\cdot k} + \frac{R^-(q^2)}{(Q-q)\cdot k} + R^0(q^2), \tag{21}$$

where $R^+(q^2)$ is linear in $F_j(q^2)$ and in e_f, μ_f, $R^-(q^2)$ is linear in $F_j(q^2)$ and in e_i, μ_i, and $R^0(q^2)$ is linear in $F_j(q^2)$ and in e_i, e_f, μ_i, μ_f.

(iii) The above assumptions about the form factors R seem to be valid for those of them (v^x_{ij}, a^x_{ij}) which are contained in tensors $V_{\mu\lambda}, A_{\mu\lambda}$ since there the dependence on $q\cdot k$ and $Q\cdot k$ enters mainly through the propagators of the (nucleon, nuclear) intermediate states. In general, however, LH is an "ad hoc" assumption subject to tests on some simple models at least. The perturbation-theory treatment for RMC on the free proton seems to be just an appropriate model case. The corresponding expression for D_μ has, however, a structure which contradicts LH. We argue then that these assumptions must not be applied to the form factors d_i. To complete the derivation of $A_{\mu\lambda}$ in absence of LH for D_μ the additional reference to perturbation theory may be needed.

4. Radiative Muon Capture on the Free Protons

Using CEC, CVC and PCAC constraints (12)-(15) and the assumptions (i)-(iii) of the preceding section we have obtained for the $p(\mu^-, \gamma\nu)n$ reaction the hadron radiating amplitude $T^{(h)}$ which is easy interpretable. In its vector part $V_{\mu\lambda}$ it repeats the result of HP1 (their eq. (26)). Derivation of the axial-vector part $A_{\mu\lambda}$ is sketched in Appendix B, the final form obtained being

$$\frac{1}{m_p}\epsilon^*_\mu A_{\mu\lambda} =$$
$$= -\bar{u}^f(p^f)\Big[\Big(F_A(q^2)\gamma_\lambda\gamma_5 + F_p(q^2)\, i\,\frac{2M(q_\lambda+k_\lambda)}{m_\pi^2}\gamma_5\Big)\frac{1}{\hat{p}^i - \hat{k} - iM}\Big(e_i + \frac{\mu_i}{2M}i\hat{k}\Big)\hat{\epsilon}^*$$

$$+\hat{\epsilon}^*\Big(e_f - \frac{\mu_f}{2M}i\hat{k}\Big)\frac{1}{\hat{p}^f + \hat{k} - iM}\Big(F_A(q^2)\gamma_\lambda\gamma_5 + F_p(q^2)\frac{i\,2M(q_\lambda+k_\lambda)}{m_\pi^2}\gamma_5\Big)$$
$$+ i\gamma_5\,\epsilon^*_\mu\,\delta_{\mu\lambda}\,F_p(q^2)\frac{2M}{m_\pi^2}\Big]\, u^i(p^i)$$

The amplitude $T^{(h)}$ corresponding to these $V_{\mu\lambda}$ and $A_{\mu\lambda}$ together with the muon-radiating part /4/

$$T^{(\ell)} = -\frac{Ge}{\sqrt{2}} < N_f(p^f)\Big|\big[V_\lambda(0) + A_\lambda(0)\big]\Big|N_i(p^i)> \cdot$$
$$\cdot\sqrt{\frac{1}{2k_0}}\,\bar{u}^\nu(p^\nu)\gamma_\lambda(1+\gamma_5)\frac{1}{\hat{p}^\mu - \hat{k} - im_\mu}\hat{\epsilon}^*\, u^\mu(p^\mu)$$

form radiative amplitude with the functional dependence identical to the one which can be derived in the "PT plus minimal coupling" treatment /2/. The radiative form factors v, a in $V_{\mu\lambda}$ and $A_{\mu\lambda}$ are, however, taken at the momentum q^2 rather then $(q+k)^2$ as is the case in PT. In HP1 it was shown that such a shift in variable can be interpreted as a result of approximate inclusion of the box diagrams and diagrams with the complex intermediate states e.g., $\mu^- p \to \mu^- X\gamma \to \nu_\mu n\gamma$, where $X = p, \Delta^+, \ldots$. We stress that the tensor $A_{\mu\lambda}$ derived in Appendix B differs from the one quoted in HP1, eq. (33).

5. Lorentz-Covariant Form of Tensors $V_{\mu\lambda}$, $A_{\mu\lambda}$ and D_μ for Integer Spin

The construction of covariants for the reaction $1+2 \to a+b+c$ was shortly described by Hearn /7/. Unlike Hwang and Primakoff we suggest that care should be exercised to choose an independent set /8/ of covariants (ISC). In particular, performing the construction with ISC we keep minimal the number of radiation form factors $v^x_{ij}, \ldots$, etc., in (18)-(20) which have to be determined. The choice of the complete set of independent covariants has turned out to be straightforward in all but one examples we have elaborated on: the only difficulty we come upon is connected with the antisymmetric rank-4 tensor $\epsilon_{\alpha\beta\gamma\delta}$ (e.g., in the vector part of the unnatural parity transitions like $0^+ \to 1^+$, $0^+ \to 2^-, \dots$). In this case several fully equivalent sets of independent covariants may be constructed. The mutual connections of these ISC can be established using the relation /9/

$$\delta_{\nu\rho}\, \epsilon_{\mu\alpha\beta\gamma} = \delta_{\mu\nu}\, \epsilon_{\rho\alpha\beta\gamma} + \delta_{\alpha\nu}\, \epsilon_{\mu\rho\beta\gamma} + \delta_{\beta\nu}\, \epsilon_{\mu\alpha\rho\gamma} + \delta_{\gamma\nu}\, \epsilon_{\mu\alpha\beta\rho} . \tag{22}$$

In principle, any choice of ISC should suit equally well our purpose. In the actual construction, however, we use the above discussed linearity hypothesis and should also refer to the perturbation theory. Two requirements are then natural to make the ISC suitable for the further work: (i) the constraint equations resulting from eqs. (12)-(15) for the chosen ISC should not contradict LH, and (ii) the necessary reference to the perturbation theory has to be minimal.

Unfortunately, the decisive criterion which would allow one to make an apriori choice of the appropriate ISC is unknown. Nevertheless we list here the practical recommendations which help us to construct the tensors (18)-(20).

First, the chosen set should not contain covariants vanishing simultaneously when conditions (12) and (14) (similarly (13) and (15)) are considered; if so, the form factors corresponding to them turn out to be irrecoverable.

Second, the set should certainly contain all covariants which may appear from the "PT plus minimal coupling" treatment.

Third, the covariants proportional to the photon momentum k_μ (μ is the photon polarization index) should be maintained in the chosen ISC. In doing so we enlighten considerably our task: the corresponding form factors (let us call them R_{k_μ}) can be taken zero without loss of generality, since these covariants will vanish in the amplitude $\epsilon^*_\mu V_{\mu\lambda}$ $(\epsilon^*_\mu A_{\mu\lambda})$. However, care should be taken to come upon no contradiction with the CVC condition. Actually: when we put zero the corresponding form factors: $R_{k_\mu} = 0$ in $V_{\mu\lambda}$, the CVC expression (14) is

$$(k_\lambda + q_\lambda) V_{\mu\lambda} = k_\mu \cdot \overline{V} + \dots$$

and the term $\overline{V}$ must vanish. Otherwise in order to fulfil the CVC condition the form factors R_{k_μ} should be kept in $V_{\mu\lambda}$ so as to compensate for the term $\overline{V}$.

The use of these three ad hoc rules will be seen in Part II where we treat RMC on targets with integer spins which bring in the "difficult" $\epsilon_{\alpha\beta\gamma\delta}$ containing covariants. We wish to stress that utilizing these rules we were able in the examples considered so far, to avoid ambiguities in the choice of ISC.

In conclusion, it is worthwhile to mention that using independent covariants for the construction of $V_{\mu\lambda}$ and $A_{\mu\lambda}$, one avoids the superfluous reference to the perturbation theory. For example, in HP2 the tensor $V_{\mu\lambda}$ for ^{12}C is expanded in terms of an overcomplete set, 16 of the covariants may be excluded from this set using the algebraic identities, eq. (22). In such a situation the authors of HP2 were forced to apply PT to fix zero values for 14 radiative form factors. At the same time, using the concept of algebraically independent covariants we have only 2 form factors to be determined through PT.

6. Summary

A method was outlined to construct the CEC, CVC, and PCAC conserving amplitude of the radiative-muon-capture reaction. Substantial for the successfull derivation was a dynamical assumption about the explicit form of the radiative form factors entering into the amplitude. The resulting amplitude is gauge invariant as can be tested by substituting k_μ for ϵ^*_μ . This is guaranteed always when eqs. (12) and (13) hold. The method is well suited for the construction of RMC amplitudes on boson-like targets. Two examples of such transitions $(0^+ \to 1^+$ and $0^+ \to 2^-)$ will be presented in Part II of the present series. We prepare also a numerical study of RMC characteristics on several experimentally feasible nuclei.

Acknowledgement

We wish to thank S.M.Bilenky, L.D.Blokhintsev, R.A.Eramzhyan, R.Mach and E.Truhlík for their interest and useful discussions.

Appendix A

The constraint equations which are of interest in this paper

$$A \cdot kQ + B \cdot kq = C, \tag{A.1}$$

can be uniquely solved using the linearity hypothesis eq. (21). Assuming for the form factors A, B, C the LH form we can rewrite eq. (A.1) as

$$\begin{aligned} &\frac{A^-(q^2)}{(Q-q)k} e_i^j \cdot kQ + \frac{A^+(q^2)}{(Q+q)k} e_f^j \cdot kQ + A^0(q^2, e_i^j, e_f^j) \cdot kQ \\ &+ \frac{B^-(q^2)}{(Q-q)k} e_i^j \cdot kq + \frac{B^+(q^2)}{(Q+q)k} e_f^j \cdot kq + B^0(q^2, e_i^j, e_f^j) \cdot kq \\ &= \frac{C^-(q^2)}{(Q-q)k} e_i^j + \frac{C^+(q^2)}{(Q+q)k} e_f^j + C^0(q^2, e_i^j, e_f^j), \end{aligned} \tag{A.2}$$

where $e^j_{i(f)}$ is the electromagnetic (charge or magnetic-momentum) form factor of initial (final) nucleus.

Taking equal the coefficients of kQ and kq gives

$$A^+ = B^+, \quad A^- = -B^-, \tag{A.3}$$

$$A^0 = B^0 = C^+ = C^- = 0, \tag{A.4}$$

$$A^+ e_i + A^- e_f = C^0. \tag{A.5}$$

As C^0 is linear in e_i and e_f, A^+ and A^- can be obtained by equating the coefficients of e_i and e_f in l.h.s. and r.h.s. of (A.5). If, in particular, C^0 is independent of e_i and e_f, using $e_i = e_f + 1$ we obtain at once

$$A^+ = -A^- = C^0. \tag{A.6}$$

Then

$$\begin{aligned} A &= C\left(\frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k}\right), \\ B &= -C\left(\frac{e_f}{(Q-q)k} + \frac{e_i}{(Q+q)k}\right). \end{aligned} \tag{A.7}$$

Appendix B

Tensor $A_{\mu\lambda}(k,q,Q)$ for the $\frac{1}{2} \rightarrow \frac{1}{2}$ radiative transitions. The weak axial-vector current is taken in the usual form

$$\langle N_f(p^f) | A_\lambda(0) | N_i(p^i) \rangle = \bar{u}_f(p^f) \left[F_A(q^2) \gamma_\lambda \gamma_5 + i F_p(q^2) \frac{2M q_\lambda \gamma_5}{m_\pi^2} \right] u_i(p^i). \tag{B.1}$$

In this Appendix we wish to express the radiative form factors a^x_{ij} and d_i of eqs. (19) and (20) in terms of F_A and F_P using the constraint equations (13), (15) and LH for a^x_{ij} but refraining from the LH form for d_i. We change here the notation to that of HP1 ($a^x_{ij} \longrightarrow G^x_{ij}$, $d_i \longrightarrow f_p, f_E, \ldots$, etc.). To determine G^x_{ij} Hwang and Primakoff[4] have constructed their eqs. (16a)-(16h) from CEC condition eq. (13). For reference we quote this system here

$$\begin{aligned} G^b_{22}\, kQ + G^b_{23}\, kq &= m_p^2\, G^a_{00} \\ G^b_{12}\, kQ + G^b_{13}\, kq &= -m_p^2\, F_A(q^2) \\ G^d_{21}\, kQ + G^d_{31}\, kq &= m_p^2 \left[i\left(G^b_{00} + G^a_{11}\right) - G^d_{00} \right] \\ G^d_{22}\, kQ + G^d_{32}\, kq &= i m_p^2\, G^a_{12} \\ G^d_{23}\, kQ + G^d_{33}\, kq &= i m_p^2\, G^a_{13} \\ G^c_{21}\, kQ + G^c_{31}\, kq &= -m_p^2\, G^c_{00} \\ G^c_{22}\, kQ + G^c_{32}\, kq &= 0 \\ G^c_{23}\, kQ + G^c_{33}\, kq &= i\, \frac{2M m_p^3}{m_\pi^2} F_p(q^2). \end{aligned} \tag{B.2}$$

To solve the system (B.2) we put according to the perturbation theory:

$$G^a_{00} = - m_p F_A \left(\frac{\mu_i}{2M} + \frac{\mu_f}{2M} \right)$$

$$G^b_{00} = m_p^2 F_A \left(\frac{e_i + \mu_i}{(Q-q)k} - \frac{e_f + \mu_f}{(Q+q)k} \right)$$

$$G^a_{11} = 2 m_p^2 F_A \frac{e_f + \mu_f}{(Q+q)k} - m_p^2 F_p \frac{2M}{m_\pi^2} \left(\frac{\mu_i}{2M} - \frac{\mu_f}{2M} \right)$$

$$G^a_{12} = 0$$

$$G^a_{13} = - m_p^2 F_p \frac{2M}{m_\pi^2} \left(\frac{\mu_i}{2M} - \frac{\mu_f}{2M} \right)$$

$$G^c_{21} = i m_p^3 \left[F_A \left(\frac{1}{(Q-q)k} \cdot \frac{\mu_i}{2M} - \frac{1}{(Q+q)k} \cdot \frac{\mu_f}{2M} \right) + F_p \frac{2M}{m_\pi^2} \left(\frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right) \right]$$

$$G^d_{00} = i m_p^2 F_A \left(\frac{e_i + \mu_i}{(Q-q)k} + \frac{e_f + \mu_f}{(Q+q)k} \right)$$

Applying PCAC, (eq. (15) another set of equations can be constructed. These are eqs. (18a)-(18j) of HP1. Since the dynamical structure of the vector D_μ is arbitrary now, this second set does not impose any additional constraint on the form factors G^x_{ij} . Instead, we should calculate the form factors G^a_{21}, G^a_{22}, and G^a_{23} via an additional reference to the perturbation theory. The result is

$$G^a_{21} = - m_p^3 F_p \cdot \frac{2M}{m_\pi^2} \left(\frac{e_i + \mu_i}{(Q-q)k} - \frac{e_f + \mu_f}{(Q+q)k} \right)$$

$$G^a_{22} = 0$$

$$G^a_{23} = - m_p^3 F_p \frac{2M}{m_\pi^2} \left(\frac{e_i + \mu_i}{(Q-q)k} - \frac{e_f + \mu_f}{(Q+q)k} \right)$$

Now, using LH in the sense of Appendix A we shall get from (B.2) at once

$$G^b_{22} = - m_p^3 F_A \left(\frac{1}{(Q-q)k} \cdot \frac{\mu_i}{2M} + \frac{1}{(Q+q)k} \frac{\mu_f}{2M} \right)$$

$$G^b_{23} = m_p^3 F_A \left(\frac{1}{(Q-q)k} \cdot \frac{\mu_i}{2M} - \frac{1}{(Q+q)k} \cdot \frac{\mu_f}{2M} \right)$$

$$G^b_{12} = - m_p^2 F_A \left(\frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$G^b_{13} = m_p^2 F_A \left(\frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

$$G^d_{21} = - i m_p^4 F_p \frac{2M}{m_\pi^2} \left(\frac{1}{(Q-q)k} \cdot \frac{\mu_i}{2M} - \frac{1}{(Q+q)k} \cdot \frac{\mu_f}{2M} \right)$$

$$G^d_{31} = i m_p^4 F_p \frac{2M}{m_\pi^2} \left(\frac{1}{(Q-q)k} \frac{\mu_i}{2M} + \frac{1}{(Q+q)k} \cdot \frac{\mu_f}{2M} \right)$$

$$G^d_{22} = 0$$

$$G^d_{32} = 0$$

$$G^d_{23} = - i m_p^4 F_p \frac{2M}{m_\pi^2} \left(\frac{1}{(Q-q)k} \cdot \frac{\mu_i}{2M} - \frac{1}{(Q+q)k} \cdot \frac{\mu_f}{2M} \right)$$

$$G^d_{33} = i m_p^4 F_p \frac{2M}{m_\pi^2} \left(\frac{1}{(Q-q)k} \cdot \frac{\mu_i}{2M} + \frac{1}{(Q+q)k} \cdot \frac{\mu_f}{2M} \right)$$

$$G^c_{31} = - i m_p^3 \left[F_A \left(\frac{1}{(Q-q)k} \cdot \frac{\mu_i}{2M} + \frac{1}{(Q+q)k} \cdot \frac{\mu_f}{2M} \right) + \right.$$
$$\left. + F_p \frac{2M}{m_\pi^2} \left(\frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right) \right]$$

$$G^c_{00} = - i m_p \left[F_A \left(\frac{\mu_i}{2M} - \frac{\mu_f}{2M} \right) + F_p \frac{2M}{m_\pi^2} \right]$$

$$G^c_{22} = 0$$

$$G^c_{32} = 0$$

$$G^c_{23} = i \frac{2M m_p^3}{m_\pi^2} F_p \left(\frac{e_i}{(Q-q)k} - \frac{e_f}{(Q+q)k} \right)$$

$$G^c_{33} = - i \frac{2M m_p^3}{m_\pi^2} F_p \left(\frac{e_i}{(Q-q)k} + \frac{e_f}{(Q+q)k} \right)$$

We do not display here the lengthy expressions for the form factors $f_P, f_E, \ldots$ entering into D_μ. One may easily obtain them using the PCAC constraints (eqs. (18a)-(18j) of HP1) since all form factors G_{ij}^{*} are already known.

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Received by Publishing Department
on June 10 1979.