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THE SUGGESTION ABOUT THE ABSENCE OF THE PHASE TRANSITION, VACUUM DEGENERATION, SPONTANEOUS SYMMETRY BREAKING AND ZERO-MASS GOLDSTONE-BOSONS IN QFT MODELS OF THE TYPE $g\varphi^4$

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Предположение об отсутствии вырождения вакуума, фазового перехода, спонтанного нарушения симметрии и голдстоуновских частиц нулевой массы покоя в моделях типа $g\phi^4$ квантовой теории поля

Рассматривается простейшее доказательство существования фазового перехода в модели $g\phi^4$. Показано, что это доказательство, как оно ни правдоподобно, является ошибочным В связи с этим высказывается предположение об отсутсвии в моделях типа $g\phi^4$ фазового перехода, вырождения вакуума, спонтанного нарушения симметрии и голдстоуновских бозонов нулевой массы покоя.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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The Suggestion about the Absence of the Phase Transition, Vacuum Degeneration, Spontaneous Symmetry Breaking and Zero-Mass Goldstone-Bosons in QFT Models of the Type $g\phi^4$

The usual proof of the phase transition existence in the model $g\phi^4$ is considered. (For $M^2 > M_0^2$ minimum of the effective potential is at $\phi(0)=0$, for $M^2 < M_0^2$ this minimum is at $\phi(0)=\pm\lambda \neq 0$, $\lambda \to +\infty$ at $M^2 \to -\infty$). This proof is shown to be wrong, thus suggesting the absence in the model considered of the phase transition, vacuum degeneration, spontaneous symmetry breaking and zero-mass Goldstone particles.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. *INTRODUCTION*

It is widely accepted now (after the work of Golstone [1]) that the model defined by the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\sum_{\alpha=1}^{d}\left(\frac{\partial\tilde{\phi}}{\partial x_\alpha}\right)^2 - \frac{1}{2}M^2\tilde{\phi}^2 - g\tilde{\phi}^4 \qquad (1)$$

undergoes a phase transition at some real value of the parameter M^2, $M^2 = M_0^2$.

1. In the region

$$M^2 > M_0^2 \qquad (2)$$

the ground-state is nondegenerated and is even in $\tilde{\phi}$ (like the Hamiltonian), whereas in the region

$$M_0^2 > M^2 \qquad (3)$$

the ground-state is doubly degenerated, so that there exists the ground-state functional even in $\tilde{\phi}$.

$$\Omega_{01}(-\tilde{\phi}) = \Omega_{01}(\tilde{\phi}), \qquad (4)$$

and the ground-state functional $\Omega_{02}(\tilde{\phi})$ odd in $\tilde{\phi}$,

$$\Omega_{02}(-\tilde{\phi}) = -\Omega_{02}(\tilde{\phi}), \qquad (5)$$

(both functionals $\Omega_{0i}(\tilde{\phi})$, $i = 1, 2$, belong to the same lowest eigenvalue of the Hamiltonian

$$(H - E_0)\Omega_{0i}(\tilde{\phi}) = 0, \quad i = 1, 2). \qquad (6)$$

1.1. The ground-state functional $\Omega_{02}(\tilde{\phi})$ has no the symmetry of the Hamiltonian under the operation $\tilde{\phi} \to -\tilde{\phi}$; one denotes this phenomenon as the spontaneous breaking of the symmetry of the Hamiltonian, the spontaneous breaking of the symmetry exists in the region (3).

1.2. Produce the operation

$$\tilde{\phi}^2 \to \sum_{\gamma=1}^{m} \tilde{\phi}_\gamma^2, \quad \sum_{\alpha=1}^{d} \left(\frac{\partial \tilde{\phi}}{\partial x_\alpha}\right)^2 \to \sum_{\alpha=1}^{d} \sum_{\gamma=1}^{m} \left(\frac{\partial \tilde{\phi}_\gamma}{\partial x_\alpha}\right)^2 \tag{7}$$

in the Lagrangian (1), the resulting Lagrangian has O(m)-symmetry. In the system with the spontaneously broken O(m) symmetry there exist the ground-state functionals $\tilde{\Omega}_0(\tilde{\phi})$ with the property

$$\lambda_i = \int \phi_i(x)\, \tilde{\Omega}_0(\tilde{\phi})^2 \delta\tilde{\phi} \,/\, \int \tilde{\Omega}_0(\tilde{\phi})^2 \delta\tilde{\phi} \neq 0. \tag{8}$$

1.2.1. The non-zero value of λ_i (8) implies the appearance of the zero-mass particles, Goldstone bosons [1,2].

1.3. Whereas in the region (2) the Green function $G(p, M^2)$

$$\int e^{-S} \phi(p_1)\phi(p_2)\delta\phi \,/ \int e^{-S} \delta\phi = \delta(p_1+p_2)\, G(p_1, M^2), \tag{9}$$

$$S = \int \mathcal{L}\, dx = \frac{1}{2} \int (p^2 + M^2)\phi(p)\phi(-p)\, dp$$

$$+ g \int \left(\prod_{i=1}^{4} \phi(p_i)\, dp_i\right) \delta\left(\sum_{i=1}^{4} p_i\right), \tag{10}$$

is continuous, in the region (3) this function has the form

$$G(p, M^2) = \gamma_0^2(M^2)\delta(p) + D(p, M^2); \tag{11}$$

here the function $\gamma_0^2(M^2)$ is positive and the function $D(p, M^2)$ is continuous.

1.3.1. In the region (2) the integral (9) is determined by the integration area

$$\phi(0) \sim O(\sqrt{\delta(0)}) \tag{12}$$

over the variable $\phi(0)$;

$$\phi(0) = (2\pi)^{-d/2} \int \tilde{\phi}(x)\, dx \,, \qquad (13)$$

$$\phi(p) = (2\pi)^{-d/2} \int \tilde{\phi}(x)\, e^{ipx}\, dx \,, \qquad (14)$$

$$\delta(p) = (2\pi)^{-d/2} \int e^{ipx}\, dx \,, \qquad (15)$$

after introducting the spatial cutoff, the quantity $\delta(0)$ becomes finite. In the region (3) the integral (9) is determined by the areas

$$|\phi(0) \pm \gamma_0 (M^2)\, \delta(0)| \sim O(\sqrt{\delta(0)}) \qquad (16)$$

of integration over the variable $\phi(0)$. These areas, unlike (12), do not contain the point $\phi(0) = 0$. Such a phenomenon was first discovered by Gross and Neveu [3]. For the model $g\phi^4$ this phenomenon was considered in our work [4].

1.4. So, we have finished the statement of the main facts of the theory of the spontaneous symmetry breaking.

1.5. The article is planned as follows.

In Sec. 2, we consider in detail the simplest proof of the statements of Sec. 1.3 [4]. In Sec. 3 we show this proof, though likely, to be wrong: the method of Sec. 2 shows the phase transition also for the ultralocal Lagrangian (37), which obviously describes no phase transition. In Sec. 3 we give also some new suggestions concerning the decompositions (46), (47), introduced in our works [5,6,8] in order to study the models $g\phi^4$, $g(\phi^*\phi)^2$ in the region (3).

1.5.1. We state the expansion (46), e.g., to give the ground state with finite error $O(e^{-\beta^2})$(for the model $g\phi^4$).

1.5.2. In Sec.4 we consider the possibilities coming from the discovered failure of the phase-transition proof. This proof was the only support for the spontaneous breaking of symmetry theory. So, the most probable result of the failure of the proof is the absence of the vacuum degeneration, the spontaneous symmetry breaking, and Goldstone bosons.

1.6. It seems to be of substantial interest to learn whether the decompositions used by Dobrushin and Minlos [9] and by Glimm, Jaffe and Spencer [10] to construct the ground-state functionals with the broken symmetry are able to withstand our criterion, the application to the ultralocal Hamiltonian (45); this Hamiltonian obviously possesses a nondegenerated ground state.

1.7. Note also the following important remark for the case $d=4$. Let us define the function $G'(p, M'^2)$ via eqs. (9), (10a). If the system undergoes phase transition at $M'^2 = M_0'^2$, then $G'(0, M_0'^2) = \infty$; if there is no phase transition in the whole region $-\infty < M'^2 < +\infty$, then $G'(0, M'^2) < \infty$ for all M'^2; $G'(0, M'^2) \to \infty$ if $M'^2 \to -\infty$. The physical Green function $G(p, M^2)$, according to equations of Sec. 3.5, equals $G(p, M^2) = G'(p/\ell, M^2/\ell^2)/\ell^2$. Introduce the squared mass operator $\Sigma(p, M^2)$ via the equation $G(p, M^2) = [p^2 + \Sigma(p, M^2)]^{-1}$ then $\Sigma(0, M^2) = \ell^2/G'(0, M'^2)$, so that the mass of particle diverges linearly; the model (1) describes, at each finite value M'^2, the infinitely heavy particles. Let us take, however, M'^2 to be the function of ℓ : $M'^2 = M'^2(\ell)$, $M'^2(\ell) \to -\infty$ if $\ell \to \infty$. Then one can choose the function $M'^2(\ell)$ so that the function $\Sigma(0, \ell^2 M'^2(\ell))$ tends to a finite or zero limit as $\ell \to \infty$. Thus, the model considered provides the possibility of renormalization and description of zero-mass particles.

2. *THE PROOF OF STATEMENTS OF SEC. 1.3.*

2. Here we shall prove the statement of Sec. 1.3 in the simplest case $d=1$. Let us introduce in eq. (10) for S the momentum and spatial cutoff:

$$S = (h/2) \sum_p (p^2 + M^2)\phi(p)\phi(-p) + g h^3 \sum_{p_1+p_2+p_3+p_4=0} \left(\prod_{i=1}^{4} \phi(p_i) \right), \tag{17}$$

where

$$p = ih, \quad i = 0, \pm 1, \pm 2, \ldots, \quad |p| < \ell, \tag{18}$$

$$\delta(0) = 1/h . \tag{19}$$

Produce in eq. (17) the change of variables

$$\phi(p) = \delta_p \gamma/h + \psi(p), \quad \psi(0) = 0, \tag{20}$$

$$\delta_p = \begin{cases} 1 & p=0 \\ 0 & p \neq 0 . \end{cases} \tag{20a}$$

Then eq. (17) takes the form

$$S = (M^2\gamma^2/2 + g\gamma^4)/h + (h/2)\sum_p (p^2+M^2+12g\gamma^2)\psi(p)\psi(-p) +$$
$$+ 4g\gamma h^2 \sum_{p_1+p_2+p_3=0} \left(\prod_{i=1}^{3}\psi(p_i)\right) + gh^3 \sum_{p_1+p_2+p_3+p_4=0} \left(\prod_{i=1}^{4}\psi(p_i)\right). \quad (21)$$

One has, evidently

$$\int \phi(p)\,\phi(-p)\,e^{-S}\,\delta\phi = \int [\delta_p(\gamma/h)^2 + \psi(p)\psi(-p)]\,e^{-S}\,\delta\psi\, d\gamma/h\,. \quad (22)$$

2.1. Consider at first the case

$$M^2 \to +\infty. \quad (23)$$

Using eq. (17) and the expansion

$$e^{-S_2 - gS_4} = e^{-S_2}[1 - gS_4 + \frac{1}{2!}(gS_4)^2 + \dots] \quad (24)$$

gives

$$\int e^{-S}\delta\phi = \prod_p \left(\frac{2\pi}{h(p^2+M^2)}\right)^{1/2} \exp\{-\frac{3g}{h}\left(\sum_p \frac{h}{p^2+M^2}\right)^2 + O(\frac{g^2}{hM^5})\},$$
$$(25)$$

$$h\int e^{-S}\,\phi(p)\phi(-p)\delta\phi \,/ \int e^{-S}\delta\phi = \frac{1}{p^2+M^2} -$$

$$- \frac{12g}{(p^2+M^2)^2}\sum_q \frac{h}{q^2+M^2} + O(\frac{g^2}{M^8}). \quad (26)$$

2.1.1. We have $d=1$, so all the integrals do converge. One sees the Green function (26) in the case (23) to be continuous, in agreement with Sec. 1.3.

In deriving eqs. (25), (26) some formulae are useful, they are listed, e.g., in Sec. 2.7 of our work [5].

2.2. Now consider the case

$$M^2 \to -\infty . \tag{27}$$

Analogously to eqs. (16), (25), (26) eq. (21) leeds to the formulae

$$\int e^{-S}\,\delta\psi = \exp\{X(\gamma^2, M^2)/h\}, \tag{28}$$

$$X(\gamma^2, M^2) = -M^2\gamma^2/2 - g\gamma^4 - \frac{1}{2}\int \ln\frac{(p^2+m^2)h}{2\pi}\,dp - 3g\left(\int\frac{dp}{p^2+m^2}\right)^2$$

$$+ \frac{(4q\gamma)^2}{2!}\,3!\int\left(\prod_{i=1}^{3}\frac{dp_i}{p_i^2+m^2}\right)\delta\left(\sum_{i=1}^{3}p_i\right)+\dots, \tag{29}$$

$$m^2 = M^2 + 12g\gamma^2, \tag{30}$$

$$h\int e^{-S}\psi(p)\psi(-p)\delta\psi \Big/ \int e^{-S}\,\delta\psi = \frac{1}{p^2+m^2} - \frac{12g}{(p^2+m^2)^2}\int\frac{dq}{q^2+m^2}$$

$$+ \frac{(4g\gamma)^2}{2!}\cdot\frac{6\cdot 3}{(p^2+m^2)^2}\int\left(\prod_{i=1}^{2}\frac{dq_i}{q_i^2+m^2}\right)\delta\left(\sum_{i=1}^{2}q_i - p\right)+\dots \tag{31}$$

Equation (29) implies the function $X(\gamma^2, M^2)$ to have a minimum at

$$-\frac{M^2}{2} - 2g\gamma^2 - 6g\int\frac{dq}{q^2+m^2} = O\left(\frac{g^2}{m^4}\right), \tag{32}$$

i.e.,

$$4g\gamma^2 = 4g\gamma_0^2 = -M^2 - \frac{12\pi g}{\sqrt{-2M^2}} + \dots \tag{32a}$$

The γ-integral in eq. (22) is determined by the regions (16), (19) around the saddle points $\gamma = \pm\gamma_0$. We have, of course

$$h \to 0. \tag{33}$$

Equations (22) and (31) are equavialent to eq. (11).

2.3. So, we have completed the proof of the results which are listed in Sec. 1.3.

2.4. The proof given seems to be quite likely. But really it is wrong. We shall prove the latter statement in Sec. 3. We shall consider a simple example, where the consideration of Sec. 2 gives obviously the wrong result.

3. THE PROOF OF SEC. 2 IS WRONG

3. Consider the system defined by the Action

$$S' = \frac{h}{2} \sum_{p} M^2 \phi(p)\phi(-p) + gh^3 \sum_{p,\lambda} \left(\prod_{i=1}^{4} \phi(p_i) \right) \delta_{\sum_{i=1}^{4} p_i - 2\lambda\ell}, \tag{34}$$

where the p -summations run over the net (18), this time we take ℓ to be of order of

$$\ell \sim g^{1/3}, \tag{35}$$

δ_q is the function (20a), in (34) the sum is taken over the integer values of λ.

3.1. Transform (34) via the substitution

$$\phi(ih) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} e^{2\pi j \frac{ik}{N}} X_k, \quad N = 2n+1, \quad nh = \ell, \quad j^2 = -1. \tag{36}$$

Then one gets the ultralocal form of S':

$$S' = \sum_{k=1}^{N} (hM^2 X_k^2/2 + 2gh^2 X_k^4). \tag{37}$$

3.2. One can obtain the expression of the Green function (9)

a) either from the representation (34) of the Action via the method of Sec. 2,

b) or directly from the representation (37) of the Action. These two expressions coincide in case (23) and do not coincide in case (27).

3.2.a. Proceeding from the representation (34), one gets

$$G'(p,M^2) = \delta_p \gamma_0^2 /h + \frac{1}{m^2} - \frac{12g}{m^4} \int_{-\ell}^{\ell} \frac{dq}{m^2} +$$

$$+\frac{(4g\gamma_0)^2}{2!\,m^4}6\cdot 3\int\left(\prod_{i=1}^{2}\frac{dq_i}{m^2}\right)\sum_\lambda \delta\left(\sum_{i=1}^{2}q_i-p-2\ell\lambda\right)\sim-\frac{M^2}{4gh}\delta_p+\frac{1}{m^2}+\dots \tag{38}$$

for, analogously to eq. (32)

$$4g\gamma_0^2=-M^2-12g\int_{-\ell}^{\ell}\frac{dq}{m^2}+\dots \tag{39}$$

Note also, that

$$\sum_\lambda \delta(x-2\ell\lambda)=\frac{1}{2\pi}\sum_n e^{in\pi x/\ell} \tag{40}$$

and

$$\int_{-\ell}^{\ell} e^{in\pi x/\ell}\,dx=0 \qquad \text{if} \quad n\neq 0. \tag{41}$$

Equations (40), (41) imply the function (38) not to depend on p.

3.2.b. Proceeding from the representation (37), one gets $G(ih,M^2)=h\int\phi(ih)\phi(-ih)e^{-S'}\delta\phi/\int e^{-S'}\delta\phi=$

$$=(h/N)\sum_{k_1,k_2=1}^{N}e^{2\pi ji(k_1-k_2)/N}\int X_{k_1}X_{k_2}\,e^{-S'}\delta X/\int e^{-S'}\delta X=$$

$$=(h/N)(N/h)\,\overline{Y^2}(M^2,g), \tag{42}$$

where

$$\overline{Y^2}(M^2,g)=\int_{-\infty}^{+\infty}dy\,y^2\exp\{-M^2y^2/2-2gy^4\}/$$

$$\int_{-\infty}^{+\infty}dy\,\exp\{-M^2y^2/2-2gy^4\}\sim -M^2/(8g) \quad \text{if} \quad M^2\to-\infty. \tag{43}$$

3.3. There are no features of similarity between eqs. (38) and (42), (43). Equation (42) is, without any doubt correct, so eq. (38) is wrong (we have done some unknown mistake in deriving eqs. (28), (29)). Meanwhile the first term of the r.h.s. of eq. (38) was the only indicator of the phase transition, which the system undergoes at some point $M^2=M_0^2$ between the cases (23) and (27).

3.4. So, the suggestion arises that the system under consideration has no phase transition at all. This suggestion is not the only possible, but it seems to be the more probable.

This suggestion is considered in more detail in Sec. 4.

3.5. It is not difficult to generalize our consideration to the cases $d=2$ and $d=3$ [4]. It can be also generalized to the case $d=4$: one has to produce the change of variables $p=\ell q$, $M^2=\ell^2 M'^2$, $\phi(p)=\phi'(q)/\ell^3$, then the Action (10) becomes:

$$\rho=\frac{1}{2}\int_{|q|<1}(q^2+M'^2)\phi'(q)\phi'(-q)d^4q + g\int_{|q_i|<1}\left(\prod_{i=1}^{4}\phi(q_i)d^4q_i\right)\delta\left(\sum_{i=1}^{4}q_i\right). \tag{10a}$$

Then, the integration regions in eqs.(29), (31) are finite, so the integrals are finite, and so on.

3.6. Another proof of the phase-transition existence in system (1) was given in our early work [5]. We have considered in [5] the ground states of the Hamiltonian

$$H=\frac{1}{2}\int dp\left[-\frac{\delta^2}{\delta\phi(p)\delta\phi(-p)}+(p^2+M^2)\phi(p)\phi(-p)\right] + g\int\left(\prod_{k=1}^{4}\phi(p_k)dp_k\right)\delta\left(\sum_{i=1}^{4}p_i\right). \tag{44}$$

The proof of work [5] is easily seen to predict the phase transition not only for the Hamiltonian (44) but also for the ultralocal Hamiltonian

$$\tilde{H}=\frac{1}{2}\int dp\left[-\frac{\delta^2}{\delta\phi(p)\delta\phi(-p)}+M^2\phi(p)\phi(-p)\right]+ g\int\left(\prod_{i=1}^{4}\phi(p_i)dp_i\right)\delta\left(\sum_{i=1}^{4}p_i\right) \tag{45}$$

which obviously has no phase transition, so the proof of [5] is a mistake.

3.7. Note by the way that the expansions of work [5]

$$\Omega_{0\pm}(\phi)=\exp\left\{-\beta^2\sum_{n=0}^{\infty}\kappa_n(\phi/\beta)\beta^{-2n}\right\},\qquad d=2, \tag{46}$$

with

$$M^2 = -4g\beta^2 + 12g \ln \frac{\ell}{\sqrt{8g\beta^2}} + \dots , \quad (47)$$

give no exact eigenfunctions of the Hamiltonian, one has to add to (46) the terms $O(e^{-\beta^2})$ in order to fulfil the conditions at $\phi = 0$ (see the problem on the two-well potential [7], chapter 7).

4. CONCLUSION

If the above suggestion about the absence of phase transition is correct, then our system (1) also has no vacuum degeneration, no spontaneous breaking symmetry, no zero-mass Goldstone besons and no zero-mass particles, which are associated with phase transition (see, however, Sec. 1.7). Our suggestion is evidently tightly connected with such matters, as Higgs mechanism and Weinberg-Salam model.

REFERENCES

1. Goldstone J. Nuovo Cimento, 1961, 19, p.154.
2. Goldstone J., Salam A., Weinberg S. Phys.Rev., 1962, 127, p.965.
3. Gross D.J., Neveu A. Phys.Rev., 1974, D10, p.3235.
4. Zastavenko L.G. JINR E2-11585, Dubna, 1978.
5. Zastavenko L.G. TMF, 1971, 7, p.20.
6. Zastavenko L.G., TMF, 1971, 9, p.355.
7. Landau L.D., Liefshitz E.M. Quantum Mechanics, Moscow, 1948.
8. Zastavenko L.G. TMF,1971, 8, p.335.
9. Dobrushin R., Minlos R. Funct.Anal.Appl., 1973, 7, p.324.
10. Glimm J., Jaffe A., Spencer T. Commun.Math.Phys., 1975, 45, p.203-216.

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