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S.B.Gerasimov

SUM RULES FOR STRUCTURE PAPAMETERS
OF HYPERONS IN BROKEN SU(3)

Правила сумм для структурных параметров гиперонов в нарушенной SU(3)

Предложены решения системы правил сумм для магнитных Моментов барионов и логарифмических наклонов дифференциальных сечений взаимодействия, согласующиеся с экспериментальными данными, которые включают большое значение магнитного момента $\Xi^{-}$-гиперона, представляющее трудность для стандартных теоретических моделей.

Работа выполнена в Лаборатории теоретической физики оияи.

Преприит Объединенного институтв ядерных исследовании. Дубна $197 \theta$

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Sum Rules for Structure Parameters of Hyperons in Broken $\operatorname{SU}(3)$

The solutions of a set of sum rules for magnetic moments of baryons and slopes of the form factors at the Po-meron-baryon vertices (i.e., the logarithmic slopes of the corresponding baryonic differential cross sections) are presented. They agree with available experimental data, in particular, with large magnetic moment of the $\Xi^{-}$-hyperon, causing difficulty for standard theoretical models. Some other consequences to be tested in experiments with high energy hyperon beams are proposed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

1. The forthcoming experiments with hyperon beams will enable one to test systematically various predictions of the broken-symmetry theory and dynamical models concerning the properties of baryons and their interactions. Recent precise measurement of the $\Lambda$-hyperon magnetic moment (m.m.) 'i exemplifies rapid progress in this field as well as apparent relevance of the nonrelativistic quark model, including the breaking of the SU(3) symmetry in the framework of hypotheses on additivity of the current operators of constituent quarks and proportionality of the single quark m.m. to the ratio of its charge to mass ${ }^{\prime 2 /}$, because the mass ratio $\mathrm{m}_{\mathrm{d}} / \mathrm{m}_{\mathrm{s}}=2 \mu(\Lambda) / \mu(\mathrm{N})$, defined from now well-known m.m.'s, corresponds surprisingly well to that determined with the aid of the mass formulas $/ 3,4 /$. Nevertheless, the large value of the $\Xi^{-}$-hyperon m.m. $\mu_{\text {exp }}\left(\Xi^{-}\right) / \mu_{\text {q. }}\left(\bar{\Pi}_{\bar{\sigma}}^{-}\right)=4.30 \pm 1.74$ causes some concern. Leaving to more prectse measurement the final decision on this matter, it seems worthwhile to attempt alternative theoretical approaches. One of our objectives in this note is to present the possible solution of a set of sum rules (s.r.) for the m.m.'s of baryons, which is consistent with all data including the large m.m. of $E^{-}$-hyperon.

From conventional assumptions on the octet properties of the e.m. current and the $\mathrm{SU}(3)$-breaking interaction, one obtains

$$
\begin{align*}
& 2 \sqrt{3} \mu\left(\Lambda \Sigma^{0}\right)-3 \mu(\Lambda)-\mu\left(\Sigma^{0}\right)+2 \mu(\mathrm{~N})+2 \mu\left(\xi^{0}\right)=0,  \tag{1}\\
& \mu(\mathrm{P})-\mu(\mathrm{N})-\mu\left(\Sigma^{+}\right)+\mu\left(\Sigma^{-}\right)+\mu\left(\Xi^{0}\right)-\mu\left(E^{-}\right)=0 . \tag{2}
\end{align*}
$$

Eq. (1) is the first-order perturbation formula with respect to the SU(3)-breaking interaction $/ 5 /$. Our choice of Eq. (2) is based on the following arguments. The s.r. for m.m.'s may, evidently, be obtained from the consideration of the mass operators in the external e.m. field. Due to similarity of the transformation properties of corresponding mass
operators with regard to the internal symmetry groups (e.g., U-spin group) the s.r.'s for the energy shifts (or mass shifts) of baryons in an external e.m. field will, formally, have the same appearance as the mass formulas taking the radiative e.m. corrections into account. Differentiating the relations thus obtained with respect to the external field and letting then the field go to zero, we get, as a result, the s.r.'s for m.m., the electric and magnetic polarizability coefficients, etc. As a short excursion, it is worth noticing, that if the baryon mass formulas take the mass-squared form, the replacement $\mu(B) \rightarrow M(B) \mu(B)$ should be made in Eqs. (1) and (2), where M(B) stands for the mass of corresponding baryon, i.e., s.r.'s should, in this case, be written down for the magnetic moments taken in the "natural" (or internal) magnetons. In what follows we, however, bear in mind more "orthodox", linear mass relations, therefore all m.m.'s will be kept in standard units, nuclear magnetons. Further, the Coleman-Glashow (CG) ${ }^{/ 6 /}$ relation for the e.m. mass difference in the baryon octet is known to agree with data very well. Derived previously in the framework of exact SU(3), this relation remains intact after introducing into the consideration of additional terms, taking partially into account the interference between the electromagnetic and "medium-strong" interaction, violating SU(3)-symmetry. That is why the s.r. for mass shifts in the external field, coinciding formally with the CG-relation is believed to be more reliable. Hence, the Eq. (2) follows. Using (1), (2), the known isotopic relation $2 \mu\left(\Sigma^{0}\right)=\mu\left(\Sigma^{+}\right)+\mu\left(\Sigma^{-}\right)$and the experimental data on other hyperons, we have $\mu\left(\xi^{-}\right)=-1.42_{-0.59}^{+0.65} \mathrm{n} . \mathrm{m}$. . demonstrating the compatibility between more general theoretical s.r.'s and the large value of $\mu_{\text {exp }}\left(E^{-}\right)=-1.85 \pm 0.75 \mathrm{n} . \mathrm{m}$. To obtain more definite results without the quark model, we use the dispersion s.r. for the anomalous magnetic moments (a.m.m.) ${ }^{18,9 /}$, saturated by the photoexcitation cross sections of the lowest decuplet and singlet baryonic resonances, the coupling constants $\mathrm{B}^{*} \rightarrow \mathrm{~B} \gamma$ being treated according to the broken SU(3). Some consequences of this set of assumptions were considered earlier by Cheng and Pagels/10/. As a basis for the further discussion we use the following relations between the a.m.m.'s $\kappa(B)$ :

$$
\begin{align*}
& \kappa(P)=-\kappa(N)  \tag{3}\\
& \kappa\left(\Sigma^{0}\right)=\frac{1}{2}\left(\kappa\left(\Sigma^{+}\right)+\kappa\left(\Sigma^{-}\right)\right)=-\kappa(\Lambda) \tag{4}
\end{align*}
$$

$$
\kappa\left(E^{-}\right)=1_{-\kappa\left(\Sigma^{-}\right)}^{+\kappa\left(\Sigma^{-}\right)}
$$

We delineate briefly the points of difference with Ref. ${ }^{/ 10}$ :
(a) On account of the stability criterion under the SU(3)--breaking we retain Eq. (4) and do not consider, at variance with/10/, other relations which may follow from the singletdecuplet saturation of the dispersion s.r. for the A -and $\mathbf{\Sigma}^{\circ}$ hyperon a.m.m.'s;
(b) As soon as $\kappa\left(\Sigma^{-}\right)$and $\kappa\left(\Xi^{-}\right)$go to zero in the exact $S U(3)$ it seems to us reasonable, in the real world of the broken symmetry, to explore both possibilities for their relative signs (the equality $\kappa^{2}\left(\Sigma^{-}\right)=\kappa^{2}(E$ ) follows only from the decuplet saturation of the dispersion s.r.).
(c) The m.m.'s are taken in nuclear magnetons. It is the relative sign of $\kappa\left(\Sigma^{-}\right)$and $\kappa\left(\Xi^{-}\right)$which presents the most important and crucial question. In this connection we note that Eqs. (4) and (5b) follow from Eq. (3) and the more restrictive set of s.r.:

$$
\begin{align*}
& \delta \Sigma=\delta \Lambda\left(1+2 \frac{\mathrm{~N}}{\mathrm{P}}\right)  \tag{6}\\
& \delta \Xi=2 \delta \Lambda\left(1+\frac{1}{2} \frac{\mathrm{~N}}{\mathrm{P}}\right) \tag{7}
\end{align*}
$$

$\delta\left(\Lambda \Sigma^{\circ}\right)=0$,
where $\delta \mathrm{Y} \equiv \mathrm{Y}-\mathrm{Y}_{\mathrm{SU}(3)}$ and the particle symbols denote the corresponding baryon a.m.m.'s. The relations (6)-(8) were derived earlier/11/ for the baryon m.m.'s within the dynamical model of SU(3) -breaking and, by construction, should be valid in any composite quark model with the electromagnetic transition operators being the sum of the single-quark operators. In view of conformity of Eq. (5b) with Eqs. (6), (7) we apt to link the choice of the opposite signs of $\kappa\left(\Sigma^{-}\right)$and $\kappa\left(\Xi^{-}\right)$, accepted also in Ref./10/ for some other reasons, to the nonexotic exchange dominance (or to the single-quark operator dominance, using language of the composite-quark model). But thịs choice leads us to the small value of $\mu(E)$ comparable to that of the standard quark model. On the other hand, if the possibility is assumed on substantial deviation from the quark additivity (or, in a more general terms, from the nonexotic, intermediate-state dominance), as might be, for example, in the quark-diquark model of baryons ${ }^{/ 12 /}$, the diquark mass being essentially different from the sum of two
constituent quark masses, then the joint analysis of s.r. (1)-(5a) appears, logically, to be acceptable. Moreover, the adequacy of Eq. (2) is suggestive on the possible role of the vector diquarks, because it includes the amplitude with the transformation properties of the $27-$ plet of the SU(3) group which may dynamically be realized via the tensor product of the diquark field operators $\left(6 \times 6^{*}=1+8+27\right)$. Using now the experimental values of the $P, N, \Lambda$ and $\Sigma^{+}$m.m.'s we get: $\mu(\Lambda \Sigma 9)=$ $1.80 \pm 0.14 ; \mu\left(\Sigma^{-}\right)=-1.6 \pm 0.25 ; \mu\left(\Xi^{\circ}\right)=-1.83 \pm 0.25 ; \mu\left(\bar{E}^{-}\right)=$ $-1.53 \pm 0.25$, where all values are in nuclear magnetons and ascribed uncertainties are due to errors in measurement of /7/ $\mu\left(\Sigma^{+}\right)$. All quantities thus obtained agree with experiment within the range of one standard deviation*.

An additional evidence to large values of $\mu\left(\Sigma^{-}\right)$and $\mu\left(\Xi^{*}\right)$ would be an observation of the unexpectedly large radiative widths of decuplets: $\Gamma\left(\Sigma^{*-}(1385), \Sigma^{-} \gamma\right)=30_{-15}^{+21} \mathrm{keV}$ and $\Gamma(E *(1530) \rightarrow$ $\left.\rightarrow E^{-} \gamma\right)=44_{-22}^{+31} \mathrm{keV}$. These estimates are obtained from the relation

$$
\begin{equation*}
\Gamma\left(B^{*} \rightarrow B \gamma\right)=\frac{1}{2}\left(\mu(B)-\frac{Q(B)}{2 M(B)}\right)^{2}\left[\frac{M^{2}\left(B^{*}\right)-M^{2}(B)}{M\left(B^{*}\right)}\right]^{3} \tag{9}
\end{equation*}
$$

which follows from the decuplet saturation of the dispersion sum rule for $\kappa\left(\Sigma^{-}\right)$and $\kappa\left(\Xi^{-}\right)$, and the above calculated values of $\mu\left(\Sigma^{-}\right)$and $\mu\left(\xi^{-}\right)$. They do not contradict rather a large upper limit $\Gamma_{\text {exp }}\left(E^{*-} \rightarrow \bar{E}^{-} \gamma\right) \leq 400 \mathrm{keV}^{/ 13 /}$ and, at the same time, exceed by more than an order of magnitude the width computed via the standard quark model including the SU(3) breaking (we remind that the radiative decays under discussion are forbidden in the exact $S U(3)$ ). The radiative widths of an order of ten's keV should be readily measurable in the reaction of the Coulomb dissociation of hyperon beams interacting with heavy nuclei.
2. The $S U(3)$-breaking may also result in a change of the electric charge and nuclear-matter spatial distribution inside hyperons comparatively to nucleons. Some estimates of these effects were undertaken earlier with the aid of the nonrelativistic quark model ${ }^{1 / 2}$. Here, we compare the logarithmic slopes of the differential cross sections $\frac{d o}{d t}(Y N)$ to that of the NN-scattering, as these quantities are presently most readily accessible to the experimental check. Let us assume that at high enough energies the baryon elas-

[^0]tic scattering, is dominated by the (approximately) factorizable Pomeron. Consider now the Pomeron-baryon vertex (PBB). The coupling constants at $t=0$ define total cross sections. Our basic observation is that the introduction of single phenomenological "spurion" describing the octet nature of the SU(3) - violating interaction into otherwise singlet PBB-vertex will change simultaneously both the dynamic (cross sections, etc.) and static (masses, radii, etc.) characteristics of particles depending on their internal quantum numbers. Hence, the corresponding s.r.'s may be written down not only for the coupling constants (i.e., total cross sections) but also for the slopes of the form factor in the PBB-vertex. With the assumed factorization, we have two relations repeating, naturally, the structure of the Gell-Mann-Okubo mass formula:
\[

$$
\begin{align*}
& \sigma_{\mathrm{t}}(\mathrm{NN})+\sigma_{\mathrm{t}}(E \mathrm{~N})-\frac{1}{2}\left(3 \sigma_{\mathrm{t}}(\Lambda \mathrm{~N})+\sigma_{\mathrm{t}}(\mathrm{\Sigma} \mathrm{~N})\right)=0,  \tag{10}\\
& \mathrm{~b}(\mathrm{~N}) \sigma_{\mathrm{t}}(\mathrm{NN})+\mathrm{b}(E) \sigma_{\mathrm{t}}(\Xi \mathrm{~N})-\frac{1}{2}\left(3 \mathrm{~b}(\Lambda) \sigma_{\mathrm{t}}(\Lambda \mathrm{~N})+\mathrm{b}(\Sigma) \sigma_{\mathrm{t}}(\Sigma \mathrm{~N})\right)=0, \tag{11}
\end{align*}
$$
\]

$\mathrm{b}(\mathrm{B})$ being the logarithmic slope of the BN -scattering differential cross section. For the simple estimation and bearing in mind a similar quark content of the $\Lambda$ - and $\Sigma$-hyperons we put, tentatively, $\sigma_{t}(\Sigma N)=\sigma_{t}(\Lambda N)$ and $b(\Lambda) \approx b(\Sigma)$. Then using the data at $19 \mathrm{GeV}^{\mathrm{t}} / \mathrm{c}^{/ 14 /}: \sigma_{\mathrm{t}}(\Lambda \mathrm{P})=\sigma_{\mathrm{t}}\left(\sum \mathrm{P}\right)=34.6 \pm 0.4 \mathrm{mb}$, $\mathrm{b}(\Lambda) / \mathrm{b}(\mathrm{P})=\mathrm{b}(\Sigma) / \mathrm{b}(\mathrm{P})=0.93 \pm 0.05$ and $\sigma(\mathrm{PP})=39.1 \pm 0.12 \mathrm{mb}$ we obtain via Eqs.(10) and (11):

$$
\begin{align*}
& \sigma_{\mathrm{t}}(E \mathrm{P})=30.1 \pm 0.8 \mathrm{mb}  \tag{12}\\
& \frac{\mathrm{~b}(E)}{\mathrm{b}(\mathrm{P})}=0.83 \pm 0.09,
\end{align*}
$$

i.e., the YN-elastic scattering is becoming less collimated with increasing strangeness of baryons. Numerically, Eqs.(12) and (13) are in accord with an empirical observation: $\mathrm{b} \sim \sigma^{\mathrm{n}}$, where $\mathrm{n}=1 / 2^{15 /}$ and also with predictions of the additive quark model for cross sections ${ }^{16 /}$, yet large experimental errors prevent discriminating between $n=1 / 2$ and $\mathrm{n}=1$ (the geometrical scaling). It would, undoubtedly, be interesting to compare Eqs. (10) and (11) with more accurate data at higher energies.

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[^0]:    *The preliminary value $\mu\left(\Xi^{\circ}\right)=-1.20 \pm 0.06$ n.m. was, however, reported in Ref. ${ }^{17 /}$

