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Распределение партонов в релятивистском адроне

Рассмотрено распределение по числу медленных партонов в адроне как функция его быстроты. В пренебрежении поправками на рекомбинацию партонных цепочек получено уравнение и найдено явное решение для этого распределения, зависящее от начального распределения при $y=y_0\approx 1$. Из сравнения с реджеонными диаграммами найдена связь между параметрами партонной модели и реджеонной теории поля. Дана партонная интерпретация правил АГК разрезания реджеонных диаграмм. Дана численная оценка параметров партонной модели, которая показывает, что при существующих энергиях плотность медленных партонов видимо близка к насыщенному значению. Следовательно, учет усиленных реджеонных графиков является принципиально важным.

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The Parton Distribution in Relativistic Hadrons

The distribution in the wee parton number in the hadron is considered as a function of its rapidity. Neglecting corrections due to the parton chain recombination the equation is derived and its explicit solution is found that describes this distribution and dependence on the initial one at $y=y_0\approx 1$. Comparison with the reggeon diagrams results in relations between the parton model and the reggeon field theory parameters. The interpretation of the AGK cutting rules in the framework of the parton model is presented. The numerical estimation of the parton model parameters is performed. It is shown that the wee parton density corresponding to accessible energies seems to be close to the saturated density. Therefore, the enhanced graphs contributions turn out to be of considerable importance.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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INTRODUCTION

The parton model [1,2] (PM) arose as a generalization of some field-theoretical models describing a hadron peripheral interaction at high energies. By the adoption of few "rules" it is possible to explain many features of hadron-lepton and hadron-hadron interactions. In addition to the reproduction of many results of the Regge-Phenomenology, the PM reflects the space-time structure of the interaction [2]. For a long time the PM has played the role of a tool for qualitative analysis being a ground for our intuition, rather than a selfconsistent theory. Recently a noticeable success however, has been achieved [3-5] in the development of the PM machinery and applications. In papers [3-5] the relationship between the PM and reggeon field theory - the only selfconsistent theoretical scheme for the elastic and inelastic hadronic reactions at high energies, has been investigated. In paper [3] the method of study of the fast hadron partonic wave function considering its behaviour under Lorentz transformations in the hadron velocity direction (z-boosts) is suggested. In spite of the apparent non-selfconsistency of the concept of the partonic wave function, it was shown in paper [5] that the PM reproduced all the results of the RFT. Therefore the PM can be used as a strict phenomenological scheme in the theory of fast hadron interactions. The PM gives a clear physical interpretation of the reggeon graphs and the reggeon coupling. For example, in ref. [4] the relation between the pomeron intercept and the triple-Regge coupling has been found. But as is shown below, the real correlation is more complicated.

In the present paper the hadronic wave function is under study from the point of view of the wee parton number distribution. This information is useful in various physical problems and of great importance in the study of hadron-nuclear interactions. The role of the tree reggeon diagrams - the analog of the parton cascade, was first understood in ref. [8], where these diagrams were studied in detail.

The present paper is composed as follows. The method for the investigation of the parton wave function by means of z-boosts [3] is discussed in section 2. A special attenion is paid to the so-called passive component introduced in ref. [3], i.e., the hadron state, which contains no wee parton. The weight of the quark passive component has been evaluated earlier [9] from the hadron-nuclear scattering data.

Section 3 is devoted to the investigation of the development of parton configurations under z-boosts. For the case, where the interaction between the parton chains can be neglected a detailed calculation is performed. The vicinity of the critical point $\alpha_P(0) \approx \alpha_P^c(0)$ is also discussed.

The comparison of the PM and the reggeon graph contributions is carried out in section 4. As a result, the relationship between the parameters of both schemes is found. It is also shown that the pomeron cut contributions have the eikonal form if the Poisson distribution in the number of fast partons in a hadron is supposed. The reggeon graph cutting rules [7] are derived in the framework of the PM.

In section 5 an attempt of numerical estimation of the PM parameters is made. The values of the parameters are found to be very sensitive to the value of P_q - the quark active component norm. The exact knowledge of the value of P_q is of need also to specify the role of the enhanced reggeon graphs.

2. THE PASSIVE COMPONENT

The parton wave function of a hadron is noninvariant under the Lorentz transformations. The z-boost generator has the form

$$L_z = K_z \cdot t - \int d^3x\, H(x) z \,. \qquad (1)$$

Here K_z is the z-component of the total momentum K; $H(x)$ is a Hamiltonian density at the point x.

The generator L_z can be divided into two terms:

$$L_z = L_z^{(0)} + L_z^{int} \qquad (2)$$

where $L_z^{(0)}$ contains only the free part of the Hamiltonian and generates the rapidity translations of the partons. L_z^{int} describes the parton interactions, i.e., it contains

the terms responsible for their decays and confluences. In the "soft" field theory which is the ground of the PM, L_z^{int} is constructed in such a way that its intensity (the rate of decays and confluences) falls off quickly, as $1/E$ at least, when the parton energy is increased.

As a result, one has the following picture: when the hadron energy is increased (this is achieved by the action of the "Hamiltonian" K_z) those partons, whose rapidity is great enough already $(y \gg 1)$, do not interact and behave like free ones, i.e., the uniform growth of their rapidity occurs only. On the contrary, in the wee-parton region $(y \approx 1)$ the decays and confluences of the partons take place in addition to the rapidity translations.

Thus, if the given parton component of a hadron having rapidity Y, contains only fast partons $(y_i \gg 1)$, then after the boosting of the hadron rapidity all the partons in this component undergo the rapidity shift only, without any decays or confluences. As such parton component does not interact with a target, it is called a passive component. On the other hand, the active component containing a number of wee-partons can turn into a passive one after the rapidity boosting because some probability exists that all the partons do not decay during the "time" of its transition to the fast part of the spectrum. From this consideration it follows that a norm w_0 of the passive component in the parton wave function is a monotonically increasing function of the hadron rapidity [3]:

$$d w_0 / d Y \geq 0. \qquad (3)$$

If the Y-value is large enough, then the decays and confluences of wee-partons proceed with the rapidity boosting irrespective of the hadron quantum numbers. In the region of $Y \lesssim 1$, however, the hadron quantum numbers can influence the y-dependence of w_0, and w_0 here may fall rather quickly with Y due to the secondary reggeon contributions.

From the above discussion it follows that the passive component of the wave function is an outstanding one, because its weight can reach a relatively large value. It is also seen that the w_0 value depends at high energy on the hadron quantum numbers.

The most pronounced manifestation of the passive component can be seen in the hadron - nuclear interaction. For this purpose an analysis of the neutron-nucleus total cross section data has been performed in ref. [9]. It has been assumed there that the valence quark scattering amplitudes are additive; the distribution in the wee-parton number in the active component has been neglected; and the mixing between the active and passive components during the passage of a hadron through the nucleus has also been neglected. It was found that the active component weight P_q for the u and d quarks is as little as about 0.5. An analogous analysis of the K_L-A total cross section data [10] showed that for a strange quark P_s is about twice smaller than P_q.

3. CALCULATION OF THE PARTON CASCADE

On the basis of the qualitative consideration given in the previous section one can write a system of equations describing the evolution of the wee-parton number distribution in a hadron with the energy growth.

Let $w_n(y)$ denote the probability of a state with n wee partons in a hadron of rapidity y. These values are normalized by the condition

$$\sum_n w_n = 1. \tag{4}$$

The case where a wee parton does not undergo any decay during its way to the fast part of the parton spectrum will be called the breaking of the parton chain. The probability of breaking per unit "time" will be denoted by γ. On the other hand, if the parton decays only once before it becomes fast enough, the parton chain will develop without breaking. If a parton decays twice, then two separate parton chains can be produced. The probability of such branching per unit "time" will be denoted by λ.

The system of equations describing the evolution of the distribution function w_n, according to the above discussion, has the form [5].

$$dw_0/dy = \gamma w_1 \tag{5}$$

$$\frac{dw_n}{dy} = -(\gamma+\lambda) n w_n + \gamma(n+1) w_{n+1} + \lambda(n-1) w_{n-1} . \tag{6}$$

The latter equation concerns only the active component.

There is also a possibility of the recombination of two chains into a single one, when they find them close to each other in the impact parameter plane. Equations (5) and (6) do not take this into account. Below we shall return to this question.

The generating function for the probabilities w_n has the form

$$F(x,y) = \sum_{n=0}^{\infty} x^n w_n(y) . \tag{7}$$

From the condition (4) it follows that

$$F(1,y) = 1 . \tag{8}$$

Equations (5) and (6) can be rewritten as follows

$$\frac{\partial F(x,y)}{\partial y} = (1-x)(\gamma-\lambda x) \frac{\partial F(x,y)}{\partial x} . \tag{9}$$

This equation can be solved by the characteristic method. The general solution is

$$F(x,y) = F\left[\frac{\gamma(1-x)-(\gamma-\lambda x)e^{-\Lambda y}}{\lambda(1-x)-(\gamma-\lambda x)e^{-\Lambda y}}, 0\right]. \tag{10}$$

The notation is used here

$$\Lambda = \lambda - \gamma . \tag{11}$$

If the initial parton distribution $w_n(0)$ is known, the average wee-parton number can be found:

$$\langle n \rangle_y = \frac{\partial}{\partial x} F(x,y)\Big|_{x=1} = \langle n \rangle_0 \, e^{\Lambda y} . \tag{12}$$

This growth of $\langle n \rangle_y$ is the reason for an increase of the hadronic total cross sections with energy and it can be associated with the pomeron intercept value [4]

$$\Lambda = \alpha_P(0) - 1 . \tag{13}$$

If only one parton exists at $y=0$, then

$$w_0(y) = [1 - P(\infty)] \frac{1 - e^{-\Delta y}}{1 - [1 - P(\infty)] e^{-\Delta y}} , \qquad (14)$$

$$w_n(y) = \frac{P^2(\infty) e^{-\Delta y} (1 - e^{-\Delta y})^{n-1}}{\{1 - [1 - P(\infty)] e^{-\Delta y}\}^{n+1}} . \qquad (15)$$

Here

$$P(\infty) = \Delta / \lambda \qquad (16)$$

is the asymptotic value of the active component weight for a single initial parton, calculated in the tree approximation.

Comparison of (14) and (15) shows that the passive component $w_0(y)$ is an outstanding one indeed, and that the distribution $w_n(y)$ in the active component is a geometrical progression. It is seen from equation (16) that the equality $\lambda = \Delta$, which has been obtained earlier /4/ is valid only in the case $\gamma = 0$. Relation (16) can be easily obtained in a way analogous to /4/ if one assumes that in the active component, having a norm $P(\infty)$, the parton number distribution is smooth, i.e., $\langle n^2 \rangle_{act} \approx \langle n \rangle^2_{act}$ and that the passive component is picked out of the whole distribution.

Let us now consider the role of the parton chain confluences. The addition of the effect makes equation (6) unsolvable. Nevertheless significant corrections to solutions (5)-(6) will apper only in the case where the wee-parton density is large enough (or at the appropriate energies). The generally accepted folklore asserts that the enhanced reggeon graphs contribution is small at modern energies. In the parton language this means that the wee-parton density is small enough (in the indicated sense). Therefore we shall assume that the solution (10) is a good approximation at moderate energies.

It is noteworthy that the passive component norm is insensitive to the inclusion of the confluence, because the conversion of the active component to the passive one takes place in the main states with small wee-parton numbers.

If $\Delta > 0$, $\langle n \rangle_y$ grows fast with the energy (see eq. (12)). The parton density becomes rather high and the effect of the confluence of the parton chains keeps back this growth. The balance of these effects results in the equilibrium wee-parton density /3,11/ in the active component. The following equation seems to be suitable to describe this process

$$\frac{\partial \rho}{\partial y} = \Delta \rho - \rho (1 - e^{-s\rho}) - \alpha' (\nabla_b^2) \rho . \qquad (17)$$

Here $\rho(y, b)$ is the wee-parton density in the impact parameter plane in the active component. The dimensional parameter s determines the recombination probability for two neighbouring partons, $1 - e^{-s\rho}$ is the probability for at least one wee parton to get into the region of area s of the impact parameter plane. After the equilibrium density ρ_0 is reached the right-hand side of eq. (17) vanishes, thus

$$\rho_0 = \frac{-\ln(1-\Delta)}{s} \approx \Delta / s . \qquad (18)$$

The parton density equals ρ_0 within the disk of radius $R \approx 2\sqrt{\alpha' \Delta} y$, where α' is the wee-parton diffusion rate in the b-plane.

In contrast to ref. /11/, where the passive component has not been taken into account, this saturated parton disk even in the active state is not "black" when it collides with the target. Indeed, the elastic scattering amplitude is

$$f_{AB}(b) = P_A(\infty)(1 - e^{-\rho_0 \sigma_B}) . \qquad (19)$$

Here σ_B is the cross section of the wee-parton interaction with the target. In the center of mass of two colliding hadrons one has:

$$f_{AB}(b) = P_A(\infty) P_B(\infty) (1 - e^{-\int d^2 b' \rho_0^2 \sigma_0}) . \qquad (20)$$

Here σ_0 is the wee parton-wee parton cross section. The exponent in eq. (20) is integrated over the overlap of the colliding disks, whose area increases with energy as y^2. Therefore the exponent in eq. (20) tends to zero and by comparing eqs. (19) and (20) one concludes that the active state amplitude of scattering on the target B is

$$f^{act}_{AB}(b) = 1 - e^{-\rho_0 \sigma_0} = P_B(\infty) < 1 . \qquad (21)$$

The considerations given above are correct only if $\Delta \gg \Delta_c$; $\Delta_c \sim r^2 \ln r^2$ being the transition value at which the critical regime occurs [12]. If $\Delta > \Delta_c$, the $P(Y)$ tends to a constant $P(\infty)$ as $Y \to \infty$; at the critical point $\Delta = \Delta_c$ $P(Y)$ falls to zero $P(Y) \underset{Y \to \infty}{\to} 0$ according to the power law in Y and, if $\Delta < \Delta_c$ the exponential fall-off of $P(Y)$ takes place [3].

However, eqs. (14)-(16) do not reflect these properties (they correspond to $\Delta_c = 0$) for the following reason. Supposing $\rho \ll \rho_0$ we have neglected recombinations. However, this supposition is obviously wrong when $\Delta \approx \Delta_c$. Indeed the value of $P(\infty) \to 0$ as $\Delta \to \Delta_c$ and Lorentz invariance (see eq. (21)) requires $\rho_0 \to 0$. In what follows the supercritical regime will be considered and the relation $\Delta \gg \Delta_c$ will be supposed to be right.

4. COMPARISION WITH THE REGGEON GRAPHS

The evolution of the distribution in the wee-parton number with the rapidity growth proceeds as it has been described above only when the rapidity is already sufficiently high, $y \gg 1$, and the hadron quantum numbers do not affect the process. The development of $w_n(y)$ at small y cannot be followed at the level accepted. Therefore, the initial distribution $w^0_n(y_0)$ at $y_0 \approx 1 \div 2$ with the generating function $F_0(x, y_0)$ will be considered as given. It will be supposed that the hadron quantum numbers affect only $F_0(x, y_0)$ and further evolution has a universal nature. Then, using eq. (10) one can obtain the wee-parton distribution in a fast hadron and study its interaction with a target by comparing the results with the RFT expressions.

The events in which only one wee parton interacts with the target and a single parton chain $1 \to 2$ disintegrates into a hadron comb. (fig.1) correspond to the pole contributions to the scattering amplitude f_{AB} of hadrons A and B (fig. 2). Only partonic configurations without recombinations of chains are relevant in this case. Comparing these contributions to σ^{AB}_{tot} one finds

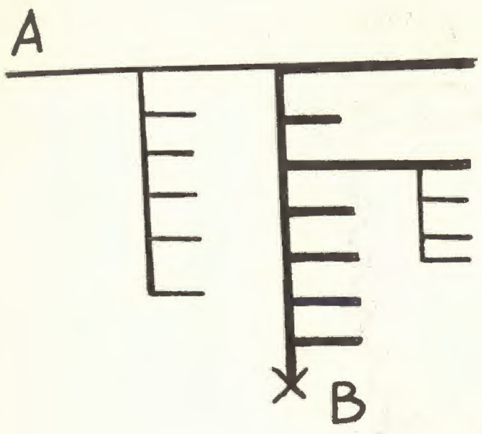

Fig. 1

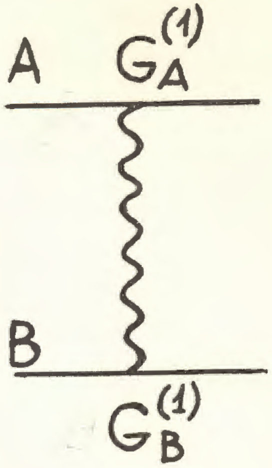

Fig. 2

$$\langle n \rangle_A \, \sigma_B = G^{(1)}_A \, G^{(1)}_B . \qquad (22)$$

The left-hand side is written in terms of parton parameters; $\langle n \rangle_A$ is the mean parton number in the initial distribution of the hadron A (at $y = y_0$); σ_B is the wee parton-hadron cross section; and $G^{(k)}$ is the k -pomeron emission vertex. The factor $\exp \Delta Y$ in the left-hand side, which reflects the growth of the mean wee-parton number is identical to that in the right-hand side, arising from the pomeron Green function.

The double pomeron exchange (fig. 3) corresponds to the case where two wee partons from two different parton chains interact with the target. This contribution to the elastic amplitude (it corresponds to inelastic processes with the double density of particles in the rapidity scale) is positive of course, but the screening terms in the amplitude with single wee-parton interaction with target when taken into account, change its sign (see below). Comparing the corresponding expressions one finds

$$\langle n(n-1) \rangle_A \, \sigma_B^2 = G^{(2)}_A \, G^{(2)}_B . \qquad (23)$$

In both sides of this relation the identical factors like $\exp 2\Delta Y$ and $(R^2 + \alpha' Y)^{-1}$ (R is the radius of the region in the b -plane, where wee partons are distributed at $y = y_0$) are dropped.

An analogous consideration of the triple-pomeron graphs shown in figs. 4a and 4b leads to the following relations

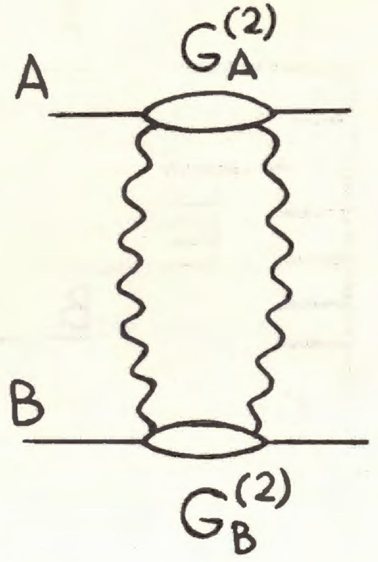

Fig. 3

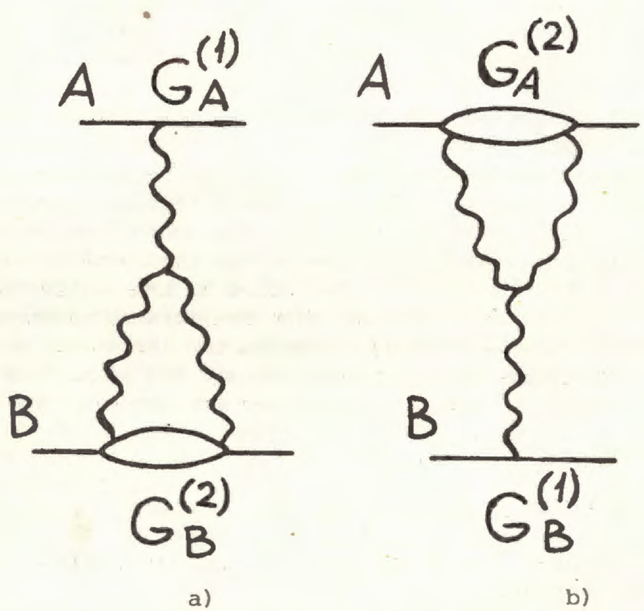

Fig. 4

$$\langle n\rangle_A \lambda\sigma_B^2 \sim G_A^{(1)} G_B^{(2)} r \, , \qquad (24)$$

$$\langle n(n-1)\rangle_A \, s\,\sigma_B \sim G_A^{(2)} G_B^{(1)} r \, , \qquad (25)$$

where r is the triple-pomeron coupling.

A comparison of more complicated graphs does not give any new relation.

Eqs. (22)-(25) result in

$$r \sim \sqrt{\lambda \cdot s} \; . \qquad (26)$$

Therefore λ does not coincide (in contrast to /4/) with the dimensionless triple-pomeron coupling constant $r_0 = r/\sqrt{\alpha'}$, but is connected with it by the relation

$$r_0 = \lambda\,(\sigma_0/\alpha')^{1/2} , \qquad (27)$$

where σ_0 is the cross section for two colliding wee partons. This equation arises when one considers the interaction of two wee parton "clouds" in the center of mass system of colliding hadrons.

It is interesting to note that the existence of the triple pomeron coupling necessarily entails the existence of a quartic coupling ($2 \to 2$ transitions). In fact, the inelastic channel in the two parton chain interaction ($2 \to 1$) with cross section s leads, as a result of the total probability conservation, to the appearance of the elastic "scattering" having the same cross section. These considerations lead to the following relation between the quartic constant t and the recombination rate s /5/:

$$t \sim s \; . \qquad (28)$$

Further, one more relation between the parameters can be obtained using eqs. (24), (25). Suppose that at $y \to y_0$ we have a Poisson distribution in the wee-parton number. Then, if $A = B$ (24) and (25) result in

$$\langle n\rangle_A \, s \simeq \lambda\sigma_A \; . \qquad (29)$$

Note that eqs. (21) and (29) are equivalent. Indeed,

$$P_A(\infty) = \sum_n w_n [1-(1-P(\infty))^n] = 1 - e^{-\Delta/\lambda <n>_A}, \quad (30)$$

where the relation $P(\infty) = \Delta/\lambda$ is used. Combining eqs. (30), (18) and (29) one arrives at (21).

Now we return to nonenhanced graphs. By comparing eqs. (22) and (23) one concludes that if the initial distribution $w_n(y_0)$ is Poissonian, i.e., $<n(n-1)>_A = <n>^2_A$, then $G^2_A = (G^{(1)}_A)^2$ and $G^{(2)}_B = (G^{(1)}_B)^2$, i.e., the contributions of different pomeron exchanges have the eikonal form: $G^{(k)} = (G^{(1)})^k$.

This point deserves more detailed consideration. At first sight there is an apparent contradiction here. The contribution of inelastic diffraction to the absorptive part of the elastic amplitude is known [13] to modify the eikonal form of the double pomeron exchange (the latter corresponds to the elastic scattering contribution to the absorptive part). Although the initial Poisson distribution leads, as is shown, to the eikonal form of the elastic amplitude the corrections should arise as a result of the inelastic diffraction, which has at fixed b, the following partial cross section

$$f^2_{dif} = <f^2> - <f>^2 = e^{-2f<n>}(e^{f^2<n>} - 1). \quad (31)$$

The distribution of partons in the b-plane is neglected in eq. (31) for the sake of simplicity. Here we consider only nonenhanced graphs and therefore, in eq. (31) the dispersion of amplitudes f_k corresponding to the scattering of k wee partons arising different "trees" should be taken into account, i.e., f now means the partial amplitude of scattering of $\exp(\Delta Y)$ wee partons (the dispersion of the amplitude within this set of partons results in the diffraction corresponding to enhanced graphs).

We note that the inelastic diffraction is of pure interference nature and arises as a result of the dispersion in the scattering amplitudes for different numbers of wee partons. Therefore the assumption that the inelastic diffraction is dominated by one-pomeron exchange is not justified. At the same time the calculation of the corrections to the eikonal form of the double-pomeron contribution should include just the one-pomeron diffraction cross section rather than the total one, as is frequently done. Therefore we extract the one-pomeron contribution from eq. (31). This corresponds to settlement $f^{(1)}_n = nf$ and one has

$$(f^2_{dif})_P = f^2(<n^2> - <n>^2) = f^2<n>. \quad (32)$$

The fact that the diffraction cross section is proportional to $<n>$ means that $(f^2_{dif})_P$ contains only planar diagrams, which are known to give no contribution to the elastic scattering amplitude at high energies. Therefore, although in the case considered the inelastic diffraction cross section (31) and its one-pomeron contribution (32) do not vanish, no correction arises to the eikonal form of multi-pomeron cuts. This resolves the contradiction mentioned.

It should be noted that the eikonal form of the nonenhanced graphs is a direct consequence of the Poisson form of the distribution in the wee-parton number at $y=y_0$. For example, the modification in the weight of passive component w^0_0 is sufficient to change the dependence of $G^{(k)}$, obtained from eqs. (22) and (23). Simultaneously nonplanar graphs become contributors to $(f^2_{dif})_P$ and it is easy to show that together with $(f^2_{el})_P$, they lead to the same modified form of $G^{(k)}$.

We also note that the Poisson form of the distribution at $y=y_0$ (and hence the eikonal expression for amplitude) is surely not true for large n, when the parton density at $y=y_0$ approaches the saturated value ρ_0 and the effect of the parton recombination suppresses the large n components. The value of n_{max}, at which this suppression occurs can be estimated as follows

$$n_{max} \approx 4\pi\alpha'_R y_0 \rho_0. \quad (33)$$

Here $\alpha'_R \approx 1\,(GeV/c)^2$ is the f-reggeon trajectory slope. It is relevant here, because the distribution of wee partons at $y \approx y_0$ is accounted for, to a great extent, by the valence quark diffusion, and this process is responsible for the f-exchange. The numerical evaluation of ρ_0 will be made in the following section.

To conclude this section we consider the interpretation of the reggeon cutting rules [7] in the parton model framework. We consider an unenhanced diagram with n pomerons including m cut ($m \geq 1$). Its contribution to the inclusive cross section is [7]

$$F_{nm} = 2^{n-m}(-1)^{n+m} C^m_n \rho^m_c \rho^{n-m} \frac{1}{n!}[G^{(n)}]^2 \quad (34)$$

Here $\rho_c = 2\rho$ is the cut pomeron Green function. In the partonic language this graph corresponds to the situation, where m wee partons (from different "trees") have interacted with the target. The amplitude of this process is

$$\sum_{k=m}^{\infty} w_k C_k^m f_c^m (1-f)^{k-m} . \qquad (35)$$

Factor $(1-f)^{k-m}$ reflects the fact that in the component with k wee partons $(k-m)$ of them should not interact with the target. The term $(-1)^{n-m} C_{k-m}^{n-m} f^{n-m}$ should be extracted from it to obtain the contribution corresponding to the n-pomeron diagram. Naturally the summation over k begins with n.

$$F_{nm} = \sum_{k=n}^{\infty} w_k^\circ C_k^m f_c^m (-1)^{n-m} f^{n-m} C_{k-m}^{n-m} . \qquad (36)$$

This expression can be rewritten in the form

$$F_{nm} = (-1)^{n-m} C_n^m f_c^m f^{n-m} \sum_{k=n}^{\infty} w_k^\circ C_k^n . \qquad (37)$$

By substituting $f_c = \rho_c \sigma_0, f = 2\rho\sigma_0$ and using the definition of $G^{(n)}$ obtained earlier $(G^{(n)})^2/n! = \sigma_0^n \langle C_k^n \rangle$ eq. (37) takes the form of eq. (34).

The cutting rules were treated also in paper [14], where the example of deuteron was considered (this case is analogous to that in our picture where only one component having $k=2$ is kept).

5. NUMERICAL EVALUATION OF THE PARAMETERS

The passive component norm of u and d quarks at the nucleon energy of 240 GeV was obtained in paper [9] analysing the total neutron-nucleon cross sections. By using this result one can evaluate the saturated parton density ρ_0.

If there is a single parton at $y=y_0$, from eqs. (14) and (16) one obtains

$$P(y) = \frac{\Delta/\lambda}{1-(1-\Delta/\lambda)e^{-\Delta y}} . \qquad (38)$$

By supposing the Poisson distribution for the number of partons at $y=y_0$ and using eq. (38) one finds the passive component weight in the quark:

$$P_q(y) = 1 - \exp\left[-\frac{\langle n\rangle_q \, \Delta/\lambda}{1-(1-\Delta/\lambda)e^{-\Delta y}}\right] . \qquad (39)$$

Eq. (18), combined with eqs. (29), (22), (23) and (39), gives

$$\rho_0 = \frac{\Delta}{\lambda}\frac{\langle n\rangle_q^2}{(G_q^{(1)})^2} = \frac{\lambda}{\Delta}\ln^2[1-P_q(y)]\,[1-(1-\frac{\Delta}{\lambda})e^{-\Delta y}]^2 \, \frac{e^{\Delta y}}{\sigma_{tot}^{qq}} . \qquad (40)$$

Here the single pomeron exchange is supposed to dominate in σ_{tot}^{qq}, i.e., $\sigma_{tot}^{qq} \simeq (G_q^{(1)})^2 \, e^{\Delta y}$. For the numerical evaluations we adopt $\sigma_{tot}^{qq} = 8$ mb.

As for the ratio λ/Δ, it is only known that $\lambda/\Delta \geq 1$. It turns out, however, that the numerical value of ρ_0 is insensitive to the magnitude of λ/Δ, because its derivative with respect to λ/Δ vanishes at $\lambda/\Delta = [e^{\Delta y} - 1]^{-1}$. Therefore ρ_0 remains practically constant when λ/Δ varies within the reasonable range of values. The values of ρ_0 corresponding to different λ/Δ and $P_q(y)$ are brought together in table 1. In accordance with [15] Δ has been taken as 0.07.

Table 1

The value of ρ_0 $(GeV/c)^2$, calculated according to eq. (40) with different λ/Δ and $P_q(y)$

$P_q(y=5)$ \ λ/Δ	1	2	4	6	8	10
0.3	0.009	0.008	0.008	0.009	0.011	0.012
0.5	0.034	0.029	0.03	0.035	0.04	0.046
0.7	0.103	0.086	0.091	0.105	0.121	0.138
0.9	0.376	0.316	0.335	0.385	0.442	0.503

The ρ_0 values quoted in the table indeed show a very week dependence on λ/Δ. On the other hand, ρ_0 is rather sensitive to $P_q(y)$. The value of $P_q(y)$ obtained in paper [9] is somewhat estimative (for different nuclei it varies from $P_q=0.4$ to $P_q=0.67$) but even within this accuracy the results given in table 1 indicate some problems.

The wee-parton density corresponding to single parton chain can be evaluted as

$$\rho^{(1)}(b) = \frac{1}{4\pi\alpha' y} e^{-b^2/4\alpha' y} . \tag{41}$$

For $\alpha'=0.25$ $(GeV/c)^{-2}$; $y=5$ and $b=0$ expression (41) gives $\rho^{(1)}(0)=0.064$. This value is of the same order of magnitude as the saturated density corresponding to $P_q=0.3 \div 0.7$. It seems therefore that the conventional view (which is held here too) that at accessible energies the interaction between parton chains is weak ($\rho^{(1)} \ll \rho_0$), i.e., the fact that enhanced graphs provide small contributions to the elastic amplitude, is wrong. In this case the true value of Δ must be substantially larger than the value $\Delta \approx 0.07$ obtained in the eikonal approximation [15], because the strong recombination suppresses the exponential growth (as $e^{\Delta y}$) of the wee-parton multiplicity.

Nevertheless, the possibility that $\rho^{(1)} \ll \rho_0$ cannot be excluded now. Firstly, the correct determination of P_q requires more accurate analysis. Secondly, the Poisson form of the distribution in the wee-parton number at $y=y_0$ is of essential use in the derivation of eq. (39). If it is not the case and the passive component weight $w_0(y=y_0) > e^{-\langle n \rangle_q}$, then the value of ρ_0 exceeds that quoted in table 1.

We note that the use of the results collected in table 1 allows one to calculate the dimensionless triple-pomeron coupling $r_0 = r/\sqrt{\alpha'}$ (r being defined by eq. (26)). The value of this coupling seems to be considerably larger than that obtained from the inelastic diffraction data [17]. The values of r_0 corresponding to the same P_q and λ/Λ as in table 1 are gathered in table 2.

Table 2

The triple pomeron coupling r_0 calculated for different values of λ/Δ and $P_q(y)$

$P_q(y=5)$ \ λ/Λ	1	2	4	6	8	10
0.3	1.09	2.18	4.36	6.53	8.71	10.89
0.5	0.29	0.58	1.15	1.73	2.31	2.88
0.7	0.10	0.19	0.38	0.57	0.76	0.95
0.9	0.03	0.05	0.10	0.16	0.21	0.26

The values of Δ and r_0 are generally suggested to satisfy the supercritical regime condition $\Delta > \Delta_c$ [12], with $\Delta_c \approx r_0^2 \ln r_0^2$ (it is implied that $r_0 \ll 1$). Table 2 however shows that for many r_0-values quoted it is not so. For example, at $P_q=0.5$ and all λ/Δ the value of r_0 corresponds to subcritical regime $\Delta < \Delta_c$. In this case all the results of Sec. 3 are irrelevant. It is another aspect of the problem mentioned above ($\rho^{(1)} \approx \rho_0$).

As noted earlier, the real Δ may considerably exceed the value 0.07 taken in calculations; the analysis should be carried out anew for this case.

The following considerations allow one to estimate λ/Δ. If ϵ is the probability that a parton decays once during its transition to the fast part of the spectrum, the total probability is unity. Therefore $\epsilon/(1-\epsilon)+\gamma=1$. Even if the parton decays twice, the partons produced can recombine and no real decay of the parton chain occurs. Therefore $\lambda < \epsilon^2$. This results in the condition $\lambda + \sqrt{\lambda}/1 - \sqrt{\lambda} - \Delta < 1$. For $\Delta=0.07$ one obtains $1 \leq \lambda/\Delta \leq 2.8$. By using this inequality and the values of ρ_0 calculated earlier one can find restrictions on the value of $\langle n \rangle_q$. Taking into account that $\langle n\rangle_q \approx [\frac{\lambda}{\Delta}\rho_0\sigma_{qq}e^{-\Delta Y}]$, for $P_q(Y)=0.5$ one gets $0.5 \leq \langle n\rangle_q \leq 0.86$.

The experimental study of hadron production on nuclei and comparison of the inclusive spectra in the beam fragmentation region for different nuclei can provide sufficient information to find $\langle n\rangle_q$.

CONCLUSION

Although all the results obtained in the PM framework can formally be derived from the RFT graphs, the parton interpretation often provides simpler and plainer qualitative considerations and allows one to obtain new results. Apart from a very important application to the theoretical calculations of the high energy hadron interactions, the analysis carried out in the present paper shows that the generally accepted view on the role of enhanced graphs can be radically wrong. The formulae obtained in the paper allow one to analyse the hadron experimental data in the PM framework taking all the partonic configurations into account. The value of the quark active component norm P_q turns out to be very important in these considerations and therefore needs a more accurate determination. It is noteworthy that this analysis allows one to find the bare triple pomeron coupling constant.

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