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О подавлении кварк-адронного формфактора

Подавление адронной волновой функции связанного состояния, необходимое для наступающего при передаче импульса $|t| \geq$ 2-3 ГэВ^2 степенного скейлинга, получено из соответствующего поведения четырехкварковых функций Грина в t, которое требуется дифракционным рассеянием.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Lewin K., Kallies W. **E2 - 12600**

About the Damping of Quark-Hadron Form Factors in Relative Quark Momentum

Damping of hadron bound state wave functions in relative quark momentum as necessary to describe the beginning of power scaling at momentum transfer $|t| \geq$ $\geq$ 2-3 GeV^2 is obtained approximately from a corresponding behaviour of four-quark Green's functions in t which is required by diffraction scattering.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The understanding of power scaling [1,2] as large momentum transfer behaviour of hadron scattering amplitudes and electromagnetic form factors in field theory presently requires phenomenological assumptions about the hadrons as bound states of two or three quarks, respectively. These bound states are assumed to exist, for instance, as solutions of a corresponding Bether-Salpeter equation [3]. If they are damped sufficiently in relative quark momentum, power scaling is obtained by counting the number of far off-shell quark propagators in tree-like diagrams. Up to now this large relative momentum behaviour cannot be deduced in vector gluon theories [4], so that the main "argument" for the damping is the assumed existence of the bound state wave function in x space at zero interquark distance (see, for instance, ref. [3]). But this argument is too general to explain the beginning of power scaling at non-asymptotic momentum transfers of a few GeV^2 which requires a sufficiently strong decrease of the bound states at relative quark momenta in the region around 1 GeV/c. A summary of "early" scaling and corresponding experimental data is contained in ref. [5] .

The aim of this work is to find arguments for a sufficient damping of the bound states at such non-asymptotic relative quark momenta. Explicit information about a corresponding behaviour of the kernel of the Bethe-Salpeter equation as two-particle irreducible four-quark Green's function is not available because of the known difficulties in quantum chromodynamics. Self energy and vertex corrections become important, exponentiation of expansions in perturbation theory as it seems to work asymptotically [6], cannot be seen. Therefore we go a more phenomenological way to obtain information about the behaviour of the four-point Green's function in momentum transfer by comparison with the known t behaviour of diffraction scattering. As is well-known, in the diffraction region the differential cross section of elastic hadron-hadron scattering strongly falls off in t independently or nearly independently of the incident energy $\sqrt{s}$ (see ref. [7]). This comparison becomes possible if quark diagrams which describe hadron-hadron scattering and contain no higher than four-quark Green's functions contribute non-negligibly to diffraction scattering. Since interaction between quarks essentially is a matter of momentum transfer and not of longitudinal momentum, the transition from large relative longitudinal momentum in diffraction scattering to a small one in the bound state should not effect the t dependent factor which governs the damping. After discussing the relation between the general four-quark Green's function and the irreducible kernel in an iterative procedure to solve the bound state equation in a restricted region, it is shown that the iterative solution exhibits the expected damping of the bound state which is needed to guarantee "early" power scaling.

Section 2 gives a short review of the role of hadron bound states in connection with power scaling. Section 3 deals with the information from diffraction scattering and in section 4 the iterative procedure is described.

2. *HADRON BOUND STATES AND POWER SCALING*

Considering, for simplicity, the meson-meson scattering amplitude T_{MM} in the center-of-mass system with $s = 2p_0^2 - 2|\vec{p}|^2$ beyond the resonance region and $s/t \sim 0(1)$, $t = q^2 = -\vec{q}^2$, one has generally

$$T_{MM} \sim \int d^4k_1 \ldots d^4k_4\, \psi^+_{\vec{p}}(k_1)\, \psi^+_{-\vec{p}}(k_2)\, M(p,q,k_1,\ldots,k_4)\, \psi_{-\vec{p}-\vec{q}}(k_3)\, \psi_{\vec{p}+\vec{q}}(k_4), \qquad (2.1)$$

where the convoluting bound states (4x4 spinor matrices)

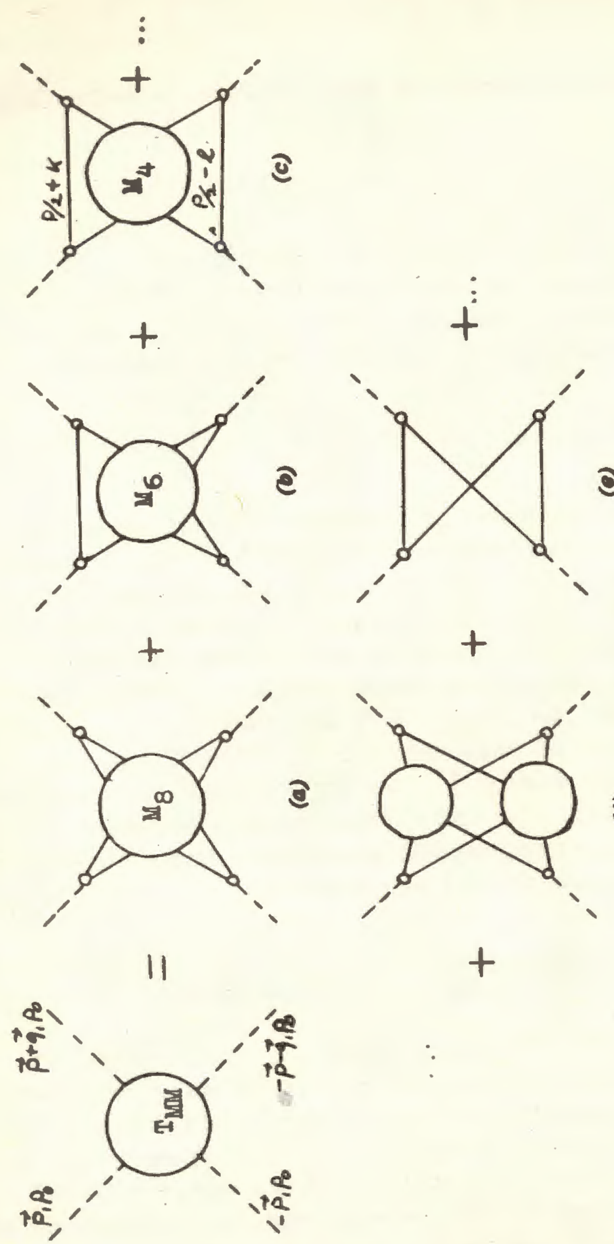

Fig. 1. Decomposition of the amplitude T_{MM} *into diagrams with* n *-quark Green's functions* M_n *of different connectivity. Concerning diagrams of the types (d) and (e) see ref.* [6] .

$$\psi_p^{\alpha\beta}(k) = \int d^4x\, e^{ikx} < 0 \,|\, T\{\psi^\alpha(0)\bar{\psi}^\beta(x)\} \,|\, p > \qquad (2.2)$$

are introduced as solutions of the homogeneous Bethe-Salpeter equation

$$\psi_p(k) = \Delta_F(\frac{p}{2} - k)[\int d^4\ell\, \psi_p(\ell)\, K(k, \ell, p)]\Delta_F(\frac{p}{2} + k), \qquad (2.3)$$

and M as a function of the four-vectors $p, q, k_1, ..., k_4$ represents the amputated full eight-quark Green's function decomposable into terms of different connectivity as is illustrated in *Fig. 1*. As usual, $\psi_p(k)$ contains the two quark legs Δ_F. The condition

$$\int d^4k\, \psi_p(k) = \tilde{\psi}_p(x=0) < \infty \qquad (2.4)$$

guarantees the existence of the wave function at zero interquark distance. The dominant contributions to the diagrams of *Fig. 1* come from regions of the loop variables belonging to small relative quark momenta at the hadron vertices. Power scaling at large t should be due to diagrams (a)-(c). The diagram (d) contains four-quark Green's functions with all quark legs near the mass shell and therefore should not be dominant because of exponentiation of infrared logarithms in QCD perturbation theory [6]. In the following we concentrate on the diagrams (a) - (c) since at intermediate momentum transfer t between asymptotic freedom and strong binding they should retain or enlarge their dominance.

3. *INFORMATION FROM DIFFRACTION SCATTERING*

The beginning of power scaling at $t \gtrsim 2\ GeV^2$ (see *Fig. 2*) requires sufficient decrease of the hadron wave functions at relative quark momenta of order 1 *GeV/c*. On the other hand, hadron-hadron elastic scattering at $t \sim O(1\ GeV^2)$ with $t \ll s$, i.e., diffraction scattering, shows damping of the differential cross section over several orders of magnitude for growing t nearly independently of the incident energy $\sqrt{s}$.

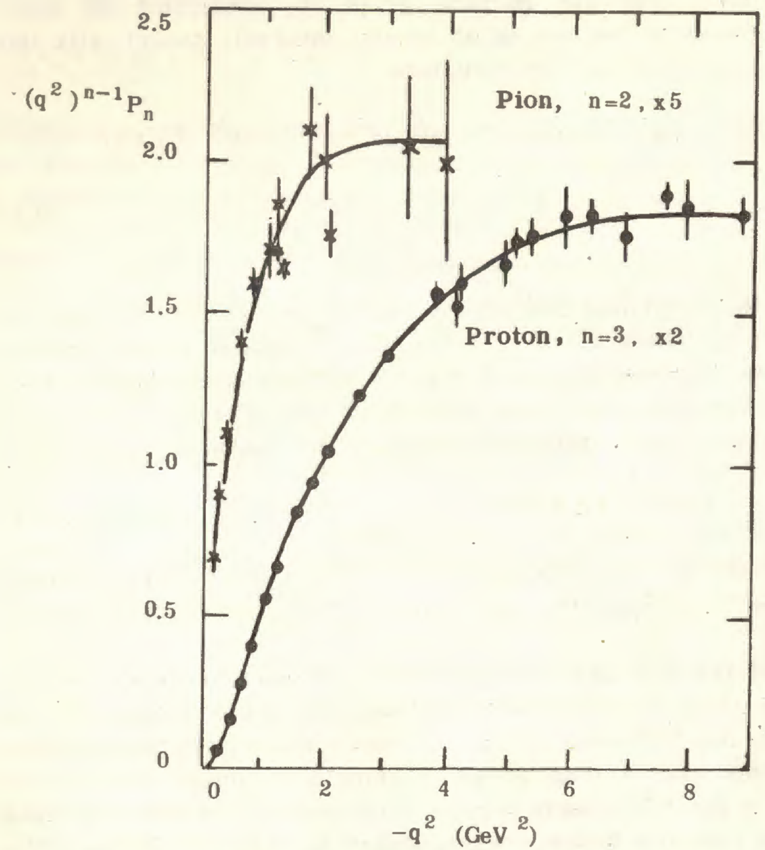

Fig.2. Power scaling of pion and proton electromagnetic form factors (see ref. [5]).

As is well known, experimental results are fitted in phenomenological work by exponentially decreasing factors $F(t)$, i.e.,

$$\frac{d\sigma}{dt} \sim F(t) \cdot G(s,t), \qquad (3.1)$$

where $G(s,t)$ only weakly depends on t compared with $F(t)$. What concerns meson-meson diffraction scattering, this damping should be reflected by all non-negligibly contributing diagrams in *Fig. 1*, especially diagrams (a) - (c). It should be a reasonable assumption that the strong damping in t is not due to coherent compensation effects among such diagrams of different connectivity. For our purpose graph (c) containing the four-quark Green's function is of special interest. Analytically this double loop graph has the structure

$$T^{(c)}_{MM} \sim \int d^4k\, d^4\ell\, \psi^{+\alpha\beta}_{\vec p}(k)\,[\Delta_F^{-1}(\tfrac{q}{2}+k)]^{\beta\gamma}\, \psi^{\gamma\delta}_{\vec p+\vec q}(\tfrac{q}{2}-k)\, M^{\delta\alpha}_{\delta'\alpha'}(p,q,k,\ell)\,\times$$
$$\times\, \psi^{+\,-\vec p}_{\alpha'\beta'}(\ell)\,[\Delta_F^{-1}(\tfrac{p}{2}-\ell)]_{\beta'\gamma'}\, \psi^{-\vec p-\vec q}_{\gamma'\alpha'}(\tfrac{q}{2}+\ell), \tag{3.2}$$

where $M^{\delta\alpha}_{\delta'\alpha'}$ denotes the spinor matrix element of M_4 and the internal variables k, ℓ, $q/2-k, q/2+\ell$ appear as half relative momenta between the quark legs of corresponding hadron vertices. The dependence on momentum transfer t, especially the behaviour of the diffraction peak, is contained in

$$M^{\delta\alpha}_{\delta'\alpha'} \sim F_1(t)\, G^{\delta\alpha}_{\delta'\alpha'}(p,q,k,\ell) \tag{3.3}$$

and in the two bound states $\psi(q/2-k)$ and $\psi(q/2+\ell)$. The relation between $F_1(t)$ and the full damping $F(t)$ turns out in the next section.

Some remarks are in order concerning the four-quark Green's function (3.3). In field theory, for example in perturbation theory of QCD, the treatment of (3.3) is much more complicated in the diffraction region than at far asymptotic momentum transfers where in the fixed angle regime exponentiation of leading graphs gives a common factor corresponding to $F_1(t)$ /5/. These difficulties essentially are due to the growing role of vertex and self energy insertions and generally to the growing effective quark coupling constant. But the following connection does not depend on the order of graphs: The function (3.3) appears in (3.2) at large longitudinal relative momentum and at $t=q^2$ in the diffraction region. Besides this situation (A) we consider a situation (B) with the same momentum transfer t but with longitudinal relative quark momentum near zero. In both cases an arbitrary Feynman graph of higher order has the analytical structure

$$\int d^4k^{(1)} \ldots d^4k^{(n)} f_1(a_i q, k^{(v)})\, f_2(b_j q \pm c_j p, d_k p, k^{(v)}), \tag{3.4}$$

$(v=1, \ldots, n)$,

where the constant factors a_i, b_i, c_i, d_i $(i=1,\ldots,n)$ normalized by the conditions

$$\sum_{v=1}^{n} a_v = \sum b_v = \sum c_v = \sum d_v = 1, \tag{3.5}$$

describe a given distribution of the external momenta p and q over the internal lines. The transition (A) → (B) only changes the arguments of f_2 and thus cannot change the factor $F_1(t)$ in (3.3). Indeed, independently of perturbation theory this expresses the fact that interaction between quarks essentially is a matter of momentum transfer. The factor $F_1(t)$ becomes important in the next section where the case (B) is studied in detail.

4. BETHE- SALPETER EQUATION IN THE DAMPING REGION

Returning to the meson bound states as introduced at the beginning we look for an approximate iterative solution of the Bethe-Salpeter equation (2.3) in a restricted region which is damped sufficiently at relative quark momentum of 0 (1 *GeV/c*). The iterative procedure starts from a "zeroth" approximation

$$\psi^{(0)}_p(k) = \begin{cases} \psi_p(k) & \text{if } k_v \lesssim \kappa \\ 0 & \text{if } k_v \gtrsim \kappa \end{cases} \tag{4.1}$$

representing the exact solution with a cutoff at some value κ near 0.2 *GeV/c* corresponding to an interquark distance near 1 *fm*. The exact solution beyond the cutoff is assumed to be a regular function of relative quark momentum, possible singularities for $k_v < \kappa$ will not be relevant for our discussion.

The first approximation

$$\psi^{(1)}_p(k) = \Delta_F(\tfrac{p}{2}-k)\,[\int d^4\ell\, \psi^{(0)}_p(\ell)\, K(k,\ell;p)]\,\Delta_F(\tfrac{p}{2}+k). \tag{4.2}$$

obtained from (2.3) and (4.1) gives nothing new if $k_v \lesssim \kappa$. We come to the intermediate region, i.e., relative quark momenta of order 1 *GeV/c*, by adding a given relative momentum q to

2k. The substitution $k \to q/2 + k$ and $p \to p + q$ $((p+q)^2 = M^2)$ in eq. (4.2) leads to

$$\psi^{(1)}_{p+q}(k+\frac{q}{2}) = \Delta_F(\frac{p}{2}-k)[\int d^4\ell\,\psi^{(0)}_{p+q}(\ell)\,K(k+\frac{q}{2},\ell,p)]\Delta_F(\frac{p}{2}+q+k) \qquad (4.3)$$

with $k_v, \ell_v \lesssim \kappa$. Improved approximations may be expressed by equations of the type (4.2) or (4.3) with iterated kernels

$$K^{(1)} = K,\ K^{(2)} = \int d^4k'\,K(k,k')\Delta_F K(k',\ell)\Delta_F,\dots \qquad (4.4)$$

where, of course, $K^{(n)}$ leads to the same solution of the exact integral equation as $K^{(1)}$. Also correspondingly normalized linear combinations

$$K^{(n)}_{a_1\dots a_n} = \sum_{i=1}^{n} a_i K^{(i)}(k,\ell),\qquad \sum_{i=1}^{n} a_i = 1 \qquad (4.5)$$

of iterated kernels of different order have the same exact solution. Thus for growing $n \to \infty$ and special coefficients $a_i \to a_i^M$ among these sums (4.4) is the s-channel quark-antiquark reducible four-point Green 's function

$$K^{(n)}_{a_1^M\dots a_n^M} \xrightarrow[(n\to\infty)]{} M_4\,. \qquad (4.6)$$

We note that generally in the iteration procedure the replacement $K^{(1)} \to K^{(n)}_{a_1\dots a_n}$ leads to an approximate solution between $\psi^{(1)}$ and $\psi^{(n)}$. In other words. An iteration with mixed kernels (4.5) converges more slowly towards the exact solution than the optimal iteration (4.4). From (4.3), (4.5) and (4.6) we obtain

$$\psi_{p+q}(\frac{q}{2}+k) = \Delta_F(\frac{p}{2}-k)[\int d^4\ell\,\psi^{(0)}_{p+q}(\ell,\kappa)M_4(\frac{q}{2}+k,\ell,p)]\Delta_F(\frac{p}{2}+k+q)$$

(4.7)

giving the original Bethe-Salpeter equation if $\kappa \to \infty$. If $\kappa \simeq 0.2$ *GeV/c* and $q^2 \simeq 0$ (1 GeV^2), then

$$(\frac{q}{2}+k)^2 \simeq \frac{q^2}{4},\qquad (\frac{q}{2}+k-\ell)^2 \simeq \frac{q^2}{4} \qquad (4.8)$$

and using the result of section 3

$$M_4 = F(\frac{t}{4})\,G'_4(p,q)$$
$$(G_4 \to G'_4 \quad \text{if} \quad (A) \to (B)) \qquad (4.9)$$

with $q^2 = t$ we have from (4.7)

$$\psi_{p+q}(\frac{q}{2}+k) \simeq F_1(\frac{q^2}{4})\Delta_F(\frac{p}{2}-k)[\int d^4\ell\,\psi^{(0)}_{p+q}(\ell)G'_4(q,p,\ell)]\Delta_F(\frac{p}{2}+k+q),$$

(4.10)

where $G'_4(p,q)$ weakly depends on q as compared with F_1. Insertion of (4.10) into the integrand of $T^{(c)}_{MM}$ (see (3.2)) gives the structure

$$T^{(c)}_{MM} \simeq F_1(t)\,F_1^2(t/4)\,G(p,q). \qquad (4.11)$$

Thus the result shows that damping of meson bound states in relative quark momentum occurs in the expected region between strong binding and the beginning of approximate asymptotic freedom at a few GeV^2. Since the function

$$F(t) \simeq F_1(t)F_1^2(t/4) \qquad (4.12)$$

falls off at least by two orders of magnitude if $t \to 2t$, in the considered region, the decrease of $F_1(t/4)$ in eq. (4.10) should be strong enough to guarantee approximate power scaling at $t \gtrsim 2$ GeV^2. The extension of this investigation to baryon wave functions and information from meson-baryon and baryon-baryon diffraction scattering should be straightforward, at least, if some simplifying assumptions are introduced about the three-particle irreducible kernel.

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