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# THE THRESHOLD BEHAVIOUR OF HADRON STRUCTURE FUNCTIONS AND CONNECTION OF CROSS-CHANNELS FOR DEEP INELASTIC PROCESSES IN QUARK MODEL WITH FACTORIZABILITY ASSUMPTION

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Пороговое поведение структурных функций адронов и связь кросс-каналов глубоконеупругих процессов в кварковой модели с гипотезой факторизуемости

Работа посвящена описанию сечений упругих и неупругих лептон-адронных взаимодействий в асимптотической области. Целью работы является определение структурных функций адронов, функций распределения и функций фрагментации валентных кварков в пороговой области, а также адронных формфакторов в области времениподобных переданных импульсов. Для этого используются динамическая модель факторизующихся кварков и соотношение локальной дуальности Блюма-Гилмана. Показано хорошее согласие полученных формул с экспериментальными данными.

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The Threshold Behaviour of Hadron Structure Functions and Connection of Cross-Channels for Deep Inelastic Processes in Quark Model with Factorizability Assumption

Based on the formula for the hadron elastic form factor derived earlier within the dynamical model of factorizing quarks, amplitudes and the relation of local duality, we get the structure functions of hadrons with spin 0, 1/2 and 1 in the threshold region of the scattering and annihilation channels. The formulae for the hadron form factors in the annihilation channel are founded, which are in good agreement with experiment. The threshold behaviour is determined for the fragmentation functions of valence quarks.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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## 1. INTRODUCTION

One of the most actual problems in high-energy physics is the study of inclusive processes of hadron-hadron scattering with the production of large-transverse-momentum particles. In the parton model, the differential cross section of such processes is defined by the formula of "hard collision"

$$E\frac{d\sigma}{dp_c}(AB\to CX)=\sum_{a,b,c}\int_{x_a^{min}}^{1}dx_a\int_{x_b^{min}}^{1}dx_b\, f_a^A(x_a) f_b^B(x_b)\frac{1}{\pi}\frac{d\hat{\sigma}}{dt}\frac{1}{z_c}D_c^C(z_c). \qquad (1.1)$$

When the hadron C with large transverse momentum ($x_\perp=\frac{2p_{c\perp}}{\sqrt{s}}\to 1$) is produced at large angle ($\theta\to\frac{\pi}{2}$), the lower limits in integrals (1.1) (see ref. [2]) and of variable $z_c$, which equals $\frac{x_\perp}{2}(\mathrm{tg}\frac{\theta}{2}+\mathrm{ctg}\frac{\theta}{2})$, are close to unity. As a result, the cross section (1.1) of the production of hadrons with maximum large transverse momenta is defined only by the cross section of a subprocess of quark-quark scattering, $d\hat{\sigma}/d\hat{t}$, and the behaviour of functions $f_q^h(x)$ and $D_q^h(z)$ in the threshold region. The same concerns the cross section of double inclusive process $AB\to CDX$ and cross sections of production of jets in the hadron scattering (see formulas in [3]).

In view of this, the important problem is the definition of the hadron structure functions near threshold. The threshold behaviour of structure functions is usually defined by using the Drell-Yan-West relation [4] which from the known behaviour of the elastic form factor of hadron A as

$$G_A(t)\sim(-t)^N, \quad -t\to\infty \qquad (1.2)$$

determines the behaviour of its structure functions near elastic threshold $(x \to 1)$

$$F_i^A(x) \sim (1-x)^p \quad , \tag{1.3}$$

connecting the powers p and N as follows

$$p = 2N - 1 . \tag{1.4}$$

The same dependence follows as well from the Bloom-Gileman relation of local duality [5].

In ref.[6] it was shown that the asymptotic behaviour of elastic form factors is given by the universal law

$$G_A(t) \sim (-t)^{-n_A + 1} \quad , \tag{1.5}$$

where $n_A$ is the number of valence quarks of hadron A. However, the results of the recent experiment on inelastic ep-scattering [17] raise the question on compatibility of the quark counting rules and Drell-Yan-West relation. Indeed, from formulae (1.4) and (1.5) it follows that $F_1^p(x) \sim (1-x)^3$ whereas the experimental data are fitted by the empirical formula $F_1^p(x_s) = a(1-x_s)^4$ ( $x_s$ is the Atwood variable) [7].

In paper [8] it was suggested that the presently investigated kinematic region can be considered as preasymptotic one. Thus, it is an interesting problem to test the possibility of describing elementary particle interaction with the use of the natural for the quark model scale parameter, - the quark mass $M_q$. To check this presumption, in refs.[8-10,40] a simple dynamical model of factorizing quarks (DMFQ) was proposed (see Sec. 2). The DMFQ well described a number of experimental data on hadron-hadron [8-12,40] and lepton-hadron interactions [12]. In this model, the data on deep inelastic ep-scattering also have been described near threshold $x \geq 0.75$ [12]. Thus, it was shown that the Drell-Yan-West relation is valid, and that in the attained region the correct choice of the "preasymptotic" behaviour of the hadron form factor is important. In the present paper we apply DMFQ to derive analogous expressions for the structure functions of hadrons with spins 0, 1/2, 1.



The next problem we discuss here concerns the study of the connection between the structure functions of electron-hadron scattering $F_i^h(x)$ and those of electron-positron annihilation into hadrons, $e^+e^- \to \bar{h}X$ , $\bar{F}_i^h(z)$. Among the most important results on the crossing problem, we mention the relation

$$\bar{F}_i(\omega) = \pm F_i(\omega) \tag{1.6}$$

found in the framework of the field-theoretical model of Drell, Levi, and Yan [13], which will be shown below to hold in other approaches, as well [14]. A possibility for analytical continuation of $F_i(\omega)$ into the annihilation channel was concluded also in papers [15-17].

Another important result is the Gribov-Lipatov relation [16-19] which has been used in the parton model to deduce the quark-hadron reciprocity relation [16] (for discussion see ref.[20] )

$$f_q^h(x) = D_q^h(z), \quad x = z, \tag{1.7}$$

which connects the distribution function $f_q^h(x)$ of quarks q in hadron h and the fragmentation function $D_q^h(z)$ of quark q in hadrons h .

It should be noted that the problem of the connection of cross-channels is not clear yet. Thus the further study of crossing problem produces the conclusion [21] that in the general case, without extra assumptions, the functions $\bar{F}_i^h(z)$ and $F_i^h(x)$ are not connected by simple analytic continuation except for the threshold region, where the connection between cross-channels takes place irrespective of the possibility of analytic continuation of the structure functions [21,27]. Note, however, that in ref.[23] it was shown, on the basis of the light cone algebra technique that in the threshold region singularities should also be taken into account when making analytic continuation.

In the DMFQ we are dealing with the explicit expression for the invariant quark-scattering amplitude that allows us to easily establish the threshold connection of structure functions of different hadrons.

The paper is organized as follows. In Sec. 2 we present main formulae of the DMFQ which are used in Sec. 3 to determine the threshold behaviour of the structure functions, and in Sec. 4 to derive formulae for the asymptotic behaviour of form factors and structure functions in the annihilation channel. In Sec. 5 the problem of the connection of cross-channels will be treated within the parton model, and in Sec. 6 the result will be applied to determine the threshold behaviour of distribution and fragmentation function of valence quarks.

## 2. *DYNAMICAL MODEL OF FACTORIZING QUARKS (DMFQ)*

Our model (DMFQ)[8-10,40] develops the model of factorizing quarks (MFQ) proposed earlier by Kawaguchi, Sumi and Yokomi[24] for describing the two particle elastic and quasielastic processes. The MFQ assumes that the quarks, constituents of a hadron, produce, during the hadron-hadron collision some (nonspecified in the MFQ) effective field, $V_{eff}$, at which they are scattering independently. Therefore, if $g_q(\theta)$ is the amplitude of scattering of an individual quark $q$ at angle $\theta$ in field $V_{eff}$, then the scattering amplitude $M_{AB\to AB}(\theta)$ of hadrons A and B at angle $\theta$ is defined by the formula

$$M_{AB\to AB}(\theta) = \prod_{q_A}^{n_A} g_q(\theta)\cdot\prod_{q_B}^{n_B} g_q(\theta), \tag{2.1}$$

which is immediate result of the known theorem of the probability theory.The probability of several statistically-independent events equals the priduct of probabilities of individual events. In paper[8] the MFQ was enlarged by the dynamical assumption that at the large-angle scattering $(-t, s\to\infty, t/s$ fixed) the quark interaction region has an effective size which is taken, for simplicity, to equal the quark Compton wave length $M_q^{-1}$ ( $M_q$ is the effective mass of an interacting quarks). Using this assumption in[8] it was obtained with the help of the expansions over the Lorentz group the following expression for the amplitude of quark scattering at the field $V_{eff}$ :

$$g_q(\theta) = \frac{\chi_q}{\mathrm{sh}\chi_q} = \frac{2M_q^2 \ln(1-t_q/2M_q^2 + 1/2M_q^2\sqrt{t_q(t_q-4M_q^2)}\,)}{\sqrt{t_q(t_q-4M_q^2)}} . \tag{2.2}$$



Here $\chi_q = \mathrm{Arch}(1-t_q/2M_q^2)$ is the rapidity corresponding to the momentum transferred to an isolated quark, $t_q \approx \frac{(p_1-p_2)^2}{n_A} = \frac{t}{n_A^2}$,

where $n_A$ is the number of quarks in a hadron A (for simplicity we assume that the total momenta transfer divides to equal portions of the transfer momentum per each quark).

The generalized Wu-Yang formula[25]

$$\frac{d\sigma}{dt}(AB\to AB) \approx \frac{1}{s^2}G_A^2(t)\,G_B^2(t) \tag{2.3}$$

(2.1) and (2.2) results is the following asymptotic behaviour of the form factor of hadron A consisting of $n_A$ valence quarks

$$G_A(t) = b_A\mu_A\left(\frac{\chi_q}{\mathrm{sh}\chi_q}\right)^{n_A} \approx \left(\left(\frac{\ln|t|\,n_A^{-2}M_q^{-2}}{|t|\,n_A^{-2}M_q^{-2}}\right)^{n_A}\right)^{n_A} \tag{2.4}$$

which can also be rewritten as the power dependence

$$G_A(t) \approx (|t|\,n_A^{-2}M_q^{-2})^{-n_A^{eff}(t)},$$

$$n_A^{eff}(t) \approx n_A - n_A\frac{\ln(\ln|t|\,n_A^{-2}M_q^{-2})}{\ln|t|\,n_A^{-2}M_q^{-2}} . \tag{2.5}$$

Formulae (2.1)-(2.5) contain the scale parameter quark mass $M_q$ to which one compares the portion of transfer momentum per one quark $\sqrt{-t_q}$ and well describe the experimental data. In particular, formula (2.4) is in good agreement with the data on the proton elastic form factor[10,12]. The analysis of recent data[26] shows that formula (2.4) well describes also the pion elastic form factor (*Fig. 1*) (data were taken in the interval $1\leq Q^2 \leq 7$ *GeV*$^2$), $\chi_{d.f.} \approx 1.33$, $M_q \approx (0.180\pm0.014)$ *GeV*, $b\ \mu$ .

It should be noted that the obtained values of quark mass coincide within errors with the values of this parameter founded earlier for the elastic form factor and structure function $F_1$ of the proton near threshold[12]. The values of the quark mass which were founded by comparing to experiment formulae for the proton structure functions $F_2$ and pion form factor in the time-like region (Sec. 4) are also in agreement with each other and those obtained earlier in[10,12]. The results of[39] also

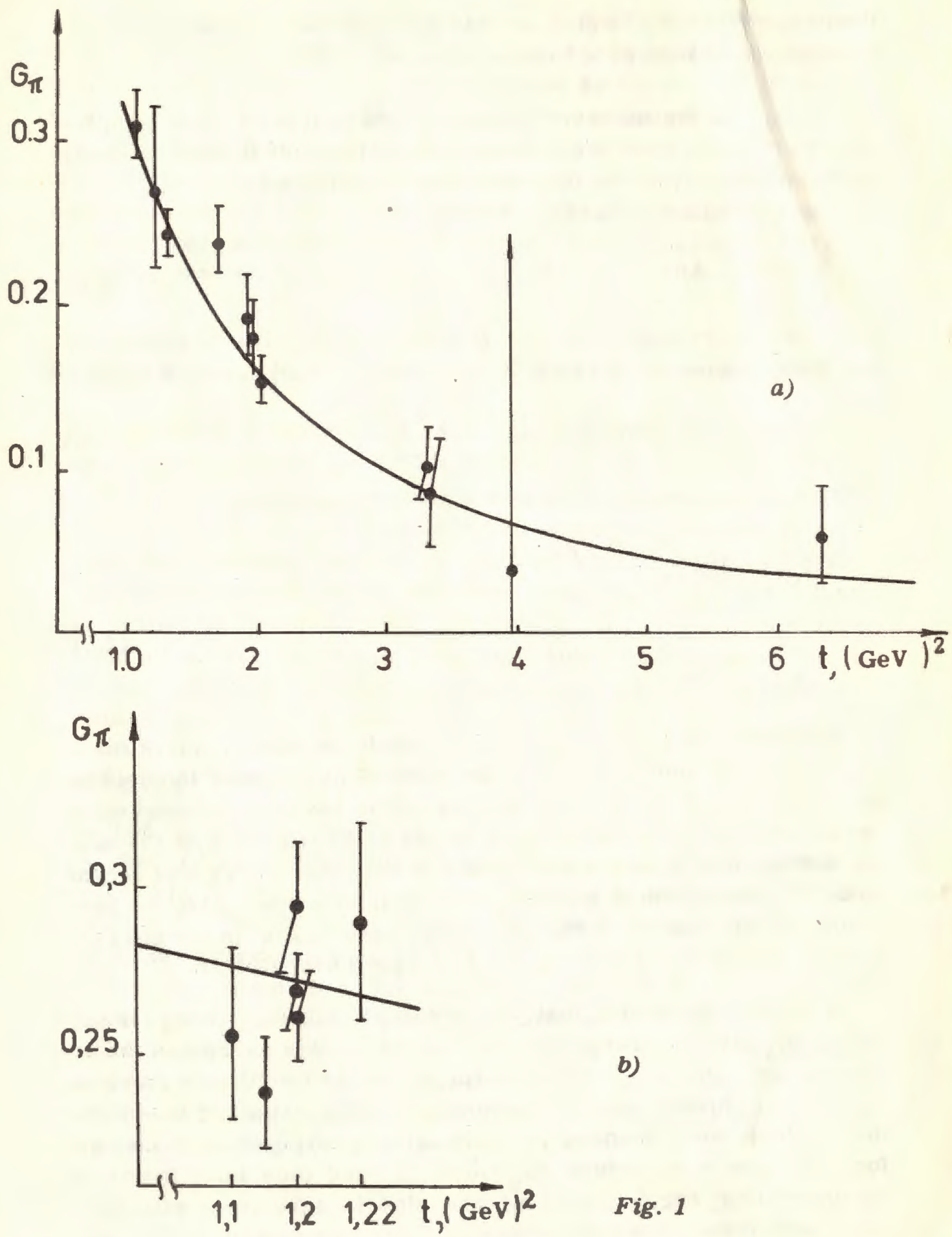


*Fig. 1*



indicate the deviation of pion and proton form factors from (1.5) law.

### 3. *THRESHOLD BEHAVIOUR OF STRUCTURE FUNCTIONS OF COMPOSITE HADRONIC SYSTEMS*

To obtain the structure functions in the threshold region, it is convenient to use the Bloom-Gilman relation of local duality in the differential form [5]

$$F_i^A(\omega_s^A) = \frac{t}{1-\omega_s^A} \frac{d}{dt} [G_i^A(t)]^2, \tag{3.1}$$

where $i=1,2$; $A=\pi, K, p, d, \ldots$, $[G_i^A]^2$ are squared combinations of elastic form factors of hadron $A$, or up to the $\delta$-function, structure functions of elastic scattering (see ref. [27,28,29]). Note that (3.1) contains the scaling variable $\omega_s^A$ (Bloom-Gilman variable) in terms of which the early scaling is observed experimentally

$$\omega_s^A = \omega - \frac{M_s^2}{t} \underset{\omega\to 1}{\to} 1 - \frac{W_A^2}{t}, \tag{3.2}$$

where

$$W_A^2 = (W_{in}^A)^2 + M_s^2 - M_A^2,$$
$$(W_{in}^A)^2 = (M_A + m_\pi)^2 \tag{3.3}$$

is the inelastic threshold of $eA$-scattering, $M_s^2$ is a fitting parameter introduced for the proton by Atwood [7] (see Appendix A).

Inserting into (3.1) the expression for the elastic form factor (2.4) we get the following formula for the structure function of hadron $A$ composed of $n_A$ quarks near threshold

$$F_2^{A(s=0)}(\omega_s^A) = C\cdot K_A (\chi_s^A \operatorname{ch}\chi_s^A - \operatorname{sh}\chi_s^A) \frac{(\operatorname{ch}\chi_s^A - 1)^2}{\operatorname{sh}^3\chi_s^A} \left(\frac{\chi_A^s}{\operatorname{sh}\chi_A^s}\right)^{2n_A-1},$$

$$K_A = 4 b_A^2 \mu_A^2 M_q^2 n_A^3 W_A^{-2}, \tag{3.4}$$

where the variable is found from $\chi_q = \mathrm{Arch}\left(1-\frac{t_q}{2M_q^2}\right)$ according to (3.2) by the change $t_q \simeq \frac{t}{n_A^2} = -\frac{W_A^2}{n_A^2(1-\omega_s^A)}$. In formula (3.3) the coefficient $C$ is introduced in order to take into account the fact that the Drell-Yan-West relation follows from (3.1) up to a proportionality coefficient of $W_{1,2}$, so that $C=1$ if the local duality holds exactly.

In the asymptotic region $\omega_s^A \simeq 1$ formula (3.3) can be represented in the form of the power law

$$F_2^A(\omega_s^A) \approx (\omega_s^A - 1)^{N_A^{eff}(\omega_s^A)} \tag{3.5}$$

with the

$$N_A^{eff}(\omega_s^A) = (2n_A - 1) - 2n_A \beta_A(\omega_s^A), \tag{3.6}$$

$$\beta_A(\omega_s^A) = \frac{\ln|\ln n_A^2 M_q^2 W_A^{-2}(\omega_s^A - 1)|}{|\ln n_A^2 M_q^2 W_A^2(\omega_s^A - 1)|}. \tag{3.7}$$

For the nucleon with formula (2.3) for the elastic magnetic form factor we get from (3.1)

$$F_1^N(\omega_s^N) = F_2^{A(s=0)}(\omega_s^A), \quad n_A = 3, \quad M_A = M_N. \tag{3.8}$$

For the proton structure function in virtue of the scaling relation between elastic electric and magnetic proton form factors [28,30] $G_M^p(t) = \mu_p \cdot G_E^p(t)$ ($\mu_p$ is the proton total magnetic moment) we find

$$F_2^p(\omega_s^p) = \frac{\mu_p^{-2} - r_p}{1 - r_p} F_1^p(\omega_s^p) + \frac{\lambda_p \cdot (\mu_p^{-2} - 1)}{4M^2(1 - r_p)^2}[G^p(\omega_s^p)]^2, \tag{3.9}$$

where changing $t$ by $\omega_s^A$ according to (3.2) and (3.4), we have

$$r_A = \frac{W_A^2}{4M^2(1-\omega_s^A)}, \quad \lambda_A = \frac{C \cdot W_A^2}{(1-\omega_s^A)^2},$$ (3.10a)



$$G^A(\omega_s^A) \approx [n_A^2 M_q^2(\omega_s^A - 1) W_A^{-2} \cdot \ln(n_A^2 M_q^2 W_A^{-2}(\omega_s^A - 1))]^{n_A}$$

$$b_\pi \mu_\pi \approx 1.43. \tag{3.10b}$$

at $n_A = 3$, $M_A = M_N$. For ultrahigh momenta transfer

$$F_2^p(\omega_s^p) \sim F_1^p(\omega_s^p) \sim (1-\omega_s^p)^{5-6\cdot\beta_p(\omega_s^p)}. \tag{3.11}$$

For the neutron $G_E^n(t) \simeq 0$ [28], and

$$F_2^n(\omega_s^n) = \frac{r_n}{r_n - 1} F_1^n(\omega_s^n) - \frac{\lambda_n}{4M^2(r_n - 1)^2}[G^n(\omega_s^n)]^2. \tag{3.12}$$

For $-t \gg 4M^2$

$$F_2^n(\omega_s^n) \sim F_1^n(\omega_s^n) \sim F_2^p(\omega_s^p). \tag{3.13}$$

As was mentioned, formula (3.9) well describes the data on the proton structure function $F_1$ in the threshold region $x \geq 0.75$: in paper [12] the following value was found for $\chi^2$ per one degree of freedom: $\chi_{d.f.} \approx 1.04$ at $M_q = 0.164$. The analysis of data [38] for $x \gtrsim 0.75$ on the proton structure function $F_2$ shows that formula (3.8) also agrees with experiment (*Fig. 2*): $\chi^2_{d.f.} \simeq 6.43/9-3$ at $M_q = (0.140 \pm 0.01)$ *GeV* ($C = 3.85 \pm 0.74$, $M_s^2 = (1.58 \pm 0.15)$ *GeV*)$^2$.

As we see, these values of the quark mass are in agreement with each other and with those obtained from the comparison of other formulae with experiment (see Secs. 2,4).

For hadron $B$ with spin 1 we assume that the elastic electric form factor is defined by expression (2.4): $G_E^B(t) = b_B(\chi_q/\mathrm{sh}\,\chi_q)^{n_B}$. We suppose also that for $-t \to \infty$ there take place the following natural relations between magnetic, quadrupole, and electric form factors

$$G_M^B(t) \approx s_M \cdot G_E^B(t), \quad G_Q^B(t) \approx \frac{s_Q}{t/4M^2} G_E^B(t), \tag{3.14}$$

where $s_M$ and $s_Q$ are some constants. Then from (3.1) we obtain the following formulae

$$F_1^B(\omega_s^B) = \frac{4}{9} s_M F_2^{A(s=0)}(\omega_s^A), \quad n_A = n_B, \quad M_A = M_B. \tag{3.15}$$

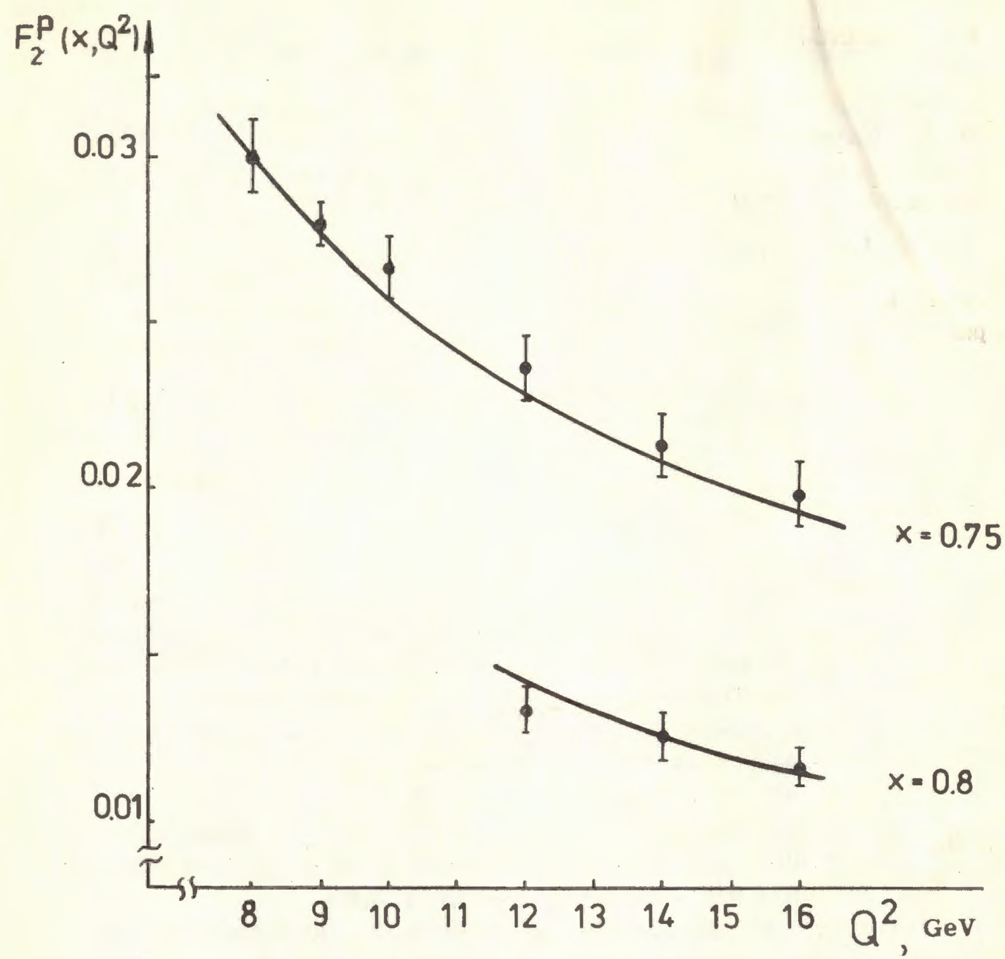


*Fig. 2*

$$F_2^B(\omega_s^B) = \frac{1-s_B \cdot r_B}{1-r_B} F_1^B(\omega_s^B) + \frac{\lambda_B(1-s_B)}{4M^2(1-r_B)}[G^B(\omega_s^B)]^2, \qquad (3.16)$$

where $s_B = \frac{2}{3}s_M - \frac{8}{9}s_Q$.



## 4. CROSSING-TRANSFORMATION IN DMFQ

Consider now the problem of the connection of cross-channels in the framework of the DMFQ. While passing from the scattering to annihilation channel the 4-momentum transferred to the quark

$$t_q = (p_1 - p_2)^2 = 2M_q^2(1-\mathrm{ch}\,\chi_q)$$

changes to the squared invariant energy of the quark-antiquark pair

$$s_q = (p_1 + p_2)^2 = 2M_q^2(1+\mathrm{ch}\,\chi_q) = 2M_q^2(1-\mathrm{ch}\,\bar{\chi}_q).$$

To this transformation in terms of rapidity there corresponds the transformation

$$\chi_q \to \bar{\chi}_q = \chi_q + i\pi \qquad (4.1)$$

and the amplitude $g_q(\theta)$ (2.2) undergoes the following transformation

$$g_q(\theta) \to \bar{g}_q(\theta) = \frac{\bar{\chi}_q}{\mathrm{sh}\,\bar{\chi}_q} = g_q(\theta) + \frac{i\pi}{\mathrm{sh}\,\chi_q}. \qquad (4.2)$$

Making the change $t \to s$, $\chi_q \to \bar{\chi}_q$ according to (4.1) in formula (2.3), we obtain the formulae for the asymptotic behaviour of the annihilation form factor of hadron A:

$$|\bar{G}_A(s)| = |C_A(\frac{\bar{\chi}_q}{\mathrm{sh}\,\bar{\chi}_q})^{n_A}|, \qquad (4.3)$$

$$|\bar{G}_A(s)| \approx (\frac{\ln s n_A^{-2} M_q^{-2}}{s n_A^{-2} M_q^{-2}})^{n_A}. \qquad (4.4)$$

The analysis of experimental data /37/ on the pion form factor in the time-like region shows a good agreement of (4.3) with experiment (*Fig. 3*). Namely formula (4.3) describes the data with $\chi^2_{d.f.} \approx 1.27$ , $M_q = (0.165 \pm 0.004)$ *GeV*, $C_\pi \approx 2.48$. As we see, within error this value of the quark mass coincides with those obtained from the analysis of data on the proton and pion form

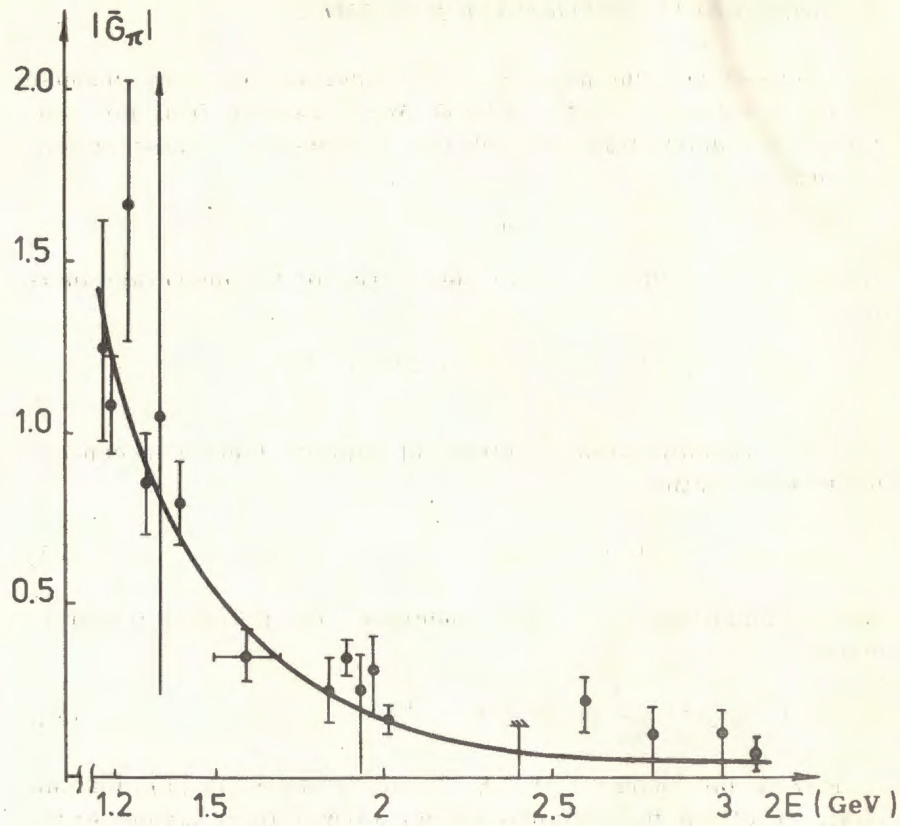


*Fig. 3*

factors in the space-time region and the proton structure functions near threshold.

The form of the structure functions near the threshold of hadron-pair $A\bar{A}$ production ($\bar{\omega}_s^A = 1 - \frac{W_A^2}{s} \to 1$) can be find by applying (4.1)

$$\bar{F}_2^{A(s=0)}(\bar{\omega}_s^A) = C \cdot K_A (\bar{\chi}_s^A \mathrm{ch}\, \bar{\chi}_s^A - \mathrm{sh}\, \bar{\chi}_s^A) \frac{(\mathrm{ch}\, \bar{\chi}_s^A - 1)^2}{\mathrm{sh}^3 \bar{\chi}_s^A} \left(\frac{\bar{\chi}_s^A}{\mathrm{sh}\, \bar{\chi}_s^A}\right)^{2n_A - 1}, \tag{4.5}$$



$$\bar{F}_1^N(\bar{\omega}_s^N) = \bar{F}_2^{A(s=0)}(\bar{\omega}_s^A), \quad n_A = 3, \quad M_A = M_N, \tag{4.6}$$

$$\bar{\chi}_s^A = \mathrm{Arch}\left(1 + \frac{W_A^2}{2 n_A^2 M_q^2 (\bar{\omega}_s^A - 1)}\right), \tag{4.7}$$

and so on. Analogously, from (3.10) and (3.12), for high energies we get

$$\bar{F}_1^p(\bar{\omega}_s^p) \sim \bar{F}_2^p(\bar{\omega}_s^p) \sim \bar{F}_1^n(\bar{\omega}_s^n) \sim \bar{F}_2^n(\bar{\omega}_s^n) \sim (1 - \bar{\omega}_s^N)^{5-6 \cdot \beta_N(\bar{\omega}_s^N)}, \tag{4.8}$$

$$\bar{F}_2^\pi(\bar{\omega}_s^\pi) \sim (1 - \bar{\omega}_s^\pi)^{3-4\beta_\pi(\bar{\omega}_s^\pi)} \tag{4.9}$$

## 5. *CROSSING TRANSFORMATION IN THE PARTON MODEL*

The quark-parton model assumes that in multiparticle and deep inelastic processes the final hadrons are products of the quark fragmentation. This process is described by the fragmentation functions $D_q^h(z)$ which can be theoretically derived by using the quark-hadron reciprocity relation (1.7) (see ref. [20]). However, the latter is based on the Gribov-Lipatov crossing relation, founded within the framework of a field-theoretical approach. But the recent data are inconsistent with this relation [32]. On the other hand, as it will be shown, information on the fragmentation functions $D_q^h(z)$ can be gained on the basis of the connection of cross-channels which can easily be established within the parton model.

So, let $M_q^h(P,k)$ be an amplitude of the probability for real hadron $h$ with the 4-momentum $P = (\mathcal{P}, 0, 0, \mathcal{P})$ to be found in a state with the parton configuration containing the quark $q$ carrying the 4-momentum $k = xP$ (see *Fig. 4*). In parton model the probability of the existence of this configuration (see [33])

$$W \approx \frac{x}{1-x} |M_q^h(P, k = xP)|^2 \tag{5.1}$$

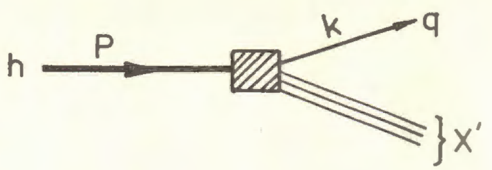


*Fig. 4*

is described by the function parton momentum distribution $f_q^h(x)$. So that we may write

$$f_q^h(x) \sim \frac{x}{1-x} \, | M_q^h(x) |^2 , \tag{5.2}$$

where according to the parton model the amplitude $M_q^h$ is assumed to be a function of the ratio of momenta $k$ to $P$ only $M_q^h(P, k = xP) = M_q^h(x)$. The transition process $q \to h + X'$ is obtained from $h \to q + X'$ via the CPT and crossing $(P \to -P, k \to -k)$ transformations, and its probability is defined as

$$\bar{W} \approx \frac{z}{1-z} \, | \bar{M}_q^h(\bar{k}, \bar{P} = z\bar{k}) |^2 . \tag{5.3}$$

The quark-hadron amplitude $\bar{M}_q^h$ is an amplitude of the probability of transition of quark $q$ with 4-momentum $\bar{k} = (\bar{K}, 0, 0, \bar{K})$ into hadron $h$ transferring the 4-momentum $\bar{P} = z\bar{k}$ with undetected final state hadrons $X'$.

In the parton model the probability $\bar{W}$ is given by the fragmentation function $D_q^h(z)$ of quarks $q$ into hadrons $h$. Thus, we have

$$D_q^h(z) \sim \frac{z}{1-z} \, | \bar{M}_q^h(z) |^2 ,$$

$$\bar{M}_q^h(\bar{k}, \bar{P} = z\bar{k}) = \bar{M}_q^h(z) . \tag{5.4}$$

Assuming the quark-hadron amplitude $M_q^h$ to be an analytic function of variable $x$, we obtain, from (5.2) and (5.4), the following relation

$$D_q^h(z) \sim \frac{1}{x} f_q^h(x), \quad z = \frac{1}{x} . \tag{5.5}$$



Eq. (5.5) coincides up to arbitrary constant with the relation (A.18) found with the help of Bethe-Salpeter equation approach at [34] and as it is easy to see, differs from (1.7).

## 6. *THRESHOLD BEHAVIOUR OF THE DISTRIBUTION AND FRAGMENTATION FUNCTIONS OF VALENCE QUARKS*

Here we will apply the obtained results to determine the threshold behaviour of the functions $f_q^h$ and $D_q^h$. For this aim, based on the charge and isotopic symmetry [24] we represent the hadron structure functions by sums of distribution function [27] and take into account that near the elastic threshold only the contribution of valence quarks survives [2]. As a result, we get *

$$f_u^{\pi^+} = f_{\bar{d}}^{\pi^+} = f_{\bar{u}}^{\pi^-} = f_d^{\pi^-} \sim (1-\omega_s^\pi)^{3-4\beta_\pi(\omega_s^\pi)} , \tag{6.1}$$

$$f_u^{K^+} = f_d^{K^0} = f_{\bar{u}}^{K^-} = f_{\bar{d}}^{\bar{K}^0} = f_s^{K^-} = f_s^{\bar{K}^0} = f_{\bar{s}}^{K^+} = f_{\bar{s}}^{K^0} \sim (1-\omega_s^K)^{3-4\beta_K(\omega_s^K)} \tag{6.2}$$

$$q = \frac{4}{9} u + \frac{1}{9} d = \frac{4}{9} f_u^p + \frac{1}{9} f_d^p = \frac{4}{9} f_d^n + \frac{1}{9} f_u^n \sim (1-\omega_s^N)^{5-6\beta_N(\omega_s^N)} \tag{6.3}$$

Further by using relation (5.5) we find from the (6.1)-(6.3) the following law of fragmentation functions behaviour near threshold

$$D_u^{\pi^+} = D_{\bar{d}}^{\pi^+} = D_{\bar{u}}^{\pi^-} = D_d^{\pi^-} \sim (1-z)^{3-4\beta_\pi(z)} , \tag{6.4}$$

$$D_u^{K^+} = D_d^{K^0} = D_{\bar{u}}^{K^-} = D_{\bar{d}}^{\bar{K}^0} = D_s^{K^-} = D_s^{\bar{K}^0} = D_{\bar{s}}^{K^+} = D_{\bar{s}}^{K^0} \sim (1-z)^{3-4\beta_k(z)} \tag{6.5}$$

$$\frac{4}{9} D_u^p + \frac{1}{9} D_d^p + = \frac{4}{9} D_d^n + \frac{1}{9} D_u^n \sim (1-z)^{5-6\beta_N(z)} \tag{6.6}$$

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* Here we use the Bloom-Gilman scaling variable, a possibility of its introducing into the parton model being discussed in Appendix.

For other distribution and fragmentation functions the threshold behaviour can be defined in a similar way.

## CONCLUSION

The dynamical model of factorizing quarks proposed earlier in paper [8] for the analysis of elastic and quasielastic processes in the preasymptotic region provides a good description for the recent experimental data on processes $pp\to pp$ [8,40], $pp\to\pi^{0}X$ [11], $ep\to ep$ [10,12], $e\pi\to e\pi$ (see [10] and Sec. 2 of the recent work) and $ep\to eX$ for $x\geq 0.75$ [12]. An important feature of the model is the presence of the explicit scale parameter, 1 quark mass to which the portion of the momentum transfer per one quark is compared.

In this paper, the DMFQ formula for the elastic hadron form factor has been used to determine the threshold behaviour of structure functions for various hadrons. To this end, we used the Bloom-Gilman relation of local duality the validity of which and, hence, of the Drell-Yan-West relation was shown earlier in paper [12] for the proton structure function $F_1$. The DMFQ contains the explicit expression for the quark scattering invariant amplitude. It enables one to easily establish the continuation of quark amplitude to the cross-channels. Thus we derive formulae for hadron form factors and structure functions behaviour in the threshold region of the annihilation channel. The formulae for form factors well describe the experimental data in the time- and space-like regions with the same value (within error) of the effective quark mass, explicit scale parameter of the model.

The relation (5.5) allows us to determine in addition to distribution functions also the fragmentation functions for valence quarks.

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## APPENDIX A

### *On Bloom-Gilman Scaling Variable in the Parton Model*

The recent experimental results indicate that the parton transverse momenta $k_\perp$ may be rather large in magnitude. Therefore, their contribution, like that of parton masses, $\mu$, should be taken into account while formulating the parton model. This step was undertaken in paper [35] (see also ref. [36]), and as a result, the nucleon structure function $W_1$, was found in the form

$$2MW_1(x,Q^2)=\sum_q Q_q^2\int\frac{d^3k}{k_0}\Phi_q^N(x,z,Q^2)\,\delta(x-x_L)\,L(x,\beta,Q^2)\times$$

$$\times[1+\frac{1}{2}\,\mathrm{tg}^2\beta],$$

$$L(x,\beta,Q^2)=[\frac{1+4x^2\mu^2/Q^2}{1+4\mu^2/Q^2+\mathrm{tg}^2\beta}]^{1/2}, \qquad \text{(A.1)}$$

where $\Phi_q^N(x,z,Q^2)$ is the function of distribution of quarks in a nucleon, and

$$z=1-2\frac{Pk}{Pq},\quad \cos\beta=\frac{1}{z},\quad x=\frac{kq}{Pq}.$$

Below, it will be shown that from (A.1) follows the possibility of the parton model description in terms of the Bloom-Gilman variable.

Following paper [35], we suppose that $\Phi_q^N(x,z,Q^2)=4x^2/Q^2F_q^N(x,z)$. Then, from (A.1) we get

$$F_1^N(x_L,Q^2)=\sum_q Q_q^2\int d\cos\beta\,\frac{x}{\sqrt{1+4\mu^2\cos^2\beta/Q^2}}\,\frac{1+\frac{1}{2}\mathrm{tg}^2\beta}{\cos^2\beta}F_q^N(x_L,z)\times$$

$$\times L(x_L,\beta,Q^2). \qquad \text{(A.2)}$$

Next, assuming the simple factorization $F_q^N(x_L,z)=f_q^N(x_L)\cdot\phi_q^N(z)$ we obtain

$$F_1^N(x_L,Q^2)=\sum_q Q_q^2 f_q^L(x_L)\cdot\eta_{1q}^N(x_L,Q^2), \tag{A.3}$$

where $\eta_{1q}^N$ is a function of $x_L$ and $Q^2$ slowly varying in asymptotics and normalized to unity. Analogous expression follows for $F_2^N(x_L,Q^2)$.

The variable $\omega_L=\frac{1}{x_L}=\frac{Pq}{kq}$ can be expressed in terms of $\omega$: $\omega_L=\omega\,\frac{Q^2}{2kq}$. In virtue of the parton-model identification in the system $\mathcal{P}\to\infty$ of variable $x$ with the parton fraction of the initial-nucleon momentum, $P=(\mathcal{P},0,0,\mathcal{P})$, we have

$$qk\approx xPq+\frac{k_\perp^2+\mu^2}{2k_{||}}q_0-k_\perp q_\perp\ . \tag{A.4}$$

Formulae (A.5) yield

$$\omega_L\approx\omega-\frac{2\omega}{t}\left(\frac{k_\perp^2+\mu^2}{2k_{||}}q_0-k_\perp q_\perp\right). \tag{A.5}$$

The quantity

$$M_s^2\equiv 2\omega\left(\frac{k_\perp^2+\mu^2}{2k_{||}}q_0-k_\perp q_\perp\right) \tag{A.6}$$

containing unknown parameters $k_\perp^2$ and $\mu^2$ can be treated as a phenomenological parameter itself. Inserting (A.7) into (A.6) we arrive at the description in terms of the Bloom-Gilman scaling variable.

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