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Динамическая реализация группы гомотопии в теории Янга-Миллса

В теории Янга-Миллса топологический индекс трактуется как динамическая переменная, описывающая кооперативное возбуждение калибровочных полей. Включение топологической степени свободы ведет к сингулярному классическому бозе-конденсату калибровочных полей /магнитным мешкам/.

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Dynamical Realization of Homotopy Group in the Yang-Mills Theory

The topological index is treated in the Yang-Mills theory as a dynamical variable describing the cooperative excitation of gauge fields. The inclusion of the topological excitation leads to singular background fields (magnetic bags).

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The important problem in the Yang-Mills theory [1] is the research of the non-Abelian gauge-field behaviour at large distances. The usual perturbation theory points only to the nonlimited growth of the effective coupling constant with increasing distance (infrared catastrophe).

As is known from the theory of critical phenomena and phase transitions, in the system with the strong coupling the long-range correlations of local excitations may appear. Such correlations are usually described by the cooperative degrees of freedom and, generally speaking, may lead to the singular Bose-condensate.

In the present paper the scheme of quantization of the Yang-Mills theory is developed which assumes from the very beginning the presence of singular gauge fields. We shall assume also that the cooperative excitation of the gauge fields appears as a Goldstone mode accompanying the topological vacuum degeneration.

II. METHOD OF QUANTIZATION [2,3]

Let us consider the Yang-Mills Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}\,;\ (F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu\ ). \qquad (1)$$

The fields A^a_0 in $\mathcal{L}$ have no canonical momenta $\pi^a_0 = (\delta\mathcal{L}/\delta\partial_0 A^a_0) \equiv 0$, therefore we express A^a_0 in terms of other fields through the Euler classical equations

$$\frac{\delta\mathcal{L}}{\delta A^a_0} = (\nabla^2_i A_0)^a - (\nabla_i\, \partial_0 A_i)^a = 0, \qquad (2)$$

where

$$\nabla_i^{ab} \equiv \nabla_i^{ab}(A_i) = \delta^{ab}\partial_i + g\epsilon^{abc}A_i^c .$$

The general solution of eq. (2)

$$A_0^a = \dot{c}\,\Phi^a + \left(\frac{1}{\nabla^2}\nabla_i\,\partial_0 A_i\right)^a \qquad (3)$$

may contain solutions of the homogeneous equation

$$(\nabla^2\Phi)^a = 0 \qquad (4)$$

($\dot{c} = \frac{dc(t)}{dt}$ is the zero mode of the operator ∇^2). Inserting eq. (3) into eq. (1), we obtain the Lagrangian

$$L = \int d^3x\,\mathcal{L} = L_{Y.-M.} + L_{Rot}$$

$$L_{Y.-M.} = \int d^3x\,\frac{1}{2}\left[(E_i^{(T)a})^2 - (B_i^a)^2\right] \qquad (5)$$

$$L_{Rot} = \int d^3x\left[\frac{1}{2}\dot{c}^2(\nabla_i\Phi)^2 - \dot{c}(\nabla_i\Phi)^a E_i^{(T)a}\right], \qquad (6)$$

where

$$E_i^{(T)a} = \left(\delta_{ij} - \frac{\nabla_i\nabla_j}{\nabla^2}\right)^{ab}\partial_0 A_j^b ;$$

$$B_i^a = \epsilon_{ijk}\left(\partial_j A_k^a + \frac{g}{2}\epsilon^{abc}A_j^b A_k^c\right). \qquad (7)$$

The fields $E^{(T)}$, B satisfy the equations

$$\nabla_i^{ab}E_i^{(T)b} = 0 ; \qquad \nabla_i^{ab}B_i^b = 0. \qquad (8)$$

There is the following alternative:

1) Suppose that the non-Abelian fields are regular. Then, owing to egs. (4),(8) and the Gauss theorem $L_{Rot} = 0$. The obtained theory is described by Lagrangian (5) with auxiliary conditions (8). The quantization scheme is equivalent to the usual one in gauge $A_0 = 0$ [4].

2) Suppose that the fields are singular.* Then $L_{Rot} \neq 0$, and new dynamical variable $\dot{c}(t)$ is introduced (as the zero mode of the operator ∇^2) which describes the cooperative excitation of the fields in the whole space.

III. REASONING OF INTRODUCTION OF THE NEW VARIABLE

The topologically nontrivial classical solutions (instantons) [6] testify to the presence of vacuum manifield. The invariance group of each of these vacua, G_0, is a subgroup of the invariance group of the Lagrangian, G .

It is known that the factor group G/G_0 is the homotopy group $\pi_3(SU_2) = Z$ (where Z is an infinite cyclic group). Here we have an example of spontaneous breaking of the vacuum symmetry. According to the ideology of the symmetry spontaneous breaking [7] one should take into account the Goldstone mode, which has the meaning of an element of the factor-space: (Lagrangian invariance group/vacuum invariance group). In our case we must consider the Goldstone mode as an element of the homotopy group $\pi_3(SU_2) = Z$. The Pontryagin index, which characterizes the nonsingular classical solutions with the finite Euclidean action:

$$\nu[A] = \frac{g^2}{64\pi^2}\int d^4x\,F_{\mu\nu}^a F_{\rho\sigma}^a\,\epsilon_{\mu\nu\rho\sigma} \qquad (9)$$

$$\nu[A_{regular}] \subseteq Z \qquad (10)$$

has just this mathematical meaning.

Generelizing eq. (9) to the singular fields (3),(7), we treat ν as the dynamical variable describing the cooperative excitation of fields in the whole space.

IV. THE BOSE-CONDENSATE EQUATIONS

Let us identify the Goldstone mode $\dot{c}$ in eq. (6) with the topological variable $\dot{\nu} = \frac{d\nu}{dt}$, inserting the general solution for A_0^a (3) into the definition of the Pontryagin index (9):

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* Note that in quantum field theory and, in particular, in electrodynamics we deal with the regular fields [5].

$$\nu = \int dt\,\dot{\nu} \;\; ; \;\; \dot{\nu} = c\,\frac{g^2}{8\pi^2}\int d^3x\,(\nabla\Phi\cdot B) - \dot{N}(A). \tag{11}$$

Here $\dot{N}(A)$ is the functional of the gauge fields. Let us express Lagrangian (6), L_{Rot}, in terms of $\tilde{\nu} = (\nu + N)$ by eq. (11):

$$L_{Rot} = \frac{(\dot{\tilde{\nu}})^2}{2}\left(\frac{8\pi^2}{g^2}\right)^2\frac{1}{\langle B\rangle^2} \mp \dot{\tilde{\nu}}\,\frac{\langle E\rangle}{\langle B\rangle}\,\frac{8\pi^2}{g^2}, \tag{12}$$

where

$$\langle D\rangle^2 = [\int d^3x\, D_i^a (\nabla_i\Phi)^a]^2 / \int d^3x\,(\nabla\Phi)^2; \quad D_i^a = E_i^a, B_i^a . \tag{13}$$

The Hamiltonian corresponding to (12) does not depend on $\dot{N}$:

$$H_{Rot}(p) = L_{Rot}(p) = \frac{1}{2}\left[p^2\left(\frac{g^2}{8\pi^2}\right)^2\langle B\rangle^2 - \langle E\rangle^2\right]. \tag{14}$$

Here

$p = \delta L_{Rot}(\dot{\nu})/\delta\dot{\nu}$.

We quantize the variable ν, $[\nu, p] = i$, requiring the state-vector covariance under transformations of the cyclic group Z

$\Psi(\nu+1) = e^{i\theta}\,\Psi(\nu)$

and obtain the momentum spectrum of the "rotator" ν :

$p = 2\pi k + \theta$,

where k is the number of the energetic zone and θ is the quasi-momentum. Finally, we get the following effective action

$$S_{eff} = S_{Y.-M.} + S_{Rot} = \int dt\, L_{eff} = \int dt\,(L_{Y.-M.} + L_{Rot})$$

$$L_{eff} = \frac{1}{2}[\int d^3x\,(E_i^a)^2 - \langle E\rangle^2] \overset{(+)}{-} \frac{1}{2}[\int d^3x\,(B_i^a)^2 - \rho\langle B\rangle^2], \tag{16}$$

$$\rho = (2\pi k + \theta)^2\left(\frac{g^2}{8\pi^2}\right)^2. \tag{17}$$

Sign (+) in eq. (16) relates to the Euclidean space. Assume that the gauge field A is a sum of the singular background field $\underset{\sim}{b}$ (Bose-condensate) and regular field $\underset{\sim}{a}$, having zero boundary conditions at the singularities

$$S_{Rot}(\underset{\sim}{b} + \underset{\sim}{a}) = S_{Rot}(\underset{\sim}{b}). \tag{18}$$

Then we have the following formal expansion in power of $\underset{\sim}{a}$:

$$S_{eff}(\underset{\sim}{a}+\underset{\sim}{b}) = S_{eff}(\underset{\sim}{b}) + S'_{Y.-M.}(\underset{\sim}{b})\underset{\sim}{a} + \frac{1}{2}S''_{Y.-M.}(\underset{\sim}{b})\underset{\sim}{a}^2 + \ldots$$

The procedure of quantization and, in particular, the variational derivative are well defined only for the regular field $\underset{\sim}{a}$ /5/. If one does not use the calculus of variations for the definition of $\underset{\sim}{b}$ the conditions of "finiteness", $S_{eff}(\underset{\sim}{b}) = 0$, and "stability", $S'_{Y.-M.}(\underset{\sim}{b})\cdot\underset{\sim}{a} = 0$, may be satisfied provided that

$$\rho = 1 , \tag{19}$$

$$\int d^3x\, D^2 = \langle D\rangle^2 \;\; ; \quad D = E, B. \tag{20}$$

Then, from the Cauchy-Bunyakovsky inequality we obtain that background fields $E(\underset{\sim}{b}), B(\underset{\sim}{b}), \nabla(\underset{\sim}{b})\Phi$ satisfy the stationary dual equations:

$$\pm E_i^a(\underset{\sim}{b}) = \nabla_i^{ab}(\underset{\sim}{b})\Phi^b \; ; \quad B_i^a(\underset{\sim}{b}) = \nabla_i^{ab}(\underset{\sim}{b})\Phi^b . \tag{21}$$

It is worth emphasizing that the charge quatization (17),(19) is necessary for the stability of the theory, as the zero-action fields (21) satisfy the classical equations beyond the singularities.

Consider the exactly solvable example of the cylindrically symmetric functions. There is the unique singular solution describing "magnetic bags" in the class of such functions $\underset{\sim}{b}_\mu^a = (\Phi^a, \underset{\sim}{b}_i^a)$

$$\Phi^a = \frac{m}{g}n^a\left[ctg(mr) - \frac{1}{mr}\right],$$

$$\underset{\sim}{b}_i^a = \frac{m}{g}\epsilon_{ia\ell}\,n^\ell\left[cosec(mr) - \frac{1}{mr}\right], \tag{22}$$

where $n^a = \frac{x'^a}{r}$; $r = \sqrt{x_i'^2}$; $x_i' = x_{oi} + x_i$; m, x_{oi} are the solution parameters. The operator $S''(\underset{\sim}{b})$ does not depend on the coupling constant g . The perturbation theory with "magnetic bags" differs qualitatively from the usual one /4/ and, to a certain extent, coincides with it only in the limit $m \to 0$ which corresponds to small distances. The equation for the quantum fields $\underset{\sim}{a}_\mu^a$ in the considered functional class in the limit $g \to 0$ reduces to the Schrödinger equation for the particle in the periodical Pöschl-Teller potential /9/ with the discrete spectrum. To find the spectrum it is sufficient to consider only one cell of the periodical potential. The penetrations are forbidden. The raial excitations are in the "trap" with dimension $\sim m^{-1}$.

V. CONCLUSION

We have expressed here the point of view according to which the true quantization of the Yang-Mills theory must take into account the non-Abelian field singularities. We also suppose that the source of such singularities is the cooperative excitation describing the long-range topological correlation of the local excitation. (In this sence the topologically nontrivial gauge theory resembles the models of statistical physics with the infinitely large correlation length).

We have shown that the presence of the singular Bose-condensate does not contradict the finiteness and stability of the theory. Just these conditions (finiteness and stability) define the Bose-condensate equations. Solutions of these equations in the class of the cylindrically symmetric functions are "magnetic bags" - the periodical singular potential. In this case the Yang-Mills theory is equivalent to the ideal isolator. The local excitations have the discrete spectrum. There are absent even zero modes which could restore the scale and translation invariances. The absence of the translation invariance for the colour excitation does not mean the absence for the colourless bound states identified with the observable hadrons. If it would be possible to prove "the freedom of motion" for the colourless states, then the colour confinement could be treated as a consequence of the dynamical realization of the homotopy group in the non-Abelian gauge theory.

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